## Y2P6 XMQs and MS

(Total: 8 marks)

1. P1\_2020 Q12. 8 marks - Y2P6 Trigonometric functions

12.	In this question you must show all stages of your working.
	Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\csc\theta - \sin\theta \equiv \cos\theta \cot\theta \qquad \theta \neq (180n)^{\circ}$$

$$\neq (180n)^{\circ} \quad n \in \mathbb{Z}$$

**(3)** 

(b) Hence, or otherwise, solve for  $0 < x < 180^{\circ}$ 

$$\csc x - \sin x = \cos x \cot (3x - 50^{\circ})$$

**(5)** 

States or uses $\cos \theta = \frac{1}{\sin \theta}$ B1       1.2 $\cos \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta}$ M1       2.1 $\frac{\cos^2 \theta}{\sin \theta} = \cos \theta \times \frac{\cos \theta}{\sin \theta} = \cos \theta \cot \theta$ * A1*       2.1         (3)         (5)         (5)         M1       3.1a $x = 25^\circ$ M1       3.1a $x = 25^\circ$ A1       1.1b         Also $\cot x = \cot(3x - 50^\circ) \Rightarrow x + 180^\circ = 3x - 50^\circ$ M1       2.1 $x = 115^\circ$ A1       1.1b         Deduces $x = 90^\circ$ B1       2.2a        (5)	Question	Scheme	Marks	AOs
$ \frac{\cos^{2}\theta}{\sin\theta} = \cos\theta \times \frac{\cos\theta}{\sin\theta} = \cos\theta \cot\theta  *  \text{A1*}  2.1 $ (b) $ \cos x - \sin x = \cos x \cot(3x - 50^{\circ}) $ $ \Rightarrow \cos x \cot x = \cos x \cot(3x - 50^{\circ}) $ $ \cot x = \cot(3x - 50^{\circ}) \Rightarrow x = 3x - 50^{\circ} $ A1 1.1b $ A \cos x + \cot(3x - 50^{\circ}) \Rightarrow x + 180^{\circ} = 3x - 50^{\circ} $ M1 2.1 $ x = 115^{\circ} $ A1 1.1b $ Deduces x = 90^{\circ} $ B1 2.2a	12 (a)	States or uses $\csc \theta = \frac{1}{\sin \theta}$	B1	1.2
(b) $\cos x - \sin x = \cos x \cot (3x - 50^{\circ})$ $\Rightarrow \cos x \cot x = \cos x \cot (3x - 50^{\circ})$ $\cot x = \cot (3x - 50^{\circ}) \Rightarrow x = 3x - 50^{\circ}$ M1 3.1a $x = 25^{\circ}$ A1 1.1b Also $\cot x = \cot (3x - 50^{\circ}) \Rightarrow x + 180^{\circ} = 3x - 50^{\circ}$ M1 2.1 $x = 115^{\circ}$ A1 1.1b Deduces $x = 90^{\circ}$ B1 2.2a		$\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1 - \sin^2\theta}{\sin\theta}$	M1	2.1
(b) $\cos x - \sin x = \cos x \cot (3x - 50^{\circ})$ $\Rightarrow \cos x \cot x = \cos x \cot (3x - 50^{\circ})$ $\cot x = \cot (3x - 50^{\circ}) \Rightarrow x = 3x - 50^{\circ}$ M1 3.1a $x = 25^{\circ}$ A1 1.1b Also $\cot x = \cot (3x - 50^{\circ}) \Rightarrow x + 180^{\circ} = 3x - 50^{\circ}$ M1 2.1 $x = 115^{\circ}$ A1 1.1b Deduces $x = 90^{\circ}$ B1 2.2a		$= \frac{\cos^2 \theta}{\sin \theta} = \cos \theta \times \frac{\cos \theta}{\sin \theta} = \cos \theta \cot \theta \qquad *$	A1*	2.1
$\Rightarrow \cos x \cot x = \cos x \cot(3x - 50^{\circ})$ $\cot x = \cot(3x - 50^{\circ}) \Rightarrow x = 3x - 50^{\circ}$ $x = 25^{\circ}$ Al 1.1b  Also $\cot x = \cot(3x - 50^{\circ}) \Rightarrow x + 180^{\circ} = 3x - 50^{\circ}$ $x = 115^{\circ}$ Al 1.1b  Deduces $x = 90^{\circ}$ Bl 2.2a  (5)			(3)	
$\cot x = \cot(3x - 50^{\circ}) \Rightarrow x = 3x - 50^{\circ}$ $x = 25^{\circ}$ Al 1.1b  Also $\cot x = \cot(3x - 50^{\circ}) \Rightarrow x + 180^{\circ} = 3x - 50^{\circ}$ M1 2.1 $x = 115^{\circ}$ Al 1.1b  Deduces $x = 90^{\circ}$ B1 2.2a  (5)	<b>(b)</b>	$\csc x - \sin x = \cos x \cot (3x - 50^{\circ})$		
$x = 25^{\circ}$ A1 1.1b  Also $\cot x = \cot(3x - 50^{\circ}) \Rightarrow x + 180^{\circ} = 3x - 50^{\circ}$ M1 2.1 $x = 115^{\circ}$ A1 1.1b  Deduces $x = 90^{\circ}$ B1 2.2a  (5)		$\Rightarrow \cos x \cot x = \cos x \cot (3x - 50^{\circ})$		
Also $\cot x = \cot(3x - 50^{\circ}) \Rightarrow x + 180^{\circ} = 3x - 50^{\circ}$ M1 2.1 $x = 115^{\circ}$ A1 1.1b  Deduces $x = 90^{\circ}$ B1 2.2a  (5)		$\cot x = \cot (3x - 50^\circ) \Rightarrow x = 3x - 50^\circ$	M1	3.1a
$x = 115^{\circ}$ A1 1.1b  Deduces $x = 90^{\circ}$ B1 2.2a  (5)		x = 25°	A1	1.1b
Deduces $x = 90^{\circ}$ B1 2.2a (5)		Also $\cot x = \cot (3x - 50^\circ) \Rightarrow x + 180^\circ = 3x - 50^\circ$	M1	2.1
(5)		x = 115°	A1	1.1b
		Deduces $x = 90^{\circ}$	B1	2.2a
			(5)	
(8 mark				(8 marks)

