Y2P8 XMQs and MS

(Total: 27 marks)

1. P1_Sample Q5 . 3 marks - Y2P8 Parametric equations
2. P1_Specimen Q14. 5 marks - Y2P8 Parametric equations
3. P1_2018 Q14. 10 marks - Y2P8 Parametric equations
4. P2_2019 Q4 . 6 marks - Y2P8 Parametric equations
5. P1_2021 Q13. 3 marks - Y2P8 Parametric equations

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5. A curve C has parametric equations

$$x = 2t - 1, \qquad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \qquad x \neq -1$$

where a and b are integers to be found.

(3)

(Total for Question 5 is 3 marks)

Question	Scheme	Marks	AOs
5	Attempts to substitute $= \dfrac{x+1}{2}$ into $y \Rightarrow y = 4\left(\dfrac{x+1}{2}\right) - 7 + \dfrac{6}{(x+1)}$	M1	2.1
	Attempts to write as a single fraction $y = \dfrac{(2x-5)(x+1)+6}{(x+1)}$	M1	2.1
	$y = \dfrac{2x^2-3x+1}{x+1}$ $a=-3, b=1$	A1	1.1b
	(3 marks)		

Notes:

M1: Score for an attempt at substituting $t = \dfrac{x+1}{2}$ or equivalent into $y = 4t - 7 + \dfrac{3}{t}$

M1: Award this for an attempt at a single fraction with a correct common denominator.

Their $4\left(\dfrac{x+1}{2}\right) - 7$ term may be simplified first

A1: Correct answer only $y = \dfrac{2x^2-3x+1}{x+1}$ $a=-3, b=1$

14.

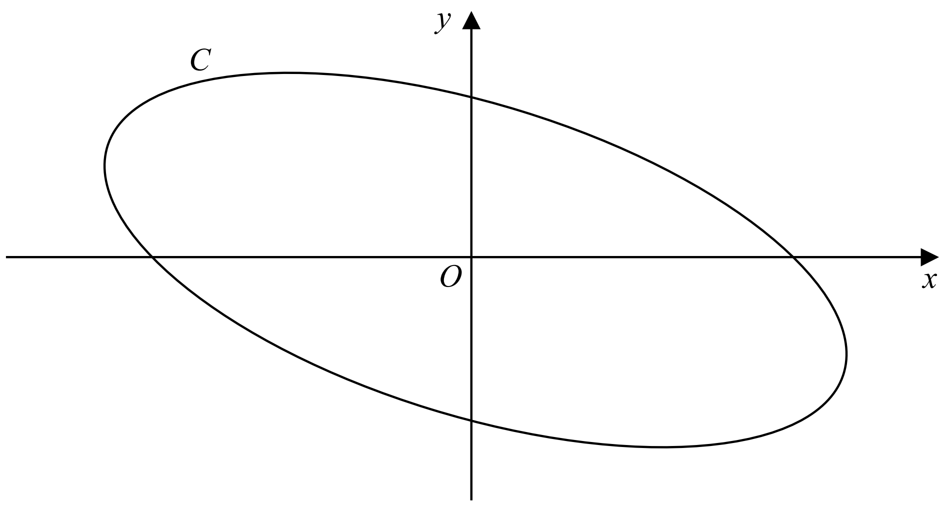

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \qquad 0 < t \leqslant 2\pi$$

Show that a Cartesian equation of C can be written in the form

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be found.

