Y2P3 XMQs and MS

(Total: 109 marks)

1. P2_Sample Q10. 4 marks - Y2P3 Sequences and series
2. P1_Specimen Q3 . 4 marks - Y2P3 Sequences and series
3. P1_Specimen Q8 . 5 marks - Y2P3 Sequences and series
4. P2_Specimen Q11. 5 marks - Y2P3 Sequences and series
5. P2_2018 Q4 . 7 marks - Y2P3 Sequences and series
6. P1_2019 Q11. 7 marks - Y2P3 Sequences and series
7. P2_2019 Q8 . 6 marks - Y2P3 Sequences and series
8. P1_2020 Q5 . 6 marks - Y2P3 Sequences and series
9. P1_2020 Q13. 7 marks - Y2P3 Sequences and series
10. P2_2020 Q15. 8 marks - Y2P3 Sequences and series
11. P1_2021 Q3 . 6 marks - Y2P3 Sequences and series
12. P1_2021 Q5 . 6 marks - Y2P3 Sequences and series
13. P2_2021 Q1 . 4 marks - Y2P3 Sequences and series
14. P2_2021 Q9 . 3 marks - Y2P3 Sequences and series
15. P1_2022 Q13. 7 marks - Y2P3 Sequences and series
16. P2_2022 Q3 . 4 marks - Y2P3 Sequences and series
17. P2_2022 Q15. 10 marks - Y2P3 Sequences and series
18. P31_2020 Q4 . 10 marks - Y1S6 Statistical distributions

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

(Total for Question 10 is 4 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme		Marks	AOs
9	$\int\left(3x^{0.5}+A\right)\mathrm{d}x = 2x^{1.5}+Ax\left(+c\right)$		M1 A1	3.1a 1.1b
	Uses limits and sets $= 2A^2 \Rightarrow \left(2\times 8+4A\right)-\left(2\times 1+A\right)=2A^2$		M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts b^2-4ac	M1	1.1b
	$\Rightarrow A=-2, \frac{7}{2}$ **and** states that there are two roots	States $b^2-4ac=121>0$ and hence there are two roots	A1	2.4
			(5 marks)	

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax\left(+c\right)$ where k is a non- zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac

A1: Either $A=-2, \frac{7}{2}$ **and** states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

Question	Scheme	Marks	AOs
10	Attempts $S_\infty=\frac{8}{7}\times S_6 \Rightarrow \frac{a}{1-r}=\frac{8}{7}\times\frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1=\frac{8}{7}\times(1-r^6)$	M1	2.1
	$\Rightarrow r^6=\frac{1}{8}\Rightarrow r=..$	M1	1.1b
	$\Rightarrow r=\pm\frac{1}{\sqrt{2}}$ (so $k=2$)	A1	1.1b
		(4 marks)	

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty=\frac{8}{7}\times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r=\pm\frac{1}{\sqrt{2}}$ giving $k=2$

3. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$

$$a_{n+1} = \frac{a_n - 3}{a_n - 2}, \qquad n \in \mathbb{N}$$

(a) Find $\displaystyle\sum_{r=1}^{100} a_r$

(3)

(b) Hence find $\displaystyle\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r$

(1)

Question	Scheme	Marks	AOs
3 (a)	$a_1 = 3$, $a_2 = 0$, $a_3 = 1.5$, $a_4 = 3$	M1	1.1b
	$\sum_{r=1}^{100} a_r = 33(4.5) + 3$	M1	2.2a
	$= 151.5$	A1	1.1b
		(3)	
(b)	$\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)(151.5) - 3 = 300$	B1ft	2.2a
		(1)	
			(4 marks)

Question 3 Notes:

(a)	
M1:	Uses the formula $a_{n+1} = \dfrac{a_n - 3}{a_n - 2}$, with $a_1 = 3$ to generate values for a_2, a_3 and a_4
M1:	Finds $a_4 = 3$ and deduces $\sum_{r=1}^{100} a_r = 33("3" + "0" + "1.5") + "3"$
A1:	which leads to a correct answer of 151.5
(b)	
B1ft:	Follow through on their periodic function. Deduces that either • $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)("151.5") - 3 = 300$ • $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = "151.5" + (33)("3" + "0" + "1.5") = 151.5 + 148.5 = 300$

8. There were 2100 tonnes of wheat harvested on a farm during 2017.

The mass of wheat harvested during each subsequent year is expected to increase by 1.2% per year.

(a) Find the total mass of wheat expected to be harvested from 2017 to 2030 inclusive, giving your answer to 3 significant figures.

(2)

Each year it costs

- £5.15 per tonne to harvest the first 2000 tonnes of wheat
- £6.45 per tonne to harvest wheat in excess of 2000 tonnes

(b) Use this information to find the expected cost of harvesting the wheat from 2017 to 2030 inclusive. Give your answer to the nearest £1000

(3)

Question	Scheme	Marks	AOs
8 (a)	Total amount $= \dfrac{2100(1-(1.012)^{14})}{1-1.012}$ or $\dfrac{2100((1.012)^{14}-1)}{1.012-1}$	M1	3.1b
	$= 31806.9948... = 31800 \text{ (tonnes) (3 sf)}$	A1	1.1b
		(2)	
	Total Cost = $5.15(2000(14)) + 6.45(31806.9948... - (2000)(14))$	M1	3.1b
		M1	1.1b
	$= 5.15(28000) + 6.45(3806.9948...) = 144200 + 24555.116...$		
	$= 168755.116... = £169000$ (nearest £1000)	A1	3.2a
		(3)	
		(5 marks)	

Question 8 Notes:

(a)	
M1:	Attempts to apply the correct geometric summation formula with either $n = 13$ or $n = 14$, $a = 2100$ and $r = 1.012$ (Condone $r = 1.12$)
A1:	Correct answer of 31800 (tonnes)
(b)	
M1:	Fully correct method to find the total cost
M1:	For either • $5.15(2000(14))\ \{= 144200\}$ • $6.45(\text{"}31806.9948...\text{"} - (2000)(14))\ \{= 24555.116...\}$ • $5.15(2000(13))\ \{= 133900\}$ • $6.45(\text{"}29354.73794...\text{"} - (2000)(13))\ \{= 21638.059...\}$
A1:	Correct answer of £169000 **Note:** Using rounded answer in part (a) gives 168710 which becomes £169000 (nearest £1000)

11. The second, third and fourth terms of an arithmetic sequence are $2k$, $5k - 10$ and $7k - 14$ respectively, where k is a constant.

