Y2P2 XMQs and MS

(Total: 110 marks)

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 1. P2_Sample    Q4 .  5 marks - Y2P2 Functions and graphs

 2. P2_Sample    Q11.  6 marks - Y2P2 Functions and graphs

 3. P1_Specimen  Q10. 13 marks - Y2P2 Functions and graphs

 4. P2_Specimen  Q4 .  6 marks - Y2P2 Functions and graphs

 5. P2_2018      Q1 .  6 marks - Y2P2 Functions and graphs

 6. P2_2018      Q3 .  5 marks - Y2P2 Functions and graphs

 7. P1_2019      Q5 . 10 marks - Y1P2 Quadratics

 8. P1_2019      Q10.  6 marks - Y1P7 Algebraic methods

 9. P2_2019      Q6 . 10 marks - Y2P2 Functions and graphs

10. P1_2020      Q4 .  5 marks - Y2P2 Functions and graphs

11. P2_2020      Q11.  7 marks - Y2P2 Functions and graphs

12. P2_2021      Q2 .  5 marks - Y2P2 Functions and graphs

13. P2_2021      Q11. 10 marks - Y2P2 Functions and graphs

14. P1_2022      Q1 .  4 marks - Y1P4 Graphs and transformations

15. P2_2022      Q1 .  4 marks - Y2P2 Functions and graphs

16. P2_2022      Q10.  8 marks - Y2P2 Functions and graphs
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4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

(Total for Question 4 is 5 marks)

Question	Scheme	Marks	AOs
4 (a)	$\mathrm{gf}(x) = 3\ln e^{x}$	M1	1.1b
	$= 3x, \ (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$\mathrm{gf}(x) = \mathrm{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln e^{x}$
An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $e^{3\ln x} =$ their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$
or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

11.

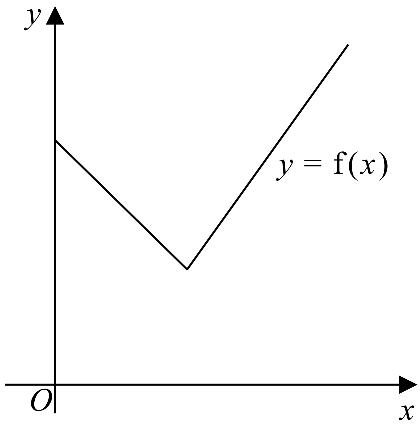


Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

Question	Scheme	Marks	AOs
11 (a)	$f(x) \geqslant 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$	M1	1.1b
	$x=\frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \leqslant 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5 < k \leqslant 11\}$	A1	2.5
		(2)	
		(6 marks)	

Notes:

(a)

B1: $f(x) \geqslant 5$ Also allow $f(x) \in [5, \infty)$

(b)

M1: Deduces that the solution to $f(x)=\frac{1}{2}x+30$ can be found by solving

$$-2(3-x)+5=\frac{1}{2}x+30$$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms

A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(c)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \leqslant 11$

A1: Correct solution only $\{k : k \in \mathbb{R}, 5 < k \leqslant 11\}$

10. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3x-5}{x+1}, \quad x \in \mathbb{R}, \ x \neq -1$$

(a) Find $\mathrm{f}^{-1}(x)$.

(3)

(b) Show that

$$\mathrm{ff}(x) = \frac{x+a}{x-1}, \quad x \in \mathbb{R}, \ x \neq \pm 1$$

where a is an integer to be found.

(4)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leqslant x \leqslant 5$$

(c) Find the value of $\mathrm{fg}(2)$.

(2)

(d) Find the range of g.

(3)

(e) Explain why the function g does not have an inverse.

