Y2P7 XMQs and MS

(Total: 126 marks)

1. P1_Sample Q9 . 5 marks - Y2P6 Trigonometric functions
2. P2_Sample Q13. 9 marks - Y2P7 Trigonometry and modelling
3. P1_Specimen Q13. 11 marks - Y2P7 Trigonometry and modelling
4. P2_Specimen Q13. 10 marks - Y2P6 Trigonometric functions
5. P1_2018 Q8 . 5 marks - Y2P7 Trigonometry and modelling
6. P2_2018 Q6 . 6 marks - Y1P7 Algebraic methods
7. P2_2018 Q7 . 9 marks - Y2P7 Trigonometry and modelling
8. P2_2018 Q12. 9 marks - Y2P7 Trigonometry and modelling
9. P1_2019 Q6 . 8 marks - Y2P7 Trigonometry and modelling
10. P2_2019 Q12. 7 marks - Y2P7 Trigonometry and modelling
11. P1_2020 Q6 . 7 marks - Y2P7 Trigonometry and modelling
12. P2_2020 Q10. 8 marks - Y2P7 Trigonometry and modelling
13. P1_2021 Q10. 8 marks - Y2P7 Trigonometry and modelling
14. P2_2021 Q15. 11 marks - Y2P7 Trigonometry and modelling
15. P1_2022 Q14. 8 marks - Y2P7 Trigonometry and modelling
16. P2_2022 Q9 . 5 marks - Y2P7 Trigonometry and modelling

9. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

(Total for Question 9 is 5 marks)

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Question	Scheme	Marks	AOs
9(a)	$\tan\theta+\cot\theta\equiv\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv\dfrac{1}{\frac{1}{2}\sin2\theta}$	M1	2.1
	$\equiv2\text{cosec}2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta+\cot\theta=1\Rightarrow\sin2\theta=2$ AND no real solutions as $-1\leqslant\sin2\theta\leqslant1$	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Writes $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$

A1: Achieves a correct intermediate answer of $\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

M1: Uses the double angle formula $\sin2\theta=2\sin\theta\cos\theta$

A1*: Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

$\tan\theta+\cot\theta\equiv\tan\theta+\dfrac{1}{\tan\theta}\equiv\dfrac{\tan^2\theta+1}{\tan\theta}\equiv\dfrac{\sec^2\theta}{\tan\theta}\equiv\dfrac{1}{\cos^2\theta\times\dfrac{\sin\theta}{\cos\theta}}\equiv\dfrac{1}{\cos\theta\times\sin\theta}$ then as the main scheme.

(b)

B1: Scored for sight of $\sin2\theta=2$ and a reason as to why this equation has no real solutions.
Possible reasons could be $-1\leqslant\sin2\theta\leqslant1$........and therefore $\sin2\theta\neq2$
or $\sin2\theta=2\Rightarrow2\theta=\arcsin2$ which has no answers as $-1\leqslant\sin2\theta\leqslant1$

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

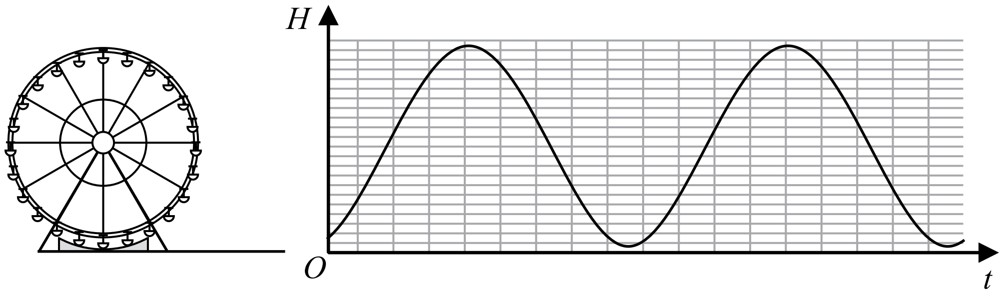

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

Question	Scheme	Marks	AOs
13(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\dfrac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos\left(\theta+16.70°\right)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos\left(80t+16.70\right)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\dfrac{540-"16.70"}{80}=\left(6.54\right)$	M1	1.1b
	t = 6 mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
			(9 marks)

Notes:

(a)

B1: $R=\sqrt{109}$ Do not allow decimal equivalents

M1: Allow for $\tan\alpha=\pm\dfrac{3}{10}$

A1: $\alpha=16.70°$

(b)(i)

B1: see scheme

(b)(ii)

B1ft: their $11+$ their $\sqrt{109}$ Allow decimals here.

(c)

M1: Sets $80t+"16.70"=540$. Follow through on their 16.70

M1: Solves their $80t+"16.70"=540$ correctly to find t

A1: t = 6 mins 32 seconds

(d)

B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

13. (a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

State the value of R and give the value of α to 4 decimal places.

(3)

Tom models the depth of water, D metres, at Southview harbour on 18th October 2017 by the formula

$$D = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \qquad 0 \leqslant t \leqslant 24$$

where t is the time, in hours, after 00:00 hours on 18th October 2017.

Use Tom's model to

(b) find the depth of water at 00:00 hours on 18th October 2017,

(1)

(c) find the maximum depth of water,

(1)

(d) find the time, in the afternoon, when the maximum depth of water occurs. Give your answer to the nearest minute.

(3)

Tom's model is supported by measurements of D taken at regular intervals on 18th October 2017. Jolene attempts to use a similar model in order to model the depth of water at Southview harbour on 19th October 2017.

Jolene models the depth of water, H metres, at Southview harbour on 19th October 2017 by the formula

$$H = 6 + 2\sin\left(\frac{4\pi x}{25}\right) - 1.5\cos\left(\frac{4\pi x}{25}\right), \qquad 0 \leqslant x \leqslant 24$$

where x is the time, in hours, after 00:00 hours on 19th October 2017.

By considering the depth of water at 00:00 hours on 19th October 2017 for both models,

(e) (i) explain why Jolene's model is not correct,

(ii) hence find a suitable model for H in terms of x.