Show that the sum of the first n terms of the sequence is a square number.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
11	Arithmetic sequence, $T_2 = 2k$, $T_3 = 5k - 10$, $T_4 = 7k - 14$		
	$(5k - 10) - (2k) = (7k - 14) - (5k - 10) \Rightarrow k = ...$	M1	2.1
	$\{3k - 10 = 2k - 4 \Rightarrow\}$ $k = 6$	A1	1.1b
	$\{k = 6 \Rightarrow\}$ $T_2 = 12$, $T_3 = 20$, $T_4 = 28$. So $d = 8$, $a = 4$	M1	2.2a
	$S_n = \frac{n}{2}\left(2(4) + (n-1)(8)\right)$	M1	1.1b
	$= \frac{n}{2}\left(8 + 8n - 8\right) = 4n^2 = (2n)^2$ which is a square number	A1	2.1
		(5)	
			(5 marks)

Question 11 Notes:

M1:	Complete method to find the value of k
A1:	Uses a correct method to find $k = 6$
M1:	Uses their value of k to deduce the common difference and the first term $(\neq T_2)$ of the arithmetic series.
M1:	Applies $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ with their $a \neq T_2$ and their d.
A1:	Correctly shows that the sum of the series is $(2n)^2$ and makes an appropriate conclusion.

4. (i) Show that $\sum_{r=1}^{16}\left(3 + 5r + 2^r\right) = 131\,798$

(4)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, ..., : u_{n+1}=\frac{1}{u_n}, u_1=\frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=\frac{16}{2}(2(8)+15(5))\;+\;\frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=728+131\,070\;=\;131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}3\;+\;\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=(3\times16)+\frac{16}{2}(2(5)+15(5))\;+\;\frac{2(2^{16}-1)}{2-1}$	M1 M1	1.1b 1.1b
	$=48+680+131\,070\;=\;131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum $=\;10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1 M1 M1 A1*	3.1a 1.1b 1.1b 2.1
		(4)	
(ii)	$\left\{u_1=\frac{2}{3}\right\},\;u_2=\frac{3}{2},\;u_3=\frac{2}{3},\;...$ (***can be implied by later working***)	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}\;50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3}\quad\left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)

Notes for Question 4	
(i)	
M1:	Uses a correct methodical strategy to enable the given sum, $\sum_{r=1}^{16}(3+5r+2^r)$ to be found Allow M1 for any of the following: • expressing the given sum as either $\sum_{r=1}^{16}(3+5r) + \sum_{r=1}^{16}(2^r)$, $\sum_{r=1}^{16}3 + \sum_{r=1}^{16}(5r) + \sum_{r=1}^{16}(2^r)$ or $\sum_{r=1}^{16}3 + 5\sum_{r=1}^{16}r + \sum_{r=1}^{16}(2^r)$ • attempting to find both $\sum_{r=1}^{16}(3+5r)$ and $\sum_{r=1}^{16}(2^r)$ separately • (3×16) and attempting to find both $\sum_{r=1}^{16}(5r)$ and $\sum_{r=1}^{16}(2^r)$ separately
M1:	**Way 1:** Correct method for finding the sum of an AP with $a=8, d=5, n=16$ **Way 2:** (3×16) and a correct method for finding the sum of an AP
M1:	Correct method for finding the sum of a GP with $a=2, r=2, n=16$
A1*:	For all steps fully shown (with correct formulae used) leading to $131\,798$
Note:	**Way 1:** Give 2nd M1 for writing $\sum_{r=1}^{16}(3+5r)$ as $\frac{16}{2}(8+83)$
Note:	**Way 2:** Give 2nd M1 for writing $\sum_{r=1}^{16}3 + \sum_{r=1}^{16}(5r)$ as $48+\frac{16}{2}(5+80)$ or $48+680$
Note:	Give 3rd M1 for writing $\sum_{r=1}^{16}(2^r)$ as $\frac{2(1-2^{16})}{1-2}$ or $2(2^{16}-1)$ or $(2^{17}-2)$
(i)	
Way 3	
M1:	At least 6 correct terms **and** 16 terms shown
M1:	At least 10 correct terms (may not be 16 terms)
M1:	At least 15 correct terms (may not be 16 terms)
A1*:	All 16 terms correct **and** an indication that the sum is $131\,798$
(ii)	
M1:	For some indication that the next two terms of this sequence are $\frac{3}{2}, \frac{2}{3}$
M1:	For deducing that the sum can be found by applying $50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$, o.e.
A1:	Obtains $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$ or an exact equivalent
Note:	Allow 1st M1 for $u_2=\frac{3}{2}$ (or equivalent) **and** $u_3=\frac{2}{3}$ (or equivalent)
Note:	Allow 1st M1 for the first 3 terms written as $\frac{2}{3}, \frac{3}{2}, \frac{2}{3}, \ldots$
Note:	Allow 1st M1 for the 2nd and 3rd terms written as $\frac{3}{2}, \frac{2}{3}, \ldots$ in the correct order
Note:	Condone $\frac{2}{3}$ written as 0.66 or awrt 0.67 for the 1st M1 mark
Note:	Give A0 for 108.3 or 108.333... without reference to $\frac{325}{3}$ or $108\frac{1}{3}$ or $108.\dot{3}$

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,
(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.
(4)

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	= 36.915 minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** th km is $6\times1.05=6\times1.05^{\mathbf{1}}$ **6**th km is $6\times1.05\times1.05=6\times1.05^{\mathbf{2}}$ **7**th km is $6\times1.05\times1.05\times1.05=6\times1.05^{\mathbf{3}}$ Hence the time for the *r* th km is $6\times1.05^{r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20} 6\times1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 th to 20th km $=\frac{6.3(1.05^{16}-1)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** Eg. Total time = 24 + $\frac{6.3(1.05^{16}-1)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

(a)

M1: For using model to calculate the total time.

Sight of 24 minutes + $6\times1.05+6\times1.05^2$ or equivalent is required. Eg 24 + 6.3 + 6.615

Alternatively in seconds 24 minutes + 378 sec (6min 18 s) +396.9 (6 min 37 s)

A1*: 36 minutes 55 seconds following 36.915, 24+ 6.3+6.615 , 24 + $6\times1.05+6\times1.05^2$ or equivalent working in seconds

(b) Must be seen in (b)

B1: As seen in scheme. For making the link between the *r* th km and the index of 1.05

Or for EXPLAINING that "the time taken per km (6 mins) only starts to increase by 5% after the first 4 km"

(c) The correct sum formula $\frac{a(r^n-1)}{r-1}$, if seen, must be correct in part (c) for all relevant marks

M1: For the overall strategy of finding the total time.