(1)

Question	Scheme	Marks	AOs
10 (a)	$y = \dfrac{3x-5}{x+1} \Rightarrow y(x+1) = 3x-5 \Rightarrow xy + y = 3x-5 \Rightarrow y+5 = 3x - xy$	M1	1.1b
	$\Rightarrow y+5 = x(3-y) \Rightarrow \dfrac{y+5}{3-y} = x$	M1	2.1
	Hence $f^{-1}(x) = \dfrac{x+5}{3-x}$, $x \in \mathbb{R},\ x \neq 3$	A1	2.5
		(3)	
(b)	$ff(x) = \dfrac{3\left(\dfrac{3x-5}{x+1}\right)-5}{\left(\dfrac{3x-5}{x+1}\right)+1}$	M1	1.1a
	$= \dfrac{\dfrac{3(3x-5)-5(x+1)}{x+1}}{\dfrac{(3x-5)+(x+1)}{x+1}}$	M1	1.1b
		A1	1.1b
	$= \dfrac{9x-15-5x-5}{3x-5+x+1} = \dfrac{4x-20}{4x-4} = \dfrac{x-5}{x-1}$ (note that $a = -5$)	A1	2.1
		(4)	
(c)	$fg(2) = f(4-6) = f(-2) = \dfrac{3(-2)-5}{-2+1}$;$= 11$	M1	1.1b
		A1	1.1b
		(2)	
(d)	$g(x) = x^2 - 3x = (x-1.5)^2 - 2.25$. Hence $g_{min} = -2.25$	M1	2.1
	Either $g_{min} = -2.25$ or $g(x) \geqslant -2.25$ or $g(5) = 25 - 15 = 10$	B1	1.1b
	$-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$	A1	1.1b
		(3)	
(e)	E.g. • the function g is many-one • the function g is not one-one • the inverse is one-many • $g(0) = g(3) = 0$	B1	2.4
		(1)	
			(13 marks)

Question 10 Notes:	
(a)	
M1:	Attempts to find the inverse by cross-multiplying and an attempt to collect all the *x*-terms (or swapped *y*-terms) onto one side
M1:	A fully correct method to find the inverse
A1:	A correct $f^{-1}(x) = \frac{x+5}{3-x}$, $x \in \mathbb{R}$, $x \neq 3$, expressed fully in function notation (including the domain)
(b)	
M1:	Attempts to substitute $f(x) = \frac{3x-5}{x+1}$ into $\frac{3f(x)-5}{f(x)+1}$
M1:	Applies a method of "rationalising the denominator" for both their numerator and their denominator.
A1:	$\frac{\frac{3(3x-5)-5(x+1)}{x+1}}{\frac{(3x-5)+(x+1)}{x+1}}$ which can be simplified or un-simplified
A1:	Shows $ff(x) = \frac{x+a}{x-1}$ where $a = -5$ or $ff(x) = \frac{x-5}{x-1}$, with no errors seen.
(c)	
M1:	Attempts to substitute the result of $g(2)$ into f
A1:	Correctly obtains $fg(2) = 11$
(d)	
M1:	Full method to establish the minimum of g. E.g. • $(x \pm \alpha)^2 + \beta$ leading to $g_{min} = \beta$ • Finds the value of *x* for which $g'(x) = 0$ and inserts this value of *x* back into $g(x)$ in order to find to g_{min}
B1:	For either • finding the correct minimum value of g (Can be implied by $g(x) \geqslant -2.25$ or $g(x) > -2.25$) • stating $g(5) = 25 - 15 = 10$
A1:	States the correct range for g. E.g. $-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$
(e)	
B1:	See scheme

4. (a) Sketch the graph with equation

$$y = |2x - 5|$$

stating the coordinates of any points where the graph cuts or meets the coordinate axes.