(3)

Question	Scheme	Marks	AOs
13 (a)	$R = 2.5$	B1	1.1b
	$\tan\alpha = \dfrac{1.5}{2}$ o.e.	M1	1.1b
	$\alpha = 0.6435$, so $2.5\sin(\theta - 0.6435)$	A1	1.1b
		(3)	
(b)	e.g. $D = 6 + 2\sin\left(\dfrac{4\pi(0)}{25}\right) - 1.5\cos\left(\dfrac{4\pi(0)}{25}\right) = 4.5\text{m}$ or $D = 6 + 2.5\sin\left(\dfrac{4\pi(0)}{25} - 0.6435\right) = 4.5\text{m}$	B1	3.4
		(1)	
(c)	$D_{max} = 6 + 2.5 = 8.5\text{ m}$	B1ft	3.4
		(1)	
(d)	Sets $\dfrac{4\pi t}{25} - \text{"}0.6435\text{"} = \dfrac{5\pi}{2}$ or $\dfrac{\pi}{2}$	M1	1.1b
	Afternoon solution $\Rightarrow \dfrac{4\pi t}{25} - \text{"}0.6435\text{"} = \dfrac{5\pi}{2} \Rightarrow t = \dfrac{25}{4\pi}\left(\dfrac{5\pi}{2} + \text{"}0.6435\text{"}\right)$	M1	3.1b
	$\Rightarrow t = 16.9052... \Rightarrow \text{Time} = 16:54$ or 4:54 pm	A1	3.2a
		(3)	
(e)(i)	• An attempt to find the depth of water at 00:00 on 19th October 2017 for at least one of either Tom's model or Jolene's model.	M1	3.4
	• At 00:00 on 19th October 2017, Tom: $D = 3.72...\text{ m}$ and Jolene: $H = 4.5\text{ m}$ and e.g. • As $4.5 \neq 3.72$ then Jolene's model is not true • Jolene's model is not continuous at 00:00 on 19th October 2017 • Jolene's model does not continue on from where Tom's model has ended	A1	3.5a
(ii)	To make the model continuous, e.g. • $H = 5.22 + 2\sin\left(\dfrac{4\pi x}{25}\right) - 1.5\cos\left(\dfrac{4\pi x}{25}\right), \quad 0 \leqslant x < 24$ • $H = 6 + 2\sin\left(\dfrac{4\pi(x+24)}{25}\right) - 1.5\cos\left(\dfrac{4\pi(x+24)}{25}\right), \quad 0 \leqslant x < 24$	B1	3.3
		(3)	
		(11 marks)	

Question	Scheme	Marks	AOs
13 (d) Alt 1	Sets $\frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$	M1	1.1b
	$\text{Period} = 2\pi \div \left(\frac{4\pi}{25}\right) = 12.5$ Afternoon solution $\Rightarrow t = 12.5 + \frac{25}{4\pi}\left(\frac{\pi}{2} + "0.6435"\right)$	M1	3.1b
	$\Rightarrow t = 16.9052... \Rightarrow \text{Time} = 16:54$ or 4:54 pm	A1	3.2a
		(3)	

Question 13 Notes:

(a)

B1: $R = 2.5$ Condone $R = \sqrt{6.25}$

M1: For either $\tan\alpha = \frac{1.5}{2}$ or $\tan\alpha = -\frac{1.5}{2}$ or $\tan\alpha = \frac{2}{1.5}$ or $\tan\alpha = -\frac{2}{1.5}$

A1: $\alpha = \text{awrt } 0.6435$

(b)

B1: Uses Tom's model to find $D = 4.5 \text{ (m)}$ at 00:00 on 18th October 2017

(c)

B1ft: Either 8.5 or follow through "6 + their R" (by using their R found in part (a))

(d)

M1: Realises that $D = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right) = 6 + "2.5"\sin\left(\frac{4\pi t}{25} - "0.6435"\right)$ and

so maximum depth occurs when $\sin\left(\frac{4\pi t}{25} - "0.6435"\right) = 1 \Rightarrow \frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$ or $\frac{5\pi}{2}$

M1: Uses the model to deduce that a p.m. solution occurs when $\frac{4\pi t}{25} - "0.6435" = \frac{5\pi}{2}$ and rearranges this equation to make $t = ...$

A1: Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm

(d) Alt 1

M1: Maximum depth occurs when $\sin\left(\frac{4\pi t}{25} - "0.6435"\right) = 1 \Rightarrow \frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$

M1: Rearranges to make $t = ...$ and adds on the period, where $\text{period} = 2\pi \div \left(\frac{4\pi}{25}\right) \{= 12.5\}$

A1: Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm

Question 13 Notes Continued:	
(e)(i)	
M1:	See scheme
A1:	See scheme
	Note: Allow Special Case M1 for a candidate who just states that Jolene's model is not continuous at 00:00 on 19th October 2017 o.e.
(e)(ii)	
B1:	Uses the information to set up a new model for H. (See scheme)

13. (a) Show that

$$\operatorname{cosec} 2x + \cot 2x \equiv \cot x, \qquad x \neq 90n°, n \in \mathbb{Z}$$

(5)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

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Question	Scheme	Marks	AOs
13(a)	$\operatorname{cosec}2x + \cot 2x \equiv \cot x,\ \ x \neq 90n°,\ \ n \in$ ¢		
	$\operatorname{cosec}2x + \cot 2x = \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$	M1	1.2
	$= \dfrac{1 + \cos 2x}{\sin 2x}$	M1	1.1b
	$= \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x} = \dfrac{2\cos^2 x}{2\sin x \cos x}$	M1	2.1
		A1	1.1b
	$= \dfrac{\cos x}{\sin x} = \cot x$ *	A1*	2.1
		(5)	
(b)	$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3};\ \ 0$ „ $\theta < 180°,$		
	$\cot(2\theta \pm ...°) = \sqrt{3}$	M1	2.2a
	$2\theta \pm ... = 30° \Rightarrow \theta = 12.5°$	M1	1.1b
		A1	1.1b
	$2\theta + 5° = 180° + PV° \Rightarrow \theta = ...°$	M1	2.1
	$\theta = 102.5°$	A1	1.1b
		(5)	
			(10 marks)

Question 13 Notes:	
(a)	
M1:	Writes $\text{cosec}2x = \dfrac{1}{\sin 2x}$ and $\cot 2x = \dfrac{\cos 2x}{\sin 2x}$
M1:	Combines into a single fraction with a common denominator
M1:	Applies $\sin 2x = 2\sin x\cos x$ to the denominator **and** applies either • $\cos 2x = 2\cos^2 x - 1$ • $\cos 2x = 1 - 2\sin^2 x$ and $\sin^2 x + \cos^2 x = 1$ • $\cos 2x = \cos^2 x - \sin^2 x$ and $\sin^2 x + \cos^2 x = 1$ to the numerator and manipulates to give a one term numerator expression
A1:	Correct algebra leading to $\dfrac{2\cos^2 x}{2\sin x\cos x}$ or equivalent.
A1*:	Correct proof with correct notation and no errors seen in working
(b)	
M1:	Uses the result in part (a) in an attempt to deduce either $2x = 4\theta + 10$ or $x = 2\theta + ...$ and uses $x = 2\theta + ...$ to write down or imply $\cot(2\theta \pm ...^{\circ}) = \sqrt{3}$
M1:	Applies $\text{arccot}\left(\sqrt{3}\right) = 30°$ or $\arctan\left(\dfrac{1}{\sqrt{3}}\right) = 30°$ and attempts to solve $2\theta \pm ... = 30°$ to give $\theta = ...$
A1:	Uses a correct method to obtain $\theta = 12.5°$
M1:	Uses $2\theta + 5 = 180 +$ their $PV°$ in a complete method to find the second solution, $\theta = ...$
A1:	Uses a correct method to obtain $\theta = 102.5°$, with no extra solutions given either inside or outside the required range $0 \leqslant \theta < 180°$