Award for adding 18, 24, 30.3 or awrt 36.9 and the sum of a geometric sequence

So award the mark for expressions such as $6\times4+\sum 6\times1.05^n$ or $24+\frac{6(1.05^{20}-1)}{1.05-1}$

The geometric sequence formula, must be used with $r=1.05$ oe but condone slips on *a* and *n*

M1: For an attempt at using a correct sum formula for a GP to find an allowable sum

The value of *r* must be 1.05 oe such as 105% but you should allow a slip on the value of *n* used for their value of *a* (See below: We are going to allow the correct value of *n* or one less)

If you don't see a calculation it may be implied by sight of one of the values seen below

Allow for $a = 6, \quad n = 17$ or 16 Eg. $\frac{6(1.05^{17}-1)}{1.05-1} = (155.0)$ or $\frac{6(1.05^{16}-1)}{1.05-1} = (141.9)$

Allow for $a = 6.3, n = 16$ or 15 Eg $\frac{6.3(1.05^{16}-1)}{1.05-1} = (149.0)$ or $\frac{6.3(1.05^{15}-1)}{1.05-1} = (135.9)$

Allow for $a = 6.615, \ n = 15$ or 14 Eg $\frac{6.615(1.05^{15}-1)}{1.05-1} = (142.7)$ or $\frac{6.615(1.05^{14}-1)}{1.05-1} = (129.6)$

A1: For a correct calculation that will find the **total time**. It does not need to be processed

Allow for example, amongst others, $24 + \frac{6.3(1.05^{16}-1)}{1.05-1}$, $18 + \frac{6(1.05^{17}-1)}{1.05-1}$, $30.3 + \frac{6.615(1.05^{15}-1)}{1.05-1}$

A1: For a total time of awrt 173 minutes and 3 seconds

This answer alone can be awarded 4 marks as long as there is some evidence of where it has come from.

....

Candidates that list values: Handy Table for Guidance

M1: For a correct overall strategy which would involve adding four sixes followed by at least 16 other values

The values which may be written in the form 6×1.05^2 or as numbers

Can be implied by $6+6+6+6+(6\times1.05)+.....+(6\times1.05^{16})$

M1: For an attempt to add the numbers from (6×1.05) to (6×1.05^{16}). This could be done on a calculator in which case expect to see awrt 149

Alternatively, if written out, look for 16 values with 8 correct or follow through correct to 1 dp

A1: Awrt 173 minutes

A1: Awrt 173 minutes and 3 seconds

Km	**Time per km**	**Total Time**
1	6.0000	
2	6.0000	12
3	6.0000	18
4	6.0000	24
5	6.3000	30.3
6	6.6150	36.915
7	6.9458	43.86075
8	7.2930	51.15379
9	7.6577	58.81148
10	8.0406	66.85205
11	8.4426	75.29465
12	8.8647	84.15939
13	9.3080	93.46736
14	9.7734	103.2407
15	10.2620	113.5028
16	10.7751	124.2779
17	11.3139	135.5918
18	11.8796	147.4714
19	12.4736	159.945
20	13.0972	173.0422

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

Question	Scheme	Marks	AOs
8 (i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + ...$		
	$= \frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \frac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \frac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40 - 37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + ... + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times...\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$ $\sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
			(6 marks)

Notes for Question 8	
(i)	**Way 1**
M1:	Applies $\dfrac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a complete strategy of applying $\dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 2**
M1:	Applies $\dfrac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\dfrac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $\dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 3**
M1:	Applies $\dfrac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\dfrac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $\dfrac{20}{1-\frac{1}{2}} - \dfrac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
Note:	Give M1 M1 A1 for a correct answer of 2.5 from no working in (i)
(ii)	**Way 1**
M1:	Some evidence of applying the addition law of logarithms as part of a valid proof
M1:	Begins to solve the problem by just writing (or by combining) at least three terms including • **either** the first two terms and the last term • **or** the first term and the last two terms
Note:	The 2nd mark can be gained by writing any of • listing $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right), \log_5\left(\frac{50}{49}\right)$ or $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{49}{48}\right), \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + ... + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + ... + \log_5\left(\frac{49}{48}\right) + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2} \times \frac{4}{3} \times ... \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}*** • $\log_5\left(\frac{3}{2} \times ... \times \frac{49}{48} \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}***
A1*:	Correct proof leading to a correct answer of 2
Note:	Do not allow the 2nd M1 if $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right)$ are listed and $\log_5\left(\frac{50}{49}\right)$ is used for the first time in their applying the formula $S_{48} = \frac{48}{2}\left(\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{50}{49}\right)\right)$
Note:	**<u>Listing all 48 terms</u>** Give M0 M1 A0 for $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \log_5\left(\frac{5}{4}\right) + ... + \log_5\left(\frac{50}{49}\right) = 2$ {lists all terms} Give M0 M0 A0 for $0.2519... + 0.1787... + 0.1386... + ... + 0.0125... = 2$ {all terms in decimals}

Notes for Question 8	
(ii)	**Way 2**
M1:	Uses the subtraction law of logarithms to give $\log_5\left(\frac{n+2}{n+1}\right) \to \log_5(n+2) - \log_5(n+1)$
M1:	Begins to solve the problem by writing at least three terms for each of $\log_5(n+2)$ and $\log_5(n+1)$ including • **either** the first two terms and the last term for both $\log_5(n+2)$ and $\log_5(n+1)$ • **or** the first term and the last two terms for both $\log_5(n+2)$ and $\log_5(n+1)$
Note:	This mark can be gained by writing any of • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 + ... + \log_5 49 + \log_5 50) - (\log_5 2 + ... + \log_5 48 + \log_5 49)$ • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 - \log_5 2) + (\log_5 4 - \log_5 3) + ... + (\log_5 50 - \log_5 49)$ • $\log_5 3 - \log_5 2,\ ......, \log_5 49 - \log_5 48,\ \log_5 50 - \log_5 49$
A1*:	Correct proof leading to a correct answer of 2
Note:	The base of 5 can be omitted for the M marks in part (ii), but the base of 5 must be included in the final line (as shown on the mark scheme) of their solution.
Note:	If a student uses a mixture of a Way 1 or Way 2 method, then award the best Way 1 mark only or the best Way 2 mark only.
Note:	Give M1 M0 A0 (1st M for implied use of subtraction law of logarithms) for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 91.8237... - 89.8237... = 2$
Note:	Give M1 M1 A1 for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$ $= \log_5(3 \times 4 \times .... \times 50) - \log_5(2 \times 3 \times .... \times 49)$ $= \log_5\left(\frac{50!}{2}\right) - \log_5(49!)$ or $= \log_5(25 \times 49!) - \log_5(49!)$ $= \log_5 25 = 2$

5. A car has six forward gears.

The fastest speed of the car

- in 1^{st} gear is 28 km h^{-1}
- in 6^{th} gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3^{rd} gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5^{th} gear.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
5 (a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2 \times "17.4" = ...$	M1	3.4
	= 62.8 (km h $^{-1}$)	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28 \times "1.3265^4" = ...$ or $\frac{115}{"1.3265"}$	M1	3.4
	= 86.7 (km h $^{-1}$)	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Translates the problem into maths using n^{th} term $= a + (n-1)d$ and attempts to find d

Look for either $115 = 28 + 5d \Rightarrow d = ...$ or an attempt at $\frac{115 - 28}{5}$ condoning slips

It is implied by use of $d = 17.4$ Note that $115 = 28 + 6d \Rightarrow d = ...$ is M0

M1: Uses the model to find the fastest speed the car can go in 3rd gear using $28 + 2"d"$ or equivalent. This can be awarded following an incorrect method of finding "d"

A1: 62.8 km/ h Lack of units are condoned. Allow exact alternatives such as $\frac{314}{5}$

(b)

M1: Translates the problem into maths using n^{th} term $= ar^{n-1}$ and attempts to find r

It must use the 1st and 6th gear and not the 3rd gear found in part (a)

Look for either $115 = 28r^5 \Rightarrow r = ...$ o.e. or $\sqrt[5]{\frac{115}{28}}$ condoning slips.