(2)

(b) Find the values of x which satisfy

$$|2x - 5| > 7$$

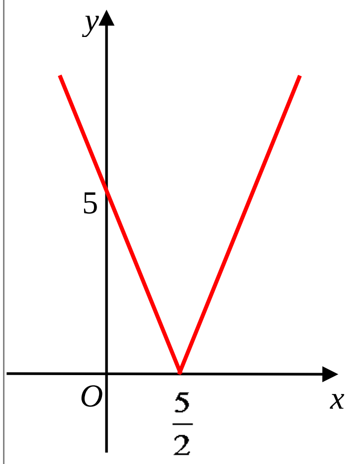
(2)

(c) Find the values of x which satisfy

$$|2x - 5| > x - \frac{5}{2}$$

Write your answer in set notation.

(2)

Question	Scheme	Marks	AOs
4(a)	 Correct graph in quadrant 1 and quadrant 2 with V on the x-axis	B1	1.1b
	States $(0, 5)$ and $\left(\frac{5}{2}, 0\right)$ **or** $\frac{5}{2}$ marked in the correct position on the x-axis **and** 5 marked in the correct position on the y-axis	B1	1.1b
		(2)	
(b)	$\lvert 2x-5 \rvert > 7$		
	$2x-5=7 \Rightarrow x=...$ **and** $-(2x-5)=7 \Rightarrow x=...$	M1	1.1b
	{critical values are $x=6, -1 \Rightarrow$ } $x<-1$ or $x>6$	A1	1.1b
		(2)	
(c)	$\lvert 2x-5 \rvert > x-\frac{5}{2}$		
	E.g. • Solves $2x-5=x-\frac{5}{2}$ to give $x=\frac{5}{2}$ and solves $-(2x-5)=x-\frac{5}{2}$ to also give $x=\frac{5}{2}$ • Sketches graphs of $y=\lvert 2x-5 \rvert$ and $y=x-\frac{5}{2}$. Indicates that these graphs meet at the point $\left(\frac{5}{2}, 0\right)$	M1	3.1a
	Hence using set notation, e.g. • $\left\{x: x<\frac{5}{2}\right\} \cup \left\{x: x>\frac{5}{2}\right\}$ • $\left\{x \in \mathbb{R}, x \neq \frac{5}{2}\right\}$ • $\mathbb{R} - \left\{\frac{5}{2}\right\}$	A1	2.5
		(2)	
	(6 marks)		

Question 4 Notes:	
(a)	
B1:	See scheme
B1:	See scheme
(b)	
M1:	See scheme
A1:	Correct answer, e.g. • $x < -1$ or $x > 6$ • $x < -1 \cup x > 6$ • $\{x: x < -1\} \cup \{x: x > 6\}$
(c)	
M1:	A complete process of finding that $y = \lvert 2x - 5 \rvert$ and $y = x - \frac{5}{2}$ meet at ***only*** one point. This can be achieved either algebraically or graphically.
A1:	See scheme. **Note:** Final answer must be expressed using set notation.

Answer ALL questions. Write your answers in the spaces provided.

1.

$$g(x) = \frac{2x + 5}{x - 3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

Question	Scheme	Marks	AOs
1	$\mathrm{g}(x)=\dfrac{2x+5}{x-3},\ x\geq 5$		
(a) **Way 1**	$\mathrm{g}(5)=\dfrac{2(5)+5}{5-3}=7.5\ \Rightarrow\ \mathrm{gg}(5)=\dfrac{2(\text{"}7.5\text{"})+5}{\text{"}7.5\text{"}-3}$	M1	1.1b
	$\mathrm{gg}(5)=\dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9}\ \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) **Way 2**	$\mathrm{gg}(x)=\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}\ \Rightarrow\ \mathrm{gg}(5)=\dfrac{2\left(\dfrac{2(5)+5}{(5)-3}\right)+5}{\left(\dfrac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$\mathrm{gg}(5)=\dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9}\ \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	{Range:} $2 < y \leq \dfrac{15}{2}$	B1	1.1b
		(1)	
(c) **Way 1**	$y=\dfrac{2x+5}{x-3}\Rightarrow\ yx-3y=2x+5\Rightarrow yx-2x=3y+5$	M1	1.1b
	$x(y-2)=3y+5\ \Rightarrow\ x=\dfrac{3y+5}{y-2}\ \left\{\text{or } y=\dfrac{3x+5}{x-2}\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x)=\dfrac{3x+5}{x-2},\quad 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
(c) **Way 2**	$y=\dfrac{2x-6+11}{x-3}\ \Rightarrow\ y=2+\dfrac{11}{x-3}\ \Rightarrow y-2=\dfrac{11}{x-3}$	M1	1.1b
	$x-3=\dfrac{11}{y-2}\ \Rightarrow\ x=\dfrac{11}{y-2}+3\ \left\{\text{or } y=\dfrac{11}{x-2}+3\right\}$	M1	2.1
	$\mathrm{g}^{-1}(x)=\dfrac{11}{x-2}+3,\quad 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
		(6 marks)	