8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)° \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

Question	Scheme	Marks	AOs
8 (a)	$D = 5 + 2\sin(30 \times 6.5)° = \text{awrt } 4.48\text{m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = awrt\ 4.48\text{m}$. The units must be seen somewhere in (a) . So allow when $D = 4.482.. = 4.5\text{ m}$

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k, \quad |k| \le 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for *t* such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of *t* for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t = \text{awrt } 10.8$

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t = \text{awrt } 13.2$

So award this mark if you see $30t = \text{invsin } their -0.6$ to give the first value of *t* where 30 *t* > 255

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate $f(2)$

(ii) Write $f(x)$ as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

Question	Scheme	Marks	AOs
6	(a) $\mathrm{f}(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$		
(a)	(i) $\{\mathrm{f}(2) = -24 + 32 - 18 + 10 \Rightarrow\}\ \mathrm{f}(2) = 0$	B1	1.1b
	(ii) $\{\mathrm{f}(x) =\}\ (x-2)(-3x^2 + 2x - 5)$ or $(2-x)(3x^2 - 2x + 5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by • explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ • or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0;\ 7\pi \le \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
			(6 marks)

Notes for Question 6

(a)(i)	
B1:	$\mathrm{f}(2) = 0$ or 0 stated by itself in part (a)(i)
(a)(ii)	
M1:	Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by • using long division to obtain either $\pm 3x^2 \pm kx + ...,\ k = \text{value} \ne 0$ or $\pm 3x^2 \pm \alpha x + \beta,\ \beta = \text{value} \ne 0,\ \alpha$ can be 0 • factorising to obtain their quadratic factor in the form $(\pm 3x^2 \pm kx \pm c),\ k = \text{value} \ne 0$, c can be 0, or in the form $(\pm 3x^2 \pm \alpha x \pm \beta),\ \beta = \text{value} \ne 0,\ \alpha$ can be 0
A1:	$(x-2)(-3x^2 + 2x - 5),\ (2-x)(3x^2 - 2x + 5)$ or $-(x-2)(3x^2 - 2x + 5)$ stated together as a product
(b)	
M1:	See scheme
A1:	See scheme. Proof must be correct ***with no errors***, e.g. giving an incorrect discriminant value
Note:	Correct calculation e.g. $(2)^2 - 4(-3)(-5),\ 4 - 60$ or -56 must be given for the first explanation
Note:	Note that M1 can be allowed for • a correct follow through calculation for the discriminant of their "$-3y^4 + 2y^2 - 5$" which would lead to a value < 0 together with an explanation that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions • **or** for the omission of < 0
Note:	< 0 must also been stated in a discriminant method for A1
Note:	Do not allow A1 for incorrect working, e.g. $(2)^2 - 4(-3)(-5) = -54 < 0$
Note:	$y^2 = 2 \Rightarrow y = \pm 2$, so 2 solutions is not allowed for A1, but can be condoned for M1
Note:	Using the formula on $-3y^4 + 2y^2 - 5 = 0$ or $-3x^2 + 2x - 5 = 0$ gives y^2 or $x = \dfrac{-2 \pm \sqrt{-56}}{-6}$ or $\dfrac{-1 \pm \sqrt{-14}}{-3}$

Notes for Question 6 Continued	
Note:	Completing the square on $-3x^2+2x-5=0$ gives $x^2-\frac{2}{3}x+\frac{5}{3}=0 \Rightarrow \left(x-\frac{1}{3}\right)^2-\frac{1}{9}+\frac{5}{3}=0 \Rightarrow x=\frac{1}{3}\pm\sqrt{\frac{-14}{9}}$
Note:	Do not recover work for part (b) in part (c)
(c)	
B1:	See scheme
Note:	Give B0 for stating $\theta=$ awrt 23.1, awrt 26.2, awrt 29.4 **without** reference to 3 solutions

7. (i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question	Scheme	Marks	AOs
7	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1-\sin^2 x) = 1$ | $16(1-\cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}$ $\left\{= \frac{2\pm\sqrt{3}}{4} \text{ or } 0.933..., 0.066...\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta-\alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$	M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$ | $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$ | $\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ | e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$	A1	2.1
	Note: Working in radians does not affect any of the first 4 marks		
		(5)	
			(9 marks)

Question	Scheme		Marks	AOs
7	(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$			
(ii) Alt 1	Complete strategy, i.e. • Attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta + \sin^2\theta = 1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$		M1	3.1a
	e.g. $25\sin^2\theta = 4 + 20\cos\theta + 25\cos^2\theta$ $\Rightarrow 25(1-\cos^2\theta) = 4 + 20\cos\theta + 25\cos^2\theta$ **or** e.g. $25\sin^2\theta - 20\sin\theta + 4 = 25\cos^2\theta$ $\Rightarrow 25\sin^2\theta - 20\sin\theta + 4 = 25(1-\sin^2\theta)$		M1	1.1b
	$50\cos^2\theta + 20\cos\theta - 21 = 0$	$50\sin^2\theta - 20\sin\theta - 21 = 0$		
	$\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e.	$\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e.	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arccos\left(\frac{-2+\sqrt{46}}{10}\right)$	e.g. $\theta = \arcsin\left(\frac{2+\sqrt{46}}{10}\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
			(5)	

Notes for Question 7

(i)	
B1:	For recalling that $\sec x = \frac{1}{\cos x}$
M1:	Correct strategy of • **Way 1:** applying $\sin 2x = 2\sin x\cos x$ and proceeding to $\sin 2x = k,\ \lvert k\rvert \le 1,\ k \ne 0$ • **Way 2:** squaring both sides, applying $\cos^2 x + \sin^2 x = 1$ and solving a quadratic equation in either $\sin^2 x$ or $\cos^2 x$ to give $\sin^2 x = k$ or $\cos^2 x = k,\ \lvert k\rvert \le 1,\ k \ne 0$
dM1:	Uses the correct order of operations to find at least one value for x in either radians or degrees
A1:	Clear reasoning to achieve both $x = \frac{\pi}{12}, \frac{5\pi}{12}$ and no other values in the range $0 \le x < \frac{\pi}{2}$
Note:	Give dM1 for $\sin 2x = \frac{1}{2} \Rightarrow$ any of $\frac{\pi}{12}, \frac{5\pi}{12}$, $15°$, $75°$, awrt 0.26 or awrt 1.3
Note:	Give special case, SC B1M0M0A0 for writing down any of $\frac{\pi}{12}, \frac{5\pi}{12}$, $15°$ or $75°$ with no working