It is implied by stating or using $r = \text{awrt } 1.33$

M1: Uses the model to find the fastest speed the car can go in 5th gear using $28 \times "r^4"$ or $\frac{115}{"r"}$ o.e.

This can be awarded following an incorrect method of finding "r"

A common misread seems to be finding the fastest speed the car can go in 3rd gear as in (a).

Providing it is clear what has been done, e.g. $u_3 = 28 \times "r^2"$ it can be awarded this mark.

A1: awrt 86.7 km/h Lack of units are condoned. Expressions must be evaluated.

13. A sequence of numbers $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1} = \dfrac{k(a_n + 2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2), a_2 = 2k, a_3 = k+1, a_4 = \dfrac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\dfrac{k(k+3)}{k+1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0$ *	A1*	2.1
		(3)	
(b)	States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times (2 + -4 + -1) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

(a)

M1: Applies the sequence formula $a_{n+1} = \frac{k\left(a_n + 2\right)}{a_n}$ seen once.

This is usually scored in attempting to find the second term. E.g. for $a_2 = 2k$ or $a_{1+1} = \frac{k\left(2+2\right)}{2}$

M1: Attempts to find $a_1 \to a_4$ and sets $a_1 = a_4$. Condone slips.

Other methods are available. E.g. Set $a_4 = 2$, work backwards to find a_3 and equate to $k+1$

There is no requirement to see either a_1 or any of the labels. Look for the correct terms in the correct order.

There is no requirement for the terms to be simplified

FYI $a_1 = 2, a_2 = 2k, a_3 = k+1, a_4 = \frac{k(k+3)}{k+1}$ and so $2 = \frac{k(k+3)}{k+1}$

A1*: Proceeds to the given answer with accurate work showing all necessary lines. See MS for minimum

(b)

B1: States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3.

Do not accept "the terms just repeat" or "it would mean all the terms of the sequence are 2"

There must be some reference to the fact that it does not have order 3. Accept it has order 1.

It is acceptable to state $a_2 = a_1 = 2$ and state that the sequence does not have order 3

(c)

B1: Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$

M1: Uses a clear strategy to find the sum to 80 terms. This will usually be found using multiples of the first three terms.

For example you may see $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{78} a_r\right) + a_{79} + a_{80} = 26\times\left(2+-4+-1\right)+2+-4$

or $\sum_{r=1}^{80} a_r = \left(\sum_{r=1}^{81} a_r\right) - a_{81} = 27\times\left(2+-4+-1\right)-\left(-1\right)$

For candidates who find in terms of *k* award for $27\times2+27\times\left(2k\right)+26\times\left(k+1\right)$ or $80k+80$

If candidates proceed and substitute $k=-2$ into $80k+80$ to get -80 then all 3 marks are scored.

A1: -80

....

Note: Be aware that we have seen candidates who find the first three terms correctly, but then find

$26\frac{2}{3}\times\left(2+-4+-1\right) = 26\frac{2}{3}\times-3$ which gives the correct answer

but it is an incorrect method and should be scored B1 M0 A0

....

15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + \ldots\ldots.. + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + \ldots\ldots.. + ar^n \Rightarrow S_n - rS_n = \ldots$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4\times\dfrac{a(1-r^5)}{1-r}$ or $4\times\dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1 - r$)	M1	3.1a
	$1 - r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5-1)(r^5-3) = 0 \Rightarrow r^5 = \ldots$ or e.g. $1 - r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = \ldots$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Notes:

(a)

B1: Writes out the sum or lists terms. Condone the omission of *S*.

The sum must include the first and last terms and (at least) two other correct terms and no incorrect terms e.g. ar^n

Note that the sum may be seen embedded within their working.

M1: For the key step in attempting to multiply the first series by *r* and subtracting.

A1: $S_n - rS_n = a - ar^n$ either way around but condone one side being prematurely factorised (but not both) **following correct work but this could follow B0 if insufficient terms were shown.**

A1*: Depends on all previous marks. Proceeds to given result showing all steps including seeing both sides unfactorised at some point in their working.

Note: If terms are <u>listed</u> rather than <u>added</u> then allow the first 3 marks if otherwise correct but withhold the final mark.

(b)

M1: For the correct strategy of producing an equation in just r^{10} and r^5 (and possibly $(1-r)$) with the "4" on either side using the result from part (a) and makes progress to at least cancel through by *a*

Some candidates retain the "$1-r$" and start multiplying out e.g. $(1-r)(1-r^{10})$ and this mark is still available as long as there is progress in cancelling the "*a*".

A1: Correct equation with the *a* and the $1-r$ cancelled. Allow any correct equation in just r^5 and r^{10}

dM1: Depends on the first M. Solves as far as $r^5 = \ldots$ by solving a 3 term quadratic in r^5 by a valid method – see general guidance or by difference of 2 squares – see above

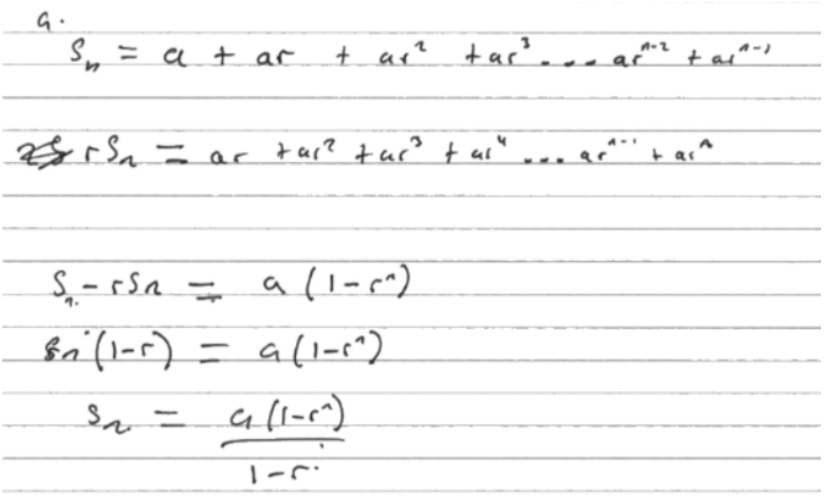
A1: $r = \sqrt[5]{3}$ oe only. The solution $r = 1$ if found must be rejected here.