(5)

Question	Scheme	Marks	AOs
14	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
	$x + y = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	M1 M1	3.1a 1.1b
	$x + y = 2\sqrt{3}\cos t$	A1	1.1b
	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	M1	3.1a
	$\frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$(x+y)^2 + 3y^2 = 12$	A1	2.1
		(5)	
14 Alt 1	$(x+y)^2 = \left(4\cos\left(t + \frac{\pi}{6}\right) + 2\sin t\right)^2$		
	$= \left(4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t\right)^2$	M1 M1	3.1a 1.1b
	$= \left(2\sqrt{3}\cos t\right)^2$ or $12\cos^2 t$	A1	1.1b
	So, $(x+y)^2 = 12(1 - \sin^2 t) = 12 - 12\sin^2 t = 12 - 12\left(\frac{y}{2}\right)^2$	M1	3.1a
	$(x+y)^2 + 3y^2 = 12$	A1	2.1
		(5)	
		(5 marks)	

Question 14 Notes:

M1:	Looks ahead to the final result and uses the compound angle formula in a full attempt to write down an expression for $x + y$ which is in terms of *t* only.
M1:	Applies the compound angle formula on their term in *x*. E.g. $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$
A1:	Uses correct algebra to find $x + y = 2\sqrt{3}\cos t$
M1:	Complete strategy of applying $\cos^2 t + \sin^2 t = 1$ on a rearranged $x + y = \text{"}2\sqrt{3}\cos t\text{"}, \; y = 2\sin t$ to achieve an equation in *x* and *y* only
A1:	Correctly proves $(x+y)^2 + ay^2 = b$ with both $a = 3, b = 12$, and no errors seen

Question 14 Notes Continued:	
Alt 1	
M1:	Apply in the same way as in the main scheme
M1:	Apply in the same way as in the main scheme
A1:	Uses correct algebra to find $(x+y)^2 = \left(2\sqrt{3}\cos t\right)^2$ or $(x+y)^2 = 12\cos^2 t$
M1:	Complete strategy of applying $\cos^2 t + \sin^2 t = 1$ on $(x+y)^2 = \left("2\sqrt{3}\cos t"\right)^2$ to achieve an equation in *x* and *y* only
A1:	Correctly proves $(x+y)^2 + ay^2 = b$ with both $a = 3, b = 12$, and no errors seen

14. A curve C has parametric equations

$$x = 3 + 2\sin t, \quad y = 4 + 2\cos 2t, \quad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2, \quad x \in \mathbb{R}$

(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

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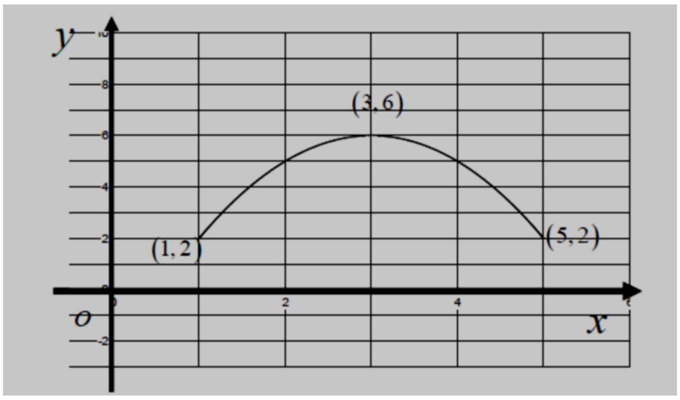
Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \dfrac{y-4}{2} = 1 - 2\left(\dfrac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \dfrac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)		M1	1.1b
	$\cap$ shaped parabola		
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \dfrac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2 - 7x + (k+3) = 0$ Or y $\quad y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow \quad 49 - 4 \times 1 \times (k+3) = 0 \Rightarrow k = \left(\dfrac{37}{4}\right)$ or $(7-2k)^2 - 4 \times 1 \times (k^2 - 6k + 3) = 0 \Rightarrow k = \left(\dfrac{37}{4}\right)$	M1	2.1
	Range of values for k = $\left\{k : 7 \leqslant k < \dfrac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)

M1: Uses $\cos 2t = 1 - 2\sin^2 t$ in an attempt to eliminate t

A1*: Proceeds to $y=6-(x-3)^2$ without any errors

Allow a proof where they start with $y=6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1,2)$ and $(5,2)$

B1: Any suitable explanation as to why *C* does not include all points of $y=6-(x-3)^2, \quad x\in\mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on *x* or *y***.