Notes for Question 7 Continued	
(ii)	
M1:	See scheme
Note:	**Alternative strategy:** Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\cos(\theta+\alpha) = -2$, finds both R and α, and proceeds to $\cos(\theta+\alpha) = k$, $\lvert k\rvert < 1$, $k \neq 0$
M1:	Either • uses $R\sin(\theta-\alpha)$ to find the values of both R and α • attempts to apply $(5\sin\theta - 5\cos\theta)^2 = 2^2$, uses $\cos^2\theta + \sin^2\theta = 1$ and proceeds to find an equation of the form $\pm\lambda \pm \mu\sin 2\theta = \pm\beta$ or $\pm\mu\sin 2\theta = \pm\beta$; $\mu \neq 0$ • attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ and uses $\cos^2\theta + \sin^2\theta = 1$ to form an equation in $\cos\theta$ only or $\sin\theta$ only
A1:	For $\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$, o.e., $\cos(\theta + 45°) = -\frac{2}{\sqrt{50}}$, o.e. or $\sin 2\theta = \frac{21}{25}$, o.e. or $\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e. or $\cos\theta =$ awrt 0.48, awrt -0.88 or $\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e., or $\sin\theta =$ awrt 0.88, awrt -0.48
Note:	$\sin(\theta - 45°), \cos(\theta + 45°), \sin 2\theta$ must be made the subject for A1
dM1:	**dependent on the first M mark** Uses the correct order of operations to find at least one value for x in either degrees or radians
Note:	dM1 can also be given for $\theta = 180° - \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ or $\theta = \frac{1}{2}\left(180° - \arcsin\left(\frac{21}{25}\right)\right)$
A1:	Clear reasoning to achieve both $\theta =$ awrt 61.4°, awrt 208.6° and no other values in the range $0 \le \theta < 360°$
Note:	Give M0M0A0M0A0 for writing down any of $\theta =$ awrt 61.4°, awrt 208.6° with no working
Note:	<u>**Alternative solutions:**</u> (to be marked in the same way as Alt 1): • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5\tan\theta - 5 = 2\sec\theta \Rightarrow (5\tan\theta - 5)^2 = (2\sec\theta)^2$ $\Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4\sec^2\theta \Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4(1+\tan^2\theta)$ $\Rightarrow 21\tan^2\theta - 50\tan\theta + 21 = 0 \Rightarrow \tan\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5 - 5\cot\theta = 2\operatorname{cosec}\theta \Rightarrow (5 - 5\cot\theta)^2 = (2\operatorname{cosec}\theta)^2$ $\Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4\operatorname{cosec}^2\theta \Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4(1+\cot^2\theta)$ $\Rightarrow 21\cot^2\theta - 50\cot\theta + 21 = 0 \Rightarrow \cot\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364..., 0.5445...$ $\Rightarrow \theta =$ awrt 61.4°, awrt 208.6° only

12. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

P 5 8 3 4 9 A 0 3 4 4 4

Question	Scheme	Marks	AOs
12	$1-\cos 2\theta \equiv \tan\theta \sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n\in\mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2}<x<\frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x=0$ **or** $(1+\tan^2 x-3\tan x-5)(1-\cos 2x)=0$ **or** $1+\tan^2 x-5=3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4=0$	A1	1.1b
	$(\tan x-4)(\tan x+1)=0 \Rightarrow \tan x = ...$	M1	1.1b
	$x = -\frac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question 12

(a)	**Way 1**
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS = $2\sin^2\theta$ and RHS = $2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Notes for Question 12 Continued

(b)	
B1:	Deduces that the given equation yields a solution $x=0$
M1:	For using the key step of $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$
Note:	Allow the use of $\pm\sec^2 x = \pm 1 \pm \tan^2 x$ for M1
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$
Note:	E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1
M1:	For a correct method of solving their 3TQ in $\tan x$
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785, awrt 1.326, $-45°$, awrt $75.964°$
A1:	Only $x = -\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Note: **Alternative Method (Alt 1)**

$(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ **or** $(\sec^2 x - 5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$			
Deduces $x=0$		B1	2.2a
$\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$ $1 - 5\cos^2 x = 3\sin x \cos x$ $1 - 5\left(\frac{1+\cos 2x}{2}\right) = \frac{3}{2}\sin 2x$	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B\cos 2x = \pm C\sin 2x$	M1	2.1
$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ $\{3\sin 2x + 5\cos 2x = -3\}$	$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ o.e.	A1	1.1b
$\sqrt{34}\sin(2x+1.03) = -3$	Expresses their answer in the form $R\sin(2x+\alpha) = k;\ k \neq 0$ with values for R and α	M1	1.1b
$\sin(2x+1.03) = -\frac{3}{\sqrt{34}}$			
$x = -\frac{\pi}{4}, 1.326$		A1 A1	1.1b 1.1b

6. (a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50°) = 9\tan(x - 25°)$$

(2)

Question	Scheme	Marks	AOs
6 (a)	$5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9\times\dfrac{\sin\theta}{\cos\theta}$ $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe	A1	1.1b
	Correct order of operations For example $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\dfrac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$	A1	1.1b
	All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$	A1	1.1b
	$\theta = 0°, \pm 180°$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25° = -18.4°$	M1	1.1b
	$x = 6.6°$	A1ft	2.2a
		(2)	
			(8 marks)

(a)

M1: Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = ...\sin\theta\cos\theta$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

A1: Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to form a correct simplified equation in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is for a correct equation in $\sin\theta$ or $\cos\theta$

dM1: Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to find at least one value for θ (Eg. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \dfrac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

A1: Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

A1: All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

B1: $\theta = 0°, \pm 180°$ This can be scored independent of method.

(b)

M1: Attempts to solve $x - 25° = "\theta"$ where θ is a solution of their part (a)

A1ft: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their θ must be chosen

12. (a) Prove

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)°,\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $90° < \theta < 180°$, the equation

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4$$

giving any solutions to one decimal place.

(3)

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Question	Scheme	Marks	AOs
12	$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) Way 1	$\{\text{LHS} = \}\ \frac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \frac{\cos(3\theta - \theta)}{\sin\theta\cos\theta}\ \left\{= \frac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS} = \}\ \frac{\cos 2\theta\cos\theta - \sin 2\theta\sin\theta}{\sin\theta} + \frac{\sin 2\theta\cos\theta + \cos 2\theta\sin\theta}{\cos\theta}$		
	$= \frac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \frac{\cos 2\theta(\cos^2\theta + \sin^2\theta)}{\sin\theta\cos\theta}\ \left\{= \frac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS} = \}\ \frac{2\cos 2\theta}{\sin 2\theta} = \frac{2\cos(3\theta - \theta)}{\sin 2\theta} = \frac{2(\cos 3\theta\cos\theta + \sin 3\theta\sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$= \frac{2(\cos 3\theta\cos\theta + \sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$= \frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	
(b) Way 1	$\left\{\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow 2\left(\frac{1}{\tan 2\theta}\right) = 4$	M1	1.1b
	Rearranges to give $\tan 2\theta = k;\ k \neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90° < \theta < 180°,\ \tan 2\theta = \frac{1}{2} \Rightarrow\right\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
(b) Way 2	$\left\{\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow \frac{2}{\tan 2\theta} = 4$	M1	1.1b
	$\frac{2}{\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right)} = 4 \Rightarrow 2(1 - \tan^2\theta) = 8\tan\theta$ $\Rightarrow \tan^2\theta + 4\tan\theta - 1 = 0 \Rightarrow \tan\theta = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$ $\{\Rightarrow \tan\theta = -2 \pm \sqrt{5}\} \Rightarrow \tan\theta = k;\ k \neq 0 \Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90° < \theta < 180°,\ \tan\theta = -2 - \sqrt{5} \Rightarrow\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
			(7 marks)