(b) Note: For candidates who use $S_5 = 4S_{10}$ expect to see:

$$4\times\frac{a\left(1-r^{10}\right)}{1-r}=\frac{a\left(1-r^{5}\right)}{1-r}\Rightarrow 4\left(1-r^{10}\right)=\left(1-r^{5}\right)\text{ M1A0}$$

$$4r^{10}-r^5-3=0\Rightarrow\left(4r^5+3\right)\left(r^5-1\right)=0\Rightarrow r^5=\ldots\text{ or }4\left(1-r^5\right)\left(1+r^5\right)=\left(1-r^5\right)\Rightarrow r^5=\ldots\text{dM1A0}$$

<u>Example for (a):</u>



a.

$S_n = a + ar + ar^2 + ar^3 \ldots ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + ar^4 \ldots ar^{n-1} + ar^n$

$S_n - rS_n = a(1-r^n)$

$S_n(1-r) = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

This scores B1M1A1A0:

B1: Writes down the sum including first and last terms and at least 2 other correct terms and no incorrect terms

M1: Multiplies by *r* and subtracts

A1: Correct equation (we allow one side to be prematurely factorised)

A0: One side was prematurely factorised

3. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
3(a)	$u_2 = k - 12,\ u_3 = k - \frac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \frac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \frac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0$*	A1*	2.1
		(3)	
(b)	$k = 6, \left(\frac{40}{3}\right)$	M1	1.1b
	k = 6 as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)10$	B1	2.2a
		(1)	
		(6 marks)	

Notes

(a)

M1: Attempts to apply the sequence formula once for either u_2 **or** u_3.

Usually for $u_2 = k - \frac{24}{2}$ o.e. but could be awarded for $u_3 = k - \frac{24}{their\ "u_2"}$

dM1: Award for

- attempting to apply the sequence formula to find both u_2 **and** u_3
- using $2 + 2"u_2" + "u_3" = 0 \Rightarrow$ an equation in k. The u_3 may have been incorrectly adapted

A1*: Fully correct work leading to the printed answer.

There must be

- (at least) one correct intermediate line between $2 + 2(k-12) + k - \frac{24}{k-12} = 0$ (o.e.) and the given answer that shows how the fractions are "removed". E.g. $(3k-22)(k-12) - 24 = 0$
- no errors in the algebra. The = 0 may just appear at the answer line.

(b)

M1: Attempts to solve the quadratic which is implied by sight of k = 6.

This may be awarded for any of

- $3k^2 - 58k + 240 = (ak \pm c)(bk \pm d) = 0$ where $ab = 3, cd = 240$ followed by k =
- an attempt at the correct quadratic formula (or completing the square)
- a calculator solution giving at least k = 6

A1: Chooses k = 6 and gives a minimal reason

Examples of a minimal reason are

- 6 because it is an integer
- 6 because it is a whole number
- 6 because $\frac{40}{3}$ or $13.\dot{3}$ is not an integer

(c)

B1: Deduces the correct value of u_3.

5. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
5(a)	$u_3 = £20\,000 \times 1.08^2 = (£)23328*$	B1*	1.1b
		(1)	
(b)	$20\,000 \times 1.08^{n-1} > 65\,000$	M1	1.1b
	$1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \frac{\ln(3.25)}{\ln(1.08)}$ or e.g. $1.08^{n-1} > \frac{13}{4} \Rightarrow n-1 > \log_{1.08}\left(\frac{13}{4}\right)$	M1	3.1b
	Year 17	A1	3.2a
		(3)	
(c)	$S_{20} = \frac{20\,000(1-1.08^{20})}{1-1.08}$	M1	3.4
	Awrt (£) 915 000	A1	1.1b
		(2)	
			(6 marks)
	Notes		

(a)

B1*: Uses a correct method to show that the Profit in Year 3 will be £23 328. Condone missing units

E.g. $£20\,000 \times 1.08^2$ or $£20\,000 \times 108\% \times 108\%$

This may be obtained in two steps. E.g $\frac{8}{100} \times 20000 = 1600$ followed by $\frac{8}{100} \times 21600 = 1728$ with the calculations $21600 + 1728 = 23328$ seen.

Condone calculations seen as 8% of 20000 = 1600.

This is a show that question and the method must be seen.

It is not enough to state Year 1 = £21 600, Year 2 = £ 23 328

(b)

M1: Sets up an inequality or an equation that will allow the problem to be solved.

Allow for example *N* or *n* for *n* – 1. So award for $20\,000\times1.08^{n-1}>65\,000$,

$20\,000\times1.08^{n}=65\,000$ or $20\,000\times(108\%)^{n}\geqslant 65\,000$ amongst others.

Condone **slips** on the 20 000 and 65 000 but the 1.08 o.e. must be correct

M1: Uses a correct strategy involving logs in an attempt to solve a type of equation or inequality of the form seen above. It cannot be awarded from a sum formula

The equation/inequality must contain an index of *n* – 1, *N*, *n* etc.

Again condone **slips** on the 20 000 and 65 000 but additionally condone an error on the 1.08, which may appear as 1.8 for example

E.g. $20\,000\times1.08^{n}=65\,000\Rightarrow n\log1.08=\log\dfrac{65000}{20000}\Rightarrow n=\ldots$

E.g. $2000\times1.8^{n}=65\,000\Rightarrow \log2000+n\log1.8=\log65000\Rightarrow n=\ldots$

A1: Interprets their decimal value and gives the correct year number. Year 17

The demand of the question dictates that solutions relying entirely on calculator technology are not acceptable, BUT allow a solution that appreciates **a correct term** formula or the entire set of calculations where you may see the numbers as part of a larger list

E.g. Uses, or implies the use of, an acceptable calculation and finds value(s)

for M1: $(n=16)\Rightarrow P=20\,000\times1.08^{15}$=awrt 63400 **or** $(n=17)\Rightarrow P=20\,000\times1.08^{16}$ = awrt 68500

M1: $(n=16)\Rightarrow P=20\,000\times1.08^{15}$=awrt 63400 **and** $(n=17)\Rightarrow P=20\,000\times1.08^{16}$ = awrt 68500

A1: 17 years following correct method and both M's

(c)

M1: Attempts to use the model with a **correct** sum formula to find the total profit for the 20 years.

You may see an attempt to find the sum of 20 terms via a list. This is acceptable provided there are 20 terms with $u_n=1.08\times u_{n-1}$ seen at least 4 times and the sum attempted.

Condone a slip on the 20 000 (e.g appearing as 2 000) and/or a slip on the 1.08 with it being the same "*r*" as in (b) . Do not condone 20 appearing as 19 for instance

A1: awrt £915 000 but condone missing unit

The demand of the question dictates that all stages of working should be seen. An answer without working scores M0 A0

1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
1(a)	$16+(21-1)\times d=24 \Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\} \Rightarrow S_{500}=\frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$=57\,900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times "0.4"=215.6 \Rightarrow S_{500}=\frac{1}{2}\times 500\{16+"215.6"\}$	M1	1.1b
	$=57\,900$	A1	1.1b
			(4 marks)

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using *a* = 16, and *n* = 21 and the 24. The method may be implied by their working.