#### (a) Condone a full proof in x (or other variable) instead of $\theta$ 's here

**B1:** States or uses  $\csc \theta = \frac{1}{\sin \theta}$  Do not accept  $\csc \theta = \frac{1}{\sin \theta}$  with the  $\theta$  missing

M1: For the key step in forming a single fraction/common denominator

E.g. 
$$\csc\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1 - \sin^2\theta}{\sin\theta}$$
. Allow if written separately  $\frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$ 

Condone missing variables for this M mark

**A1\*:** Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

#### (b) Condone $\theta$ 's instead of x's here

**M1:** Uses part (a), cancels or factorises out the  $\cos x$  term, to establish that one solution is found when  $x = 3x - 50^{\circ}$ .

You may see solutions where  $\cot A - \cot B = 0 \Rightarrow \cot(A - B) = 0$  or  $\tan A - \tan B = 0 \Rightarrow \tan(A - B) = 0$ . As long as they don't state  $\cot A - \cot B = \cot(A - B)$  or  $\tan A - \tan B = \tan(A - B)$  this is acceptable

**A1:**  $x = 25^{\circ}$ 

M1: For the key step in realising that  $\cot x$  has a period of  $180^{\circ}$  and a second solution can be found by solving  $x+180^{\circ}=3x-50^{\circ}$ . The sight of  $x=115^{\circ}$  can imply this mark provided the step  $x=3x-50^{\circ}$  has been seen. Using reciprocal functions it is for realising that  $\tan x$  has a period of  $180^{\circ}$ 

A1:  $x = 115^{\circ}$  Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

**B1:** Deduces that a solution can be found from  $\cos x = 0 \Rightarrow x = 90^{\circ}$ . Ignore additional values here.

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Solutions with limited working. The question demands that candidates show all stages of working.

SC: 
$$\cos x \cot x = \cos x \cot (3x - 50^\circ) \Rightarrow \cot x = \cot (3x - 50^\circ) \Rightarrow x = 25^\circ, 115^\circ$$

They have shown some working so can score B1, B1 marked on epen as 11000

### Alt 1- Right hand side to left hand side

Question	Scheme	Marks	AOs
12 (a)	States or uses $\cot \theta = \frac{\cos \theta}{\sin \theta}$	B1	1.2
	$\cos\theta\cot\theta = \frac{\cos^2\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \frac{1}{\sin \theta} - \sin \theta = \csc \theta - \sin \theta \qquad *$	A1*	2.1
		(3)	

#### Alt 2- Works on both sides

Question	Scheme	Marks	AOs
12 (a)	States or uses $\cot \theta = \frac{\cos \theta}{\sin \theta}$ or $\csc \theta = \frac{1}{\sin \theta}$	B1	1.2
	$LHS = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta}$ $RHS = \cos \theta \cot \theta = \frac{\cos^2 \theta}{\sin \theta}$	M1	2.1
	States a conclusion E.g.  "HENCE TRUE",  "QED"  or $\cos \theta - \sin \theta = \cos \theta \cot \theta$ o.e. (condone = for $\equiv$ )	A1*	2.1
		(3)	

# Alt (b)

Question	Scheme	Marks	AOs
	$\cot x = \cot(3x - 50^\circ) \Rightarrow \frac{\cos x}{\sin x} = \frac{\cos(3x - 50^\circ)}{\sin(3x - 50^\circ)}$		
	$\sin(3x - 50^{\circ})\cos x - \cos(3x - 50^{\circ})\sin x = 0$ $\sin((3x - 50^{\circ}) - x) = 0$	M1	3.1a
	$2x - 50^{\circ} = 0$		
	x = 25°	A1	1.1b
	Also $2x - 50^{\circ} = 180^{\circ}$	M1	2.1
	x = 115°	A1	1.1b
	Deduces $\cos x = 0 \Rightarrow x = 90^{\circ}$	B1	2.2a
		(5)	