Notes for Question 12	
(a)	**Way 1 and Way 2**
M1:	Correct valid method forming a common denominator of $\sin\theta\cos\theta$ i.e. correct process of $\dfrac{(...)\cos\theta+(...)\sin\theta}{\cos\theta\sin\theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta-\theta)$ or $\cos 2\theta$
dM1:	**dependent on the previous M mark** Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$
A1*	Correct proof
Note:	Writing $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta} + \dfrac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$
Note:	Allow 2nd M1 for stating a correct $\sin 2\theta = 2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$
(a)	**Way 3**
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$
dM1:	**dependent on the previous M mark** Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1st M1 1st A1 (together) for any of LHS $\to \dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ or LHS $\to \dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or LHS $\to \cos 2\theta(\cot\theta+\tan\theta)$ or LHS $\to \cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ (i.e. where $\cos 2\theta$ has been factorised out)
Note:	Allow 1st M1 1st A1 for progressing as far as $\text{LHS} = ... = \cot x - \tan x$
Note:	The following is a correct alternative solution $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ $= \dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *
Note:	E.g. going from $\dfrac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ with no intermediate working is 1st A0

Notes for Question 12 Continued	
(b)	**Way 1**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Rearranges to give $\tan 2\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta = \text{awrt } 103.3°$
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
Note:	1st M1 can be implied by seeing $\tan 2\theta = \frac{1}{2}$
Note:	Condone 2nd M1 for applying $\frac{1}{2}\arctan\left(\frac{1}{2}\right)\ \{= 13.28...\}$
(b)	**Way 2**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Applies $\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta = \text{awrt } 103.3°$
Note:	Give M1 dM1 A1 for no working leading to $\theta = \text{awrt } 103.3°$ and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta = \text{awrt } 103.3°$ and other solutions which can be either outside or inside the range $90° < \theta < 180°$

6. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

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Question	Scheme	Marks	AOs
6 (a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha = 2 \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12}+1.107-3=\frac{\pi}{2}\Rightarrow t=$	M1	3.1b
	$t=$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)
Notes:			

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value of α from $\tan\alpha = \pm 2, \tan\alpha = \pm\frac{1}{2}, \sin\alpha = \pm\frac{2}{"R"}$ OR $\cos\alpha = \pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

A1: $\alpha =$ awrt 1.107

(b)

B1ft: Deduces that the maximum temperature is $\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C Remember to isw

Condone a lack of units. Follow through on their value of R so allow $\left(5+"R"\right)$ °C

(c)

M1: An complete strategy to find t from $\frac{\pi t}{12}\pm 1.107-3=\frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t\pm 63.4-171.9=90$

A1: awrt $t=13.2$

A1: The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

....

It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$\frac{d\theta}{dt}=\frac{\pi}{12}\cos\left(\frac{\pi t}{12}-3\right)-\frac{2\pi}{12}\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow \tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ scores M1 A1 A1

$\frac{d\theta}{dt}=\cos\left(\frac{\pi t}{12}-3\right)-2\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow \tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ they can score M1 A0 A1 (SC)

A value of $t=1.23$ implies the minimum value has been found and therefore incorrect method M0.

....

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leqslant x \leqslant 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

Question	Scheme	Marks	AOs
10 (a)	$\cos 3A = \cos(2A + A) = \cos 2A\cos A - \sin 2A\sin A$	M1	3.1a
	$= \left(2\cos^2 A - 1\right)\cos A - \left(2\sin A\cos A\right)\sin A$	dM1	1.1b
	$= \left(2\cos^2 A - 1\right)\cos A - 2\cos A\left(1 - \cos^2 A\right)$	ddM1	2.1
	$= 4\cos^3 A - 3\cos A$ *	A1*	1.1b
		(4)	
(b)	$1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x\left(4\cos^2 x - \cos x - 3\right) = 0$ $\Rightarrow \cos x\left(4\cos x + 3\right)\left(\cos x - 1\right) = 0$ $\Rightarrow \cos x = \ldots$	dM1	3.1a
	Two of $-90°$, 0, $90°$, awrt $139°$	A1	1.1b
	All four of $-90°$, 0, $90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)

Allow a proof in terms of *x* rather than A

M1: Attempts to use the compound angle formula for $\cos(2A + A)$ or $\cos(A + 2A)$
Condone a slip in sign

dM1: Uses correct double angle identities for $\cos 2A$ and $\sin 2A$
$\cos 2A = 2\cos^2 A - 1$ must be used. If either of the other two versions are used expect to see an attempt to replace $\sin^2 A$ by $1 - \cos^2 A$ at a later stage.
Depends on previous mark.

ddM1: Attempts to get all terms in terms of $\cos A$ using correct and appropriate identities.
Depends on both previous marks.

A1*: A completely correct and rigorous proof including correct notation, no mixed variables, missing brackets etc.

Alternative right to left is possible:

$$4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right) = \cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right) = \cos A\left(\cos 2A - 2\sin^2 A\right)$$

$$= \cos A\cos 2A - 2\sin A\cos A\sin A = \cos A\cos 2A - \sin 2A\sin A = \cos(2A + A) = \cos 3A$$

Score M1: For $4\cos^3 A - 3\cos A = \cos A\left(4\cos^2 A - 3\right)$

dM1: For $\cos A\left(2\cos^2 A - 1 + 2\left(1 - \sin^2 A\right) - 2\right)$ (Replaces $4\cos^2 A - 1$ by $2\cos^2 A - 1$ and $2\left(1 - \sin^2 A\right)$)

ddM1: Reaches $\cos A\cos 2A - \sin 2A\sin A$

A1: $\cos(2A + A) = \cos 3A$

(b)

M1: For an attempt to produce an equation just in $\cos x$ using both part (a) and the identity $\sin^2 x = 1 - \cos^2 x$
Allow one slip in sign or coefficient when copying the result from part (a)

dM1: Dependent upon the preceding mark. It is for taking the cubic equation in $\cos x$ and making a valid attempt to solve. This could include factorisation or division of a $\cos x$ term followed by an attempt to solve the 3 term quadratic equation in $\cos x$ to reach at least one non zero value for $\cos x$.
May also be scored for solving the cubic equation in $\cos x$ to reach at least one non zero value for $\cos x$.