If the AP term formula is quoted it must be correct, so use of e.g. $u_n=a+nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20}, \frac{4}{10}, \frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with *a* = 16, *n* = 500 and their numerical *d* from part (a)

If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their *l*

A1: Correct value

Note that some candidates are showing implied use of $u_n=a+nd$ by showing the following:

(a) $d=\frac{24-16}{21}=\frac{8}{21}$ (b) $S_{500}=\frac{1}{2}\times 500\left\{2\times 16+499\times\frac{8}{21}\right\}=55523.80952...$

This scores (a) M0A0 (b) M1A0

9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^{n} \cos(180n)^{\circ} = \frac{9}{28}$$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
9	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=...$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
		(3)	
	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=...$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-...\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=...$	M1	3.1a
	$=\frac{9}{28}*$	A1*	1.1b
			(3 marks)

Notes

B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be seen as part of them writing down the sequence but must be the **first** term.

M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a = \frac{9}{16}$ and $r = \pm\frac{3}{4}$

A1*: Correct proof

Alternative 1:

B1: Deduces the correct value for the sum to infinity (starting at n = 1) or the common ratio

M1: Calculates the required value by subtracting the first term from their sum to infinity

A1*: Correct proof

Alternative 2:

B1: Deduces the correct value of the **first** term or the common ratio.

M1: Splits the series into “odds” and “evens”, attempts the sum of both parts and calculates the required value by adding both sums

A1*: Correct proof

Alternative 3:

B1: Deduces the correct value of the **first** term

M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for “S”

A1*: Correct proof

13. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
13 (i)	States that $S = a + (a+d) + \dots\dots + (a + (n-1)d)$	B1	1.1a
	$S = \quad a + \quad . \quad (a+d) + \quad (a+(n-1)d)$ $S = (a+(n-1)d) + (a+(n-2)d) + \dots\dots + a$	M1	3.1a
	Reaches $2S = n \times (2a + (n-1)d)$ And so proves that $S = \frac{n}{2}[2a + (n-1)d]$ *	A1*	2.1
		(3)	
(ii)	(a) $S = 10 + 9.20 + 8.40 + \dots$		
	$64 = \frac{n}{2}(20 - 0.8(n-1))$ o.e	M1	3.1b
	$128 = 20n - 0.8n^2 + 0.8n$ $0.8n^2 - 20.8n + 128 = 0$ $n^2 - 26n + 160 = 0$ *	A1*	2.1
		(2)	
	(b) $n = 10, 16$	B1	1.1b
		(1)	
	(c) 10 weeks with a minimal correct reason. E.g. • He has saved up the amount by 10 weeks so he would not save for another 6 weeks • You would choose the smaller number • He starts saving negative amounts (in week 14) so 16 does not make sense	B1	2.3
		(1)	
			(7 marks)
Notes:			

(i)

B1: Correctly writes down an expression for the key terms S or S_n including $S =$ or $S_n =$

Allow a minimum of 3 correct terms including the first and last terms, and no incorrect terms.

Score for S or $S_n = a + (a+d) + \dots\dots + (a+(n-1)d)$ with + signs, not commas

If the series contains extra terms that should not be there E.g

$S = a + (a+d) + \dots\dots(\boldsymbol{a + nd}) + (a+(n-1)d)$ score B0

M1: For the key step in reversing the terms and adding the two series.

Look for a minimum of two terms, including a and $a+(n-1)d$, the series reversed with evidence of adding, for example $2S =$ Condone the extra incorrect terms (see above) appearing.

Can be scored when terms are separated by commas

A1*: Shows correct work (no errors) with all steps shown leading to given answer.

There should be no incorrect terms. A minimum of 3 terms should be shown in each sum

The solution below is a variation of this.

$S = a + (a+d) + ... + l$

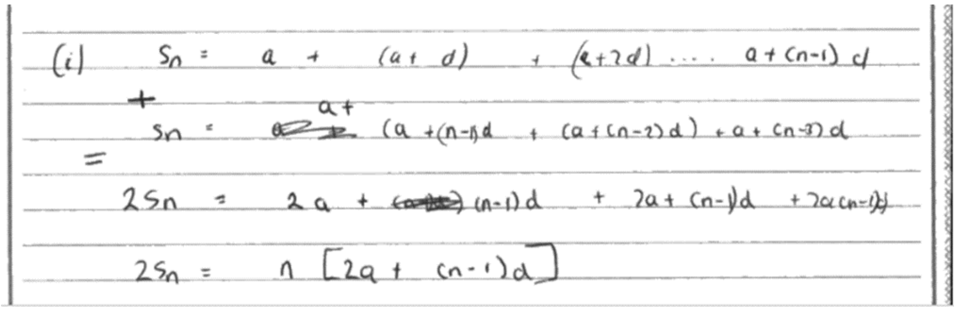
$S = l + (l-d) + ... + a$

$2S = n(a+l)$

$S = \frac{n}{2}(a+l) = \frac{n}{2}(a+a+(n-1)d) = \frac{n}{2}(2a+(n-1)d)$

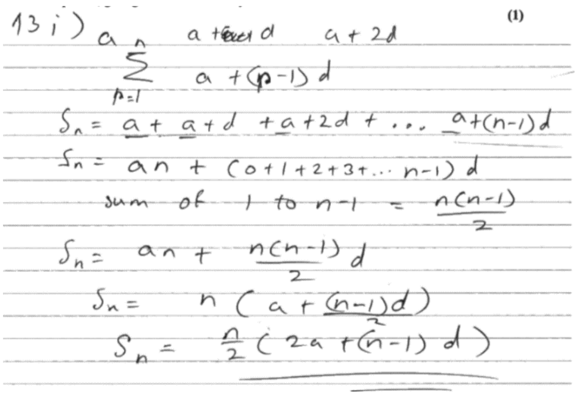
B1 and A1 are not scored until the last line, M scored on line 3

The following scores B1 M0 A0 as the terms in the second sum are not reversed


(i) $S_n = a + (a+d) + (a+2d) \dots a+(n-1)d$
$+$
$S_n = a + (a+(n-1)d + (a+(n-2)d) + a+(n-3)d$
$=$
$2S_n = 2a + (n-1)d + 2a+(n-1)d + 2a(n-1)d$
$2S_n = n[2a+(n-1)d]$

SC in (a) Scores B1 M0 A0.

They use $0+1+2+....+(n-1) = \frac{n(n-1)}{2}$ which relies on the quoted proof.