Notes for Question 1

(a)	
M1:	Full method of attempting $\mathrm{g}(5)$ and substituting the result into g
Note:	**Way 2:** Attempts to substitute $x=5$ into $\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}$, o.e. Note that $\mathrm{gg}(x)=\dfrac{9x-5}{14-x}$
A1:	Obtains $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or 4.444... without reference to $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$

Notes for Question 1 Continued	
(b)	
B1:	States $2 < y \leq \frac{15}{2}$ Accept any of $2 < \mathrm{g} \leq \frac{15}{2}$, $2 < \mathrm{g}(x) \leq \frac{15}{2}$, $\left(2, \frac{15}{2}\right]$
Note:	Accept $\mathrm{g}(x) > 2$ **and** $\mathrm{g}(x) \leq \frac{15}{2}$ o.e.
(c) Way 1	
M1:	Correct method of cross multiplication followed by an attempt to collect terms in *x* or terms in a swapped *y*
M1:	A complete method (i.e. as above and also factorising and dividing) to find the inverse
A1ft:	Uses correct notation to correctly define the inverse function g^{-1} , where the domain of g^{-1} stated correctly or correctly followed through (using correct notation) on the values shown in their range in part (b). Allow $\mathrm{g}^{-1} : x \rightarrow$. Condone $\mathrm{g}^{-1} = ...$ Do not accept $y = ...$
Note:	Correct notation is required when stating the domain of $\mathrm{g}^{-1}(x)$. Allow $2 < x \leq \frac{15}{2}$ or $\left(2, \frac{15}{2}\right]$ Do not allow any of e.g. $2 < \mathrm{g} \leq \frac{15}{2}$, $2 < \mathrm{g}^{-1}(x) \leq \frac{15}{2}$
Note:	Do not allow A1ft for following through their range in (b) to give a domain for g^{-1} as $x \in \mathbb{R}$
(c) Way 2	
M1:	Writes $y = \frac{2x+5}{x-3}$ in the form $y = 2 \pm \frac{k}{x-3}$, $k \neq 0$ and rearranges to isolate *y* and 2 on one side of their equation. **Note:** Allow the equivalent method with *x* swapped with *y*
M1:	A complete method to find the inverse
A1ft:	As in Way 1
Note:	If a candidate scores no marks in part (c), but • states the domain of g^{-1} correctly, **or** • states a domain of g^{-1} which is correctly followed through on the values shown in their range in part (b) then give special case (SC) M1 M0 A0

3. (a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

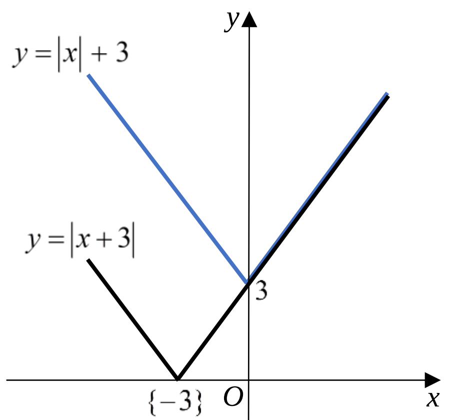
Disprove this statement by means of a counter example.