For example

'As $-1\le \sin t\le 1$ then $1\leqslant x\leqslant 5$' Condone in words '*x* lies between 1 and 5' and strict inequalities

'As $\sin t\le 1$ then $x\leqslant 5$' Condone in words '*x* is less than 5'

'As $-1\le \cos(2t)\le 1$ then $2\leqslant y\leqslant 6$ ' Condone in words '*y* lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2\leqslant x\leqslant 5$"

Do not allow a statement on the top limit of *y* as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k=7$ This can be found by substituting into $(5,2)$

  $x+y=k \Rightarrow k=7$ or substituting $x=5$ into $x^2-7x+(k+3)=0 \Rightarrow 25-35+k+3=0$
  $\Rightarrow k=7$
- or deduces that $k<\dfrac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for *k*.

Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method.

Eg. Sets $k-x=6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2-7x+(k+3)=0$ The = 0 may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2=4ac$ oe or $b^2 ... 4ac$ where ... is any inequality leading to a critical value for *k*. Eg. one root $\Rightarrow 49-4\times1\times(k+3)=0 \Rightarrow k=\dfrac{37}{4}$

A1: Range of values for *k* = $\left\{k: 7\leqslant k<\dfrac{37}{4}\right\}$ Accept $k\in\left[7,\dfrac{37}{4}\right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y=6-(x-3)^2$ equal to -1	M1	3.1a
	$-2x+6=-1 \Rightarrow x=3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of *k*. $y=6-(3.5-3)^2=5.75$ Hence using $k=3.5+5.75=9.25$	M1	2.1
	Range of values for *k* = $\{k: 7\leqslant k<9.25\}$	A1	2.5

4.

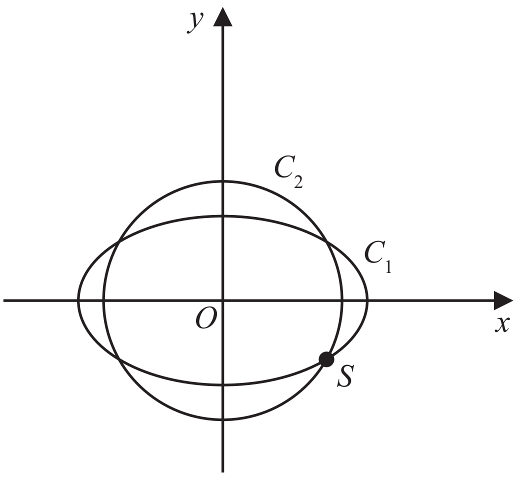


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

Question	Scheme		Marks	AOs
4	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\quad 0 \le t < 2\pi;\quad C_2: x^2 + y^2 = 66$			
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$	$100\cos^2 t + 32(1-\cos^2 t) = 66$	M1	2.1
			A1	1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = ...$	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the *x*-coordinate and value of the corresponding *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1\ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$		M1	3.1a
	$\frac{x^2}{100} + \frac{66 - x^2}{32} = 1$	$\frac{66 - y^2}{100} + \frac{y^2}{32} = 1$	M1	2.1
			A1	1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $x^2 = 50 \Rightarrow x = ...$	$2112 - 32y^2 + 100y^2 = 3200$ $y^2 = 16 \Rightarrow y = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding *x*-coordinate or *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 3	$\{C_2 : x^2 + y^2 = 66 \Rightarrow\}\ x = \sqrt{66}\cos\alpha,\ y = \sqrt{66}\sin\alpha$ $\{C_1 = C_2 \Rightarrow\}\ 10\cos t = \sqrt{66}\cos\alpha,\ 4\sqrt{2}\sin t = \sqrt{66}\sin\alpha$ $\{\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow\}\ \left(\frac{10\cos t}{\sqrt{66}}\right)^2 + \left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2 = 1$		M1	3.1a
	then continue with applying the mark scheme for Way 1			
Way 4	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100\left(\frac{1+\cos 2t}{2}\right) + 32\left(\frac{1-\cos 2t}{2}\right) = 66$		M1	2.1
			A1	1.1b
	$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \Rightarrow 34\cos 2t + 66 = 66$ $\Rightarrow \cos 2t = ...$		dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the *x*-coordinate and value of the *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
	Note: Give final A0 for writing $x = 5\sqrt{2},\ y = -4$ followed by $S = (-4, 5\sqrt{2})$			
				(6 marks)