13 i) $a_n \quad a+d \quad a+2d$
$\sum_{p=1}^{n} a+(p-1)d$
$S_n = a + a+d + a+2d + \dots a+(n-1)d$
$S_n = an + (0+1+2+3+\dots n-1)d$
sum of 1 to $n-1 = \frac{n(n-1)}{2}$
$S_n = an + \frac{n(n-1)}{2}d$
$S_n = n\left(a + \frac{(n-1)d}{2}\right)$
$S_n = \frac{n}{2}(2a+(n-1)d)$

(ii) (a)

M1: Uses the information given to set up a correct equation in n.

The values of S, a and d need to be correct and used within a correct formula

Possible ways to score this include unsimplified versions $64 = \frac{n}{2}(2\times10+(n-1)\times-0.8)$,

$64 = \frac{n}{2}(10+10+(n-1)\times-0.8)$ or versions using pence rather than £'s $6400 = \frac{n}{2}(2000+(n-1)\times-80)$

Allow recovery for both marks following $64 = \frac{n}{2}(2\times10+(n-1)-0.8)$ with an invisible ×

A1*: Proceeds without error to the given answer. (Do not penalise a missing final trailing bracket)
Look for at least a line with the brackets correctly removed as well as a line with the terms in n correctly combined

E.g. $64 = \frac{n}{2}(20+(n-1)\times-0.8) \Rightarrow 64 = 10n - 0.4n^2 + 0.4n \Rightarrow 0.4n^2 - 10.4n + 64 = 0 \Rightarrow n^2 - 26n + 160 = 0$

(ii)(b)

B1: $n = 10, 16$

(ii)(c)

B1: Chooses 10 (weeks) and gives a minimal acceptable reason. The reason must focus on why the answer is 10 (weeks) rather than 16(weeks) or alternatively why it would not be 16 weeks.

3. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
3(a)(i)	$a_1 = 3,\ a_2 = 5,\ a_3 = 3\ ...$	B1	1.1b
(ii)	2	B1	1.1b
		(2)	
(b)	$\sum_{n=1}^{85} a_n = 42\times(3+5)+3$ o.e.	M1	3.1a
	$= 339$	A1	1.1b
		(2)	
			(4 marks)
Notes:			

(a)(i) Mark (a)(i) and (a)(ii) together.

B1: States the values of at least $a_2 = 5$ and $a_3 = 3$. This is sufficient but if more terms are given they must be correct. There is no need to see e.g. $a_2 = ...,\ a_3 = ...$ just look for values.

Allow an algebraic approach e.g. $a_{n+1} = 8 - a_n,\ a_{n+2} = 8-(8-a_n) = a_n$

A conclusion is **not** needed.

(a)(ii)

B1: States that the order of the periodic sequence is 2

Allow "second order", "it repeats every 2 numbers" or equivalent statements that convey the idea of the period being 2.

Note that ±2 is B0

(b)

M1: Attempts a **correct** method to find $\sum_{n=1}^{85} a_n$

For example $\sum_{n=1}^{85} a_n = 42\times(3+5)+3$, $\sum_{n=1}^{85} a_n = \frac{84}{2}\times 3 + 42\times 5 + 3$ or $\sum_{n=1}^{85} a_n = 43\times(3+5)-5$

or $\sum_{n=1}^{85} a_n = 43\times 3 + 42\times 5$ or $\sum_{n=1}^{85} a_n = \frac{85}{2}\times 8 - 1$

There may be other methods e.g. "Chunking": 5×(3 + 5) = 40, 40 × 8 = 320, 320 + 3×3 +2×5 = 339

A1: 339. Correct answer only scores both marks.

Attempts to use an AP formula score M0

15. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5 + 2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

(3)

Given that θ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of θ

(2)

(c) show that the sum to infinity of the series can be expressed in the form

$$k\left(1 - \sqrt{3}\right)$$

where k is a constant to be found.

(5)

P 6 9 6 0 2 A 0 4 0 4 8

Question	Scheme	Marks	AOs
15(a)	Uses the common ratio $\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{6\tan\theta}{5+2\sin\theta}$ o.e.	M1	3.1a
	Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\times12\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(a) Alt	**(a) Alternative example:**		
	Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ $\Rightarrow 12\cos\theta\left(\dfrac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$	M1	3.1a
	Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(b)	$4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\dfrac{1}{2}\left(,\dfrac{25}{2}\right)$	M1	1.1b
	$\theta=\dfrac{5\pi}{6}$	A1	1.2
		(2)	
(c)	Attempts a value for either a or r e.g. $a=12\cos\theta=\ 12\times-\dfrac{\sqrt{3}}{2}$ **or** $r=\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$	M1	3.1a
	$"a"=-6\sqrt{3}$ **and** $"r"=-\dfrac{1}{\sqrt{3}}$ o.e.	A1	1.1b
	Uses $S_\infty=\dfrac{a}{1-r}=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$	dM1	2.1
	Rationalises denominator $S_\infty=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\dfrac{-18}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$	ddM1	1.1b
	$(S_\infty=)9(1-\sqrt{3})$	A1	2.1
		(5)	
			(10 marks)

Notes:

(a)

M1: For the key step in using the ratio of $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}$

dM1: Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Alternative:

M1: Expresses the 2nd and 3rd terms in terms of the first term and the common ratio and eliminates "r"

dM1: Multiplies up and uses $\tan\theta \times \cos\theta = \sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

> Other approaches may be seen in (a) and can be marked in a similar way e.g. M1 for correctly obtaining an equation in θ using the GP, M1 for applying $\tan\theta \times \cos\theta = \sin\theta$ or equivalent and eliminating fractions, A1 as above
>
> Example: $u_2 = \frac{u_1 \times u_3}{u_2} \Rightarrow 5 + 2\sin\theta = \frac{12\cos\theta \times 6\tan\theta}{5 + 2\sin\theta}$ **M1**
>
> $\Rightarrow (5 + 2\sin\theta)^2 = 72\sin\theta$ **dM1**
>
> $25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$
>
> $\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0$ * **A1**

(b)

M1: Attempts to solve $4\sin^2\theta - 52\sin\theta + 25 = 0$. Must be clear they have found $\sin\theta$ and not e.g. just x from $4x^2 - 52x + 25 = 0$. Working does not need to be seen but see general guidance for solving a 3TQ if necessary. Note that the $\frac{25}{2}$ does not need to be seen.

A1: $\theta = \frac{5\pi}{6}$ and no other values unless they are rejected or the $\frac{5\pi}{6}$ clearly selected here and not in (c)

A minimum requirement in (b) is e.g. $\sin\theta = \frac{1}{2}, \quad \theta = \frac{5\pi}{6}$

Do **not** allow 150° for $\frac{5\pi}{6}$

PTO for the notes to part (c)

(c) Allow full marks in (c) if e.g. $\theta=\frac{\pi}{6}$ is their answer to (b) but $\theta=\frac{5\pi}{6}$ is used here.

or if e.g. $\theta=\frac{5\pi}{6}$ is their answer to (b) but $\theta=\frac{\pi}{6}$ is used here allow the M marks only.