A1: Two of $-90°$, 0, $90°$, awrt $139°$ **Depends on the <u>first</u> method mark**.

A1: All four of $-90°$, 0, $90°$, awrt $139°$ with no extra solutions offered within the range.

Note that this is an alternative approach for obtaining the cubic equation in (b):

$$1 - \cos 3x = \sin^2 x \Rightarrow 1 - \cos 3x = \frac{1}{2}\left(1 - \cos 2x\right)$$

$$\Rightarrow 2 - 2\cos 3x = 1 - \cos 2x$$

$$\Rightarrow 1 = 2\cos 3x - \cos 2x$$

$$\Rightarrow 1 = 2\left(4\cos^3 x - 3\cos x\right) - \left(2\cos^2 x - 1\right)$$

$$\Rightarrow 0 = 4\cos^3 x - 3\cos x - \cos^2 x$$

The M1 will be scored on the penultimate line when they use part (a) and use the correct identity for $\cos 2x$

10. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

(4)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

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Question	Scheme	Marks	AOs
10(a)	$\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\cos 2\theta+\sin 2\theta}$ or $\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	M1	2.1
	$\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	A1	1.1b
	$=\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\dfrac{2\sin\theta\left(\sin\theta+\cos\theta\right)}{2\cos\theta\left(\cos\theta+\sin\theta\right)}$	dM1	2.1
	$=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$ *	A1*	1.1b
		(4)	
(b)	$\dfrac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x}=3\sin 2x\Rightarrow\tan 2x=3\sin 2x$ o.e	M1	3.1a
	$\Rightarrow\sin 2x-3\sin 2x\cos 2x=0$ $\Rightarrow\sin 2x\left(1-3\cos 2x\right)=0$ $\Rightarrow\left(\sin 2x=0,\right)\ \cos 2x=\dfrac{1}{3}$	A1	1.1b
	$x=90°$, awrt $35.3°$, awrt $144.7°$	A1 A1	1.1b 2.1
		(4)	
			(8 marks)

Notes

(a)

M1: Attempts to use a correct double angle formulae for both $\sin 2\theta$ and $\cos 2\theta$ (seen once).
The application of the formula for $\cos 2\theta$ must be the one that cancels out the "1"
So look for $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator or $\cos 2\theta = 2\cos^2\theta - 1$ in the denominator
Note that $\cos 2\theta = \cos^2\theta - \sin^2\theta$ may be used as well as using $\cos^2\theta + \sin^2\theta = 1$

A1: $\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$ or $\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$

dM1: Factorises numerator and denominator in order to demonstrate cancelling of $(\sin\theta + \cos\theta)$

A1*: Fully correct proof with no errors.

You must see an intermediate line of $\dfrac{2\sin\theta\cancel{\left(\sin\theta+\cos\theta\right)}}{2\cos\theta\cancel{\left(\cos\theta+\sin\theta\right)}}$ or $\dfrac{\sin\theta}{\cos\theta}$ or even $\dfrac{2\sin\theta}{2\cos\theta}$

Withhold this mark if you see, within the body of the proof,

- notational errors. E.g. $\cos 2\theta = 1 - 2\sin^2$ or $\cos\theta^2$ for $\cos^2\theta$
- mixed variables. E.g. $\cos 2\theta = 2\cos^2 x - 1$

(b)

M1: Makes the connection with part (a) and writes the lhs as $\tan 2x$. Condone $x \leftrightarrow \theta$ $\tan 2\theta = 3\sin 2\theta$

A1: Obtains $\cos 2x = \dfrac{1}{3}$ o.e. with $x \leftrightarrow \theta$. You may see $\sin^2 x = \dfrac{1}{3}$ or $\cos^2 x = \dfrac{2}{3}$ after use of double angle formulae.

A1: Two "correct" values. Condone accuracy of awrt $90°,\ 35°,\ 145°$
Also condone radian values here. Look for 2 of awrt 0.62, 1.57, 2.53

A1: All correct (allow awrt) and no other values in range. Condone $x \leftrightarrow \theta$ if used consistently

....

Answers without working in (b): Just answers and no working score 0 marks.
If the first line is written out, i.e. $\tan 2x = 3\sin 2x$ followed by all three correct answers score 1100 .

15. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

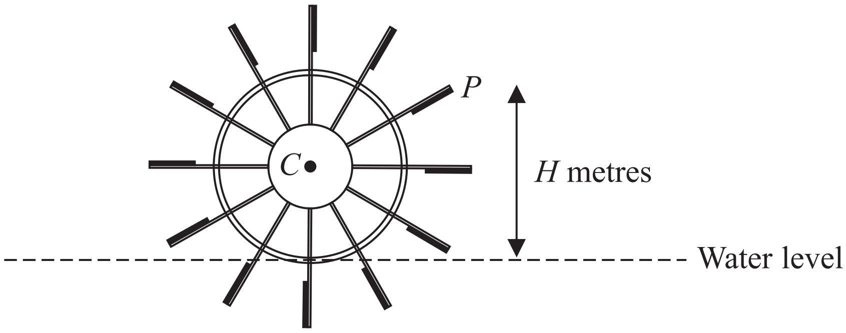

Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point C.

The point P is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, H metres, of P above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where t is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of P above the water level,

(ii) the value of t when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, P is below the water level for a total of T seconds.

According to the model,

(c) find the value of T giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable*.)

(4)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

(1)

Question	Scheme	Marks	AOs
15(a)	$R = \sqrt{5}$	B1	1.1b
	$\tan\alpha = \frac{1}{2}$ or $\sin\alpha = \frac{1}{\sqrt{5}}$ or $\cos\alpha = \frac{2}{\sqrt{5}} \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 0.464$	A1	1.1b
		(3)	
(b)(i)	$3 + 2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t + 0.464) = 1 \Rightarrow 0.5t + 0.464 = 2\pi$ $\Rightarrow t = ...$	M1	3.4
	$t = 11.6$	A1	1.1b
		(3)	
(c)	$3 + 2\sqrt{5}\cos(0.5t + 0.464) = 0$ $\cos(0.5t + 0.464) = -\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t + 0.464) = -\frac{3}{2\sqrt{5}} \Rightarrow 0.5t + 0.464 = \cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t = 2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right) - 0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977... - 0.464) - 2(2.306... - 0.464)$	dM1	3.1b
	= 3.34	A1	1.1b
		(4)	
(d)	e.g. the “3” would need to vary	B1	3.5c
		(1)	
			(11 marks)

Notes

(a)

B1: $R = \sqrt{5}$ only.