Notes for Question 4

	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $\sin t = ...$ where $-1 \le \sin t \le 1$ or $\cos t = ...$ where $-1 \le \cos t \le 1$ Condone 3rd M1 for $\sin^2 t = \frac{1}{2} \Rightarrow \sin t = \frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$
	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t + \sin^2 t \equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of x only or y only not involving trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $x = ...$ or $y = ...$ their x^2 or their y^2 must be >0 for this mark
M1: **Note:**	See scheme their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$
	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2 \alpha + \sin^2 \alpha \equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1
	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1: **Note:**	Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t = ...$ where $-1 \le \cos 2t \le 1$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question	Scheme	Marks	AOs

4	$C_1:\ x=10\cos t\ ,\ \ y=4\sqrt{2}\sin t,\ \ \ 0\le t<2\pi;\ \ C_2: x^2+y^2=66$		
Way 5	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66$	M1	3.1a
	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66(\sin^2 t+\cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t+32\sin^2 t=66\sin^2 t+66\cos^2 t\ \Rightarrow 34\cos^2 t=34\sin^2 t$ $\Rightarrow\ \tan t=...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the *x*-coordinate and value of the corresponding *y*-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S=(5\sqrt{2},-4)$ or $x=5\sqrt{2},\ y=-4$ or $S=(\text{awrt } 7.07,-4)$	A1	3.2a
		(6)	

	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. *t*) only.
M1:	Uses the identity $\sin^2 t+\cos^2 t\equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t=...$
M1:	See scheme
A1:	Selects the correct coordinates for *S* Allow either $S=(5\sqrt{2},-4)$ or $S=(\text{awrt } 7.07,-4)$ or $S=(\sqrt{50},-4)$ or $S=\left(\frac{10}{\sqrt{2}},-4\right)$

13. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \qquad y = \frac{4t}{t^2 + 1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(3)

Question	Scheme	Marks	AOs
13	$(x-3)^2+y^2=\left(\frac{t^2+5}{t^2+1}-3\right)^2+\left(\frac{4t}{t^2+1}\right)^2$	M1	3.1a
	$=\frac{(2-2t^2)^2+16t^2}{(t^2+1)^2}=\frac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\frac{4(t^4+2t^2+1)}{(t^2+1)^2}=\frac{4(t^2+1)^2}{(t^2+1)^2}=4$*	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.
dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.
A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
Alt	$x=\frac{t^2+5}{t^2+1}\Rightarrow xt^2+x=t^2+5\Rightarrow t^2=\frac{5-x}{x-1}$ $y=\frac{4t}{t^2+1}\Rightarrow y^2=\frac{16t^2}{(t^2+1)^2}=\frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2}$	M1	3.1a
	$y^2=\frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2}=16\left(\frac{5-x}{x-1}\right)\times\left(\frac{(x-1)}{5-x+x-1}\right)^2\Rightarrow y^2=(5-x)(x-1)$	dM1	1.1b
	$y^2=(5-x)(x-1)\Rightarrow y^2=6x-x^2-5$ $\Rightarrow y^2=4-(x-3)^2$ or other intermediate step $\Rightarrow(x-3)^2+y^2=4$*	A1*	2.1
		(3)	
			(3 marks)
	Notes		

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme.
Other methods exist which also lead to an appropriate equation. E.g using $t=\frac{y}{x-1}$
dM1: Uses correct processing to eliminate the fractions and start to simplify
A1*: Fully correct proof showing all key steps