M1: For attempting a value (exact or decimal) for either *a* or *r* using **their** θ

E.g. $a=12\cos\theta=\left(12\times-\frac{\sqrt{3}}{2}\right)$ or $r=\frac{5+2\sin\theta}{12\cos\theta}=\left(\frac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}\right)$ oe e.g. $r=\frac{6\tan\theta}{5+2\sin\theta}=\left(\frac{6\times-\frac{1}{\sqrt{3}}}{5+2\times\frac{1}{2}}\right)$

A1: Finds both $a=-6\sqrt{3}$ and $r=-\frac{1}{\sqrt{3}}$ which can be left unsimplified but $\sin\theta=\frac{1}{2},\cos\theta=-\frac{\sqrt{3}}{2}$

and $\tan\theta=-\frac{\sqrt{3}}{3}$ (if required) must have been used.

dM1: Uses both **values** of "*a*" and "*r*" with the equation $S_\infty=\frac{a}{1-r}=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ to create an expression

involving surds where *a* and *r* have come from appropriate work and $|r|<1$

Depends on the first method mark.

ddM1: Rationalises denominator. The denominator must be of the form $p\pm q\sqrt{3}$ oe e.g. $p+\frac{q}{\sqrt{3}}$

Depends on both previous method marks.

Note that stating e.g. $\frac{k}{p+q\sqrt{3}}\times\frac{p-q\sqrt{3}}{p-q\sqrt{3}}$ or $\frac{k}{p+\frac{q}{\sqrt{3}}}\times\frac{p-\frac{q}{\sqrt{3}}}{p-\frac{q}{\sqrt{3}}}$ is sufficient.

A1: Obtains $(S_\infty=)9\left(1-\sqrt{3}\right)$

Note that full marks are available in (c) for the use of $\theta=150°$

Note also that marks may be implied in (c) by e.g.

$$S_\infty=\frac{a}{1-r}=\frac{12\cos\theta}{1-\frac{5+2\sin\theta}{12\cos\theta}}=\frac{144\cos^2\theta}{12\cos\theta-5-2\sin\theta}=\frac{144\cos^2\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}-5-2\sin\frac{5\pi}{6}}$$

$$=\frac{108}{-6-6\sqrt{3}}=\frac{108}{-6-6\sqrt{3}}\times\frac{-6+6\sqrt{3}}{-6+6\sqrt{3}}=\frac{-648+648\sqrt{3}}{-72}=9\left(1-\sqrt{3}\right)$$

Scores M1A1 implied dM1 ddM1 A1

See next page for some other cases in (c) and how to mark them:

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} \quad \text{or e.g.} \quad S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}}$$

And nothing else

scores M1A0dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}} = 9\left(1+\sqrt{3}\right)$$

Scores M1A0dM1ddM0A0

$S_\infty = 9\left(1-\sqrt{3}\right)$ with no working scores no marks

4. The discrete random variable D has the following probability distribution

d	10	20	30	40	50
$\mathrm{P}(D = d)$	$\frac{k}{10}$	$\frac{k}{20}$	$\frac{k}{30}$	$\frac{k}{40}$	$\frac{k}{50}$

where k is a constant.

(a) Show that the value of k is $\frac{600}{137}$

(2)

The random variables D_1 and D_2 are independent and each have the same distribution as D.

(b) Find $\mathrm{P}(D_1 + D_2 = 80)$
Give your answer to 3 significant figures.

(3)

A single observation of D is made.

The value obtained, d, is the common difference of an arithmetic sequence.

The first 4 terms of this arithmetic sequence are the angles, measured in degrees, of quadrilateral Q

(c) Find the exact probability that the smallest angle of Q is more than 50°

(5)

Qu 4	Scheme	Marks	AO
(a)	$\frac{k}{10}+\frac{k}{20}+\frac{k}{30}+\frac{k}{40}+\frac{k}{50}=1$ or $\frac{1}{600}\left(60k+30k+20k+15k+12k\right)=1$	M1	1.1b
	So $k=\frac{600}{137}$ (*)	A1cso	1.1b
		(2)	
(b)	(Cases are:) $D_1=30, D_2=50$ and $D_1=50, D_2=30$ and $D_1=40, D_2=40$	M1	2.1
	$\text{P}\left(D_1+D_2=80\right)=\frac{k}{50}\times\frac{k}{30}\times 2 + \left(\frac{k}{40}\right)^2$	M1	3.4
	= 0.0375619… awrt **<u>0.0376</u>**	A1	1.1b
		(3)	
(c)	Angles are: a, $a+d$, $a+2d$, $a+3d$	M1	3.1a
	$S_4 = a + (a+d) + (a+2d) + (a+3d) = 360$	M1	2.1
	$2a + 3d = 180$ (o.e.)	A1	2.2a
	Smallest angle is $a > 50$ consider cases: $d = 10$ so $a = 75$ <u>or</u> $d = 20$ so $a = 60$ [$d = 30$ gives $a = 45$ no good]	M1	3.1b
	P(D = 10 or 20) = $\frac{3k}{20} = \frac{90}{137}$	A1	1.1b
		(5)	
		(10 marks)	

	Notes
(a)	M1 for clear use of sum of probabilities = 1 (all terms seen) A1 cso (*) M1 scored and no incorrect working seen.
Verify	**(Assume $k = \frac{600}{137}$)** to score the final A1 they must have a <u>final</u> comment "$\therefore k = \frac{600}{137}$"
(b)	1st M1 for selecting at least 2 of the relevant cases (may be implied by their correct probs) e.g. allow 30, 50 and 50,30 i.e. D_1 and D_2 labels not required 2nd M1 for using the model to obtain a correct expression for two different probabilities. May use letter k or their value for k. Allow for $\frac{k}{50}\times\frac{k}{30} + \left(\frac{k}{40}\right)^2$ <u>or</u> $2\times\left(\frac{k}{50}\times\frac{k}{30} + \left(\frac{k}{40}\right)^2\right)$ A1 for awrt 0.0376 (exact fraction is $\frac{705}{18769}$)
(c)	1st M1 for recognising the 4 angles and finding expressions in terms of d and their a 2nd M1 for using property of quad with these 4 angles (equation can be un-simplified) Allow these two marks for use of a (possible) value of d e.g. $a + a + 10 + a + 20 + a + 30 = 360$ (If at least 3 cases seen allow A1 for e.g. $4a = 300$) <u>or</u> allow M1M1 for a set of 4 angles with sum 360 and possible value of d (3 cases for A1) e.g. (for $d = 20$) 60, 80, 100, 120 1st A1 for $2a + 3d = 180$ condition (o.e.) [Must be in the form $pa + qd = N$] 3rd M1 for examining cases and getting $d = 10$ and $d = 20$ only 2nd A1 for $\frac{90}{137}$ or exact equivalent The correct answer and no obviously incorrect working will score 5/5 A final answer of awrt 0.657 (0.65693…) with no obviously incorrect working scores 4/5