M1: Proceeds to a value for α from $\tan\alpha = \pm\frac{1}{2}$ or $\sin\alpha = \pm\frac{1}{"R"}$ or $\cos\alpha = \pm\frac{2}{"R"}$

It is implied by either awrt 0.464 (radians) or awrt 26.6 (degrees)

A1: $\alpha =$ awrt 0.464

(b)(i)

B1ft: For $(3 + 2\sqrt{5})$ m or awrt 7.47 m and remember to isw. Condone lack of units.

Follow through on their R value so allow 3 + 2×Their R. (Allow in decimals with at least 3sf accuracy)

(b)(ii)

M1: Uses $0.5t \pm "0.464" = 2\pi$ to obtain a value for t

Follow through on their 0.464 but this angle must be in radians.

It is possible in degrees but only using $0.5t \pm "26.6" = 360$

A1: Awrt 11.6

Alternative for (b):

$$H = 3+4\cos(0.5t)-2\sin(0.5t) \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}t} = -2\sin(0.5t)-\cos(0.5t)=0$$

$$\Rightarrow \tan(0.5t) = -\frac{1}{2} \Rightarrow 0.5t = 2.677...,5.819... \Rightarrow t = 5.36,\ 11.6$$

$$t = 11.6 \Rightarrow H = 7.47$$

Score as follows:

M1: For a complete method:

Attempts $\frac{\mathrm{d}H}{\mathrm{d}t}$ and attempts to solve $\frac{\mathrm{d}H}{\mathrm{d}t}=0$ for t

A1: For t = awrt 11.6

B1ft: For awrt 7.47 or 3 + 2×Their R

(c)

M1: Uses the model and sets $3+2"\sqrt{5}"\cos(...)=0$ and proceeds to $\cos(...)=k$ where $|k|<1$.

Allow e.g. $3+2"\sqrt{5}"\cos(...)<0$

dM1: Solves $\cos(0.5t \pm "0.464")=k$ where $|k|<1$ to obtain at least one value for t

This requires e.g. $2\left(\pi+\cos^{-1}(k)\pm\tan^{-1}\left(\frac{1}{2}\right)\right)$ or e.g. $2\left(\pi-\cos^{-1}(k)\pm\tan^{-1}\left(\frac{1}{2}\right)\right)$

Depends on the previous method mark.

dM1: A fully correct strategy to find the required duration. E.g. finds 2 consecutive values of t when H = 0 and subtracts. Alternatively finds t when H is minimum and uses the times found correctly to find the required duration.

Depends on the previous method mark.

Examples:

Second time at water level – first time at water level:

$$2\left(\pi+\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)-2\left(\pi-\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)=7.02685...-3.68492...$$

2×(first time at minimum point – first time at water level):

$$2\left(2\left(\pi-\tan^{-1}\left(\frac{1}{2}\right)\right)-2\left(\pi-\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right)\right)=2(5.35589...-3.68492...)$$

Note that both of these examples equate to $4\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ **which is not immediately obvious but may be seen as an overall method.**

There may be other methods – if you are not sure if they deserve credit send to review.

A1: Correct value. Must be 3.34 (**not** awrt).

Special Cases in (c):

Note that if candidates have an incorrect α and have e.g. $3+2\sqrt{5}\cos(0.5t-0.464)$, this has no impact on the final answer. So for candidates using $3+2\sqrt{5}\cos(0.5t\pm\alpha)$ in (c) allow all the marks including the A mark as a correct method should always lead to 3.34

Some values to look for:

$$0.5t \pm "0.464" = \pm 2.306,\ \pm 3.977,\ \pm 8.598,\ \pm 10.26$$

(d)

B1: Correct refinement e.g. As in scheme. If they suggest a specific function to replace the "3" then it must be sensible e.g. a trigonometric function rather than e.g. a quadratic/linear one.

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)

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P 6 9 6 0 1 A 0 3 8 4 8

Question	Scheme	Marks	AOs
14(a)	Attempts to use both $\sin(x-60°)=\pm\sin x\cos 60°\pm\cos x\sin 60°$ $\cos(x-30°)=\pm\cos x\cos 30°\pm\sin x\sin 30°$	M1	2.1
	Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$	A1	1.1b
	Either uses $\frac{\sin x}{\cos x}=\tan x$ and attempts to make $\tan x$ the subject E.g. $(2\cos 60°-\sin 30°)\tan x=\cos 30°+2\sin 60°$ Or attempts $\sin 30°$ etc with at least two correct and collects terms in $\sin x$ and $\cos x$ E.g. $\left(2\times\frac{1}{2}-\frac{1}{2}\right)\sin x=\left(2\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\cos x$,	M1	2.1
	Proceeds to given answer showing all key steps E.g. $\frac{1}{2}\tan x=\frac{3\sqrt{3}}{2}\Rightarrow\tan x=3\sqrt{3}$ *	A1*	1.1b
		(4)	
(b)	Deduces that $x=2\theta+60°$	B1	2.2a
	$\tan(2\theta+60°)=3\sqrt{3}\Rightarrow 2\theta+60°=79.1°,259.1°,.....$	M1	1.1b
	Correct method to find one value of θ E.g $\theta=\frac{79.1°-60°}{2}$	dM1	1.1b
	$\theta=$ awrt $9.6°, 99.6°$ (See note)	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)

M1: Attempts to use both compound angle expansions to set up an equation in $\sin x$ and $\cos x$
The terms must be correct but condone sign errors and a slip on the multiplication of 2

A1: Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$ o.e.
Note that $\cos 60°=\sin 30°$ and $\cos 30°=\sin 60°$
Also allow this mark for candidates who substitute in their trigonometric values "early"

$$2\sin x\times\frac{1}{2}-2\cos x\times\frac{\sqrt{3}}{2}=\cos x\times\frac{\sqrt{3}}{2}+\sin x\times\frac{1}{2}\quad\text{o.e.}$$

M1: Shows the necessary progress towards showing the given result.
There are three key moves, two of which must be shown for this mark.

- uses $\frac{\sin x}{\cos x}=\tan x$ to form an equation in just $\tan x$.
- uses exact numerical values for $\sin 30°, \sin 60°, \cos 30°, \cos 60°$ with at least two correct
- collects terms in $\sin x$ and $\cos x$ or alternatively in $\tan x$

A1*: Proceeds to the given answer with accurate work showing all necessary lines.