(2)

(b) (i) Sketch the graph of $y = |x| + 3$

(ii) Explain why $|x| + 3 \geqslant |x + 3|$ for all real values of x.

(3)

Question	Scheme	Marks	AOs
3	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."		
(a)	E.g. $m = \sqrt{3},\ n = \sqrt{12}$	M1	1.1b
	$\{mn =\}\ \left(\sqrt{3}\right)\left(\sqrt{12}\right) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational or 6 is rational	A1	2.4
		(2)	
(b)(i), (ii) Way 1	 V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret (0, 3) or 3 stated or marked on the positive y-axis	B1	1.1b
	Superimposes the graph of $y = \lvert x+3 \rvert$ on top of the graph of $y = \lvert x \rvert + 3$	M1	3.1a
	the graph of $y = \lvert x \rvert + 3$ is either the same or above the graph of $y = \lvert x+3 \rvert$ {for corresponding values of x} **or** when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \lvert x \rvert + 3$ is above the graph of $y = \lvert x+3 \rvert$	A1	2.4
		(3)	
(b)(ii) Way 2	**<u>Reason 1</u>** When $x \geq 0$, $\lvert x \rvert + 3 = \lvert x+3 \rvert$ **<u>Reason 2</u>** When $x < 0$, $\lvert x \rvert + 3 > \lvert x+3 \rvert$ Any one of Reason 1 or Reason 2	M1	3.1a
	Both Reason 1 and Reason 2	A1	2.4
		(5 marks)	

Notes for Question 3

(a)	
M1:	States or uses any pair of ***different*** numbers that will disprove the statement. E.g. $\sqrt{3},\ \sqrt{12}$; $\sqrt{2},\ \sqrt{8}$; $\sqrt{5},\ -\sqrt{5}$; $\frac{1}{\pi},\ 2\pi$; $3\text{e},\ \frac{4}{5\text{e}}$;
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion
Note:	Writing $(3\text{e})\left(\frac{4}{5\text{e}}\right) = \frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1
(b)(i)	
B1:	See scheme
(b)(ii)	
M1:	For constructing a method of comparing $\lvert x \rvert + 3$ with $\lvert x+3 \rvert$. See scheme.
A1:	Explains fully why $\lvert x \rvert + 3 \geq \lvert x+3 \rvert$. See scheme.
Note:	Do not allow either $x > 0$, $\lvert x \rvert + 3 \geq \lvert x+3 \rvert$ or $x \geq 0$, $\lvert x \rvert + 3 \geq \lvert x+3 \rvert$ as a valid reason
Note	$x = 0$ (or where necessary $x = -3$) need to be considered in their solutions for A1
Note:	Do not allow an incorrect statement such as $x \leq 0$, $\lvert x \rvert + 3 > \lvert x+3 \rvert$ for A1

Notes for Question 3 Continued	
(b)(ii)	
Note:	Allow M1A1 for $x > 0,\ |x| + 3 = |x + 3|$ and for $x \le 0,\ |x| + 3 \ge |x + 3|\geqslant$
Note:	Allow M1 for any of • x is positive, $|x| + 3 = |x + 3|$ • x is negative, $|x| + 3 > |x + 3|$ • $x > 0,\ |x| + 3 = |x + 3|$ • $x \le 0,\ |x| + 3 \ge |x + 3|$ • $x > 0,\ |x| + 3$ and $|x + 3|$ are equal • $x \ge 0,\ |x| + 3$ and $|x + 3|$ are equal • when $x \ge 0$, both graphs are equal • for positive values $|x| + 3$ and $|x + 3|$ are the same Condone for M1 • $x \le 0,\ |x| + 3 > |x + 3|$ • $x < 0,\ |x| + 3 \ge |x + 3|$

(b)(ii) **Way 3**	• For $x > 0,\ |x| + 3 