Examples of two proofs showing all necessary lines

E.g. I $2\sin x\cos 60° - 2\cos x\sin 60° = \cos x\cos 30° + \sin x\sin 30°$

$\sin x(2\cos 60° - \sin 30°) = \cos x(\cos 30° + 2\sin 60°)$ 1. collect terms

$(2\cos 60° - \sin 30°)\tan x = \cos 30° + 2\sin 60°$ 2. $\frac{\sin x}{\cos x} = \tan x$ so M1

$\tan x = \frac{\cos 30° + 2\sin 60°}{2\cos 60° - \sin 30°} = \frac{\frac{\sqrt{3}}{2} + \sqrt{3}}{1 - \frac{1}{2}} = 3\sqrt{3}$ 3..uses values and completes proof A1*

E.g II

$2\sin x \times \frac{1}{2} - 2\cos x \times \frac{\sqrt{3}}{2} = \cos x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{1}{2}$ 1.uses values

$\Rightarrow \frac{1}{2}\sin x = \frac{3\sqrt{3}}{2}\cos x$ 2.collects terms so M1

$\Rightarrow \tan x = 3\sqrt{3}$

3. $\frac{\sin x}{\cos x} = \tan x$ completes proof A1*

(b) Hence

B1: Deduces that $x = 2\theta + 60°$ o.e such as $\theta = \frac{x - 60°}{2}$

This is implied for sight of the equation $\tan(2\theta + 60°) = 3\sqrt{3}$

M1: Proceeds from $\tan(2\theta \pm \alpha°) = 3\sqrt{3} \Rightarrow 2\theta \pm \alpha° =$ one of $79.1°, 259.1°, ...$ where $\alpha \neq 0$

One angle for $\arctan(3\sqrt{3})$ **must** be correct in degrees or radians(3sf). FYI radian answers 1.38, 4.52

dM1: Correct method to find one value of θ from their $2\theta \pm \alpha° = 79.1°$ to $\theta = \frac{79.1° \mp \alpha°}{2}$

This is dependent upon one angle being correct, which must be in degrees, for $\arctan(3\sqrt{3})$

$\tan(2\theta + 60°) = 3\sqrt{3} \Rightarrow \theta = 9.6°$ would imply B1 M1 dM1

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range

Otherwise: Via the use of $\cos(2\theta + 30°) = \cos 2\theta\cos 30° - \sin 2\theta\sin 30°$.

$2\sin 2\theta = \cos(2\theta + 30°) \Rightarrow \tan 2\theta = \frac{\sqrt{3}}{5} \Rightarrow \theta = 9.6°, 99.6°$

The order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta = \frac{\sqrt{3}}{5}$ o.e so allow $\tan 2\theta =$ awrt 0.346 or $\tan 2\theta = \frac{\cos 30°}{2 + \sin 30°}$

Or via double angle identities $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ o.e.

M1: Attempts to use the compound angle identities to reach a form $\tan 2\theta = k$ where k is a constant not $3\sqrt{3}$ (or expression in trig terms such as cos 30 etc as seen above)

Or via double angle identities reaches a 3TQ in $\tan\theta$

dM1: Correct order of operations from $\tan 2\theta = k$ leading to $\theta = ...$

Correctly solves their $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ leading to $\theta = ...$

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range.

Note that $\tan(2\theta + 60°) = 3\sqrt{3} \Rightarrow \theta = 9.6°, 99.6°$ is acceptable for full marks

9.

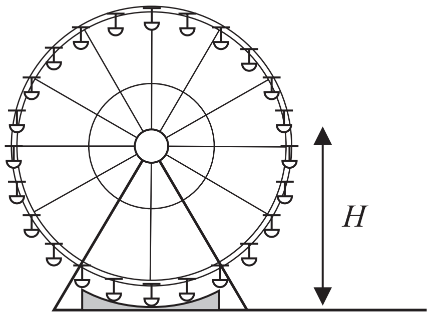

Figure 4

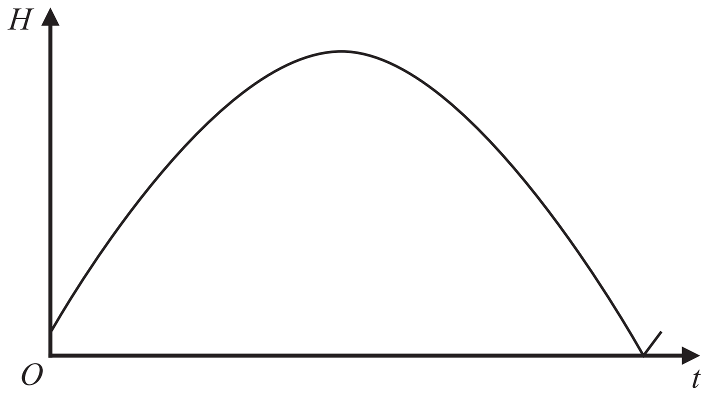

Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, H m, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = \left|A\sin(bt + \alpha)^\circ\right|$$

where A, b and α are constants.

Figure 5 shows a sketch of the graph of H against t, for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A, the exact value of b and the value of α to 3 significant figures.

(4)

(b) Explain why an equation of the form

$$H = \left|A\sin(bt + \alpha)^\circ\right| + d$$

where d is a positive constant, would be a more appropriate model.

(1)

Question	Scheme	Marks	AOs
9(a)	Deduces that $A=\pm 50$ **or** $b=\frac{1}{4}$	B1	3.4
	Deduces that $A=\pm 50$ **and** $b=\frac{1}{4}$	B1	3.4
	Uses $t=0, H=1 \Rightarrow \alpha = ...$ E.g. $1="50"\sin(\alpha)° \Rightarrow \alpha = ...$	M1	3.4
	$H=\left\lvert \pm 50\sin\left(\frac{1}{4}t+1.15\right)^{\circ}\right\rvert$	A1	3.3
		(4)	
(b)	E.g. the minimum height above the ground of the passenger on the original model was 0 m or Adding "*d*" means the passenger does not touch the ground.	B1	3.5b
		(1)	
			(5 marks)
Notes:			

(a) Note that B0B1 is not possible

B1: Uses the equation of the given model to deduce that $A=\pm 50$ **or** $b=\frac{1}{4}$ o.e.

May be seen embedded within their equation.

B1: Uses the equation of the given model to deduce that $A=\pm 50$ **and** $b=\frac{1}{4}$ o.e.

May be seen embedded within their equation.

M1: Uses $t = 0$ and $H = 1$ in the equation of the model to find a value for α .

Follow through on their value for A. Allow for $\pm 1="50"\sin(\alpha)° \Rightarrow \alpha = ...$ where α is in degrees or radians.

Note that in radians $\sin^{-1}\left(\frac{1}{50}\right) \approx \frac{1}{50}$ (0.0200…) which may appear incorrect but is in fact ok.

Also in degrees a value of e.g. 1.14 (truncated) would indicate the method.

A1: Writes down the correct full equation of the model: $H=\left\lvert "\pm" 50\sin\left(\frac{1}{4}t+1.15\right)^{\circ}\right\rvert$ o.e.

Condone omission of degrees symbol and allow awrt 1.15 for α.
Allow if a correct equation is seen anywhere in their solution.

(b)

B1: Gives a suitable explanation with no contradictory statements.

Condone "so that pod/capsule/seat/passenger/ferris wheel/it etc. will not hit/touch the ground"

Responses that focus on the starting point of the model are likely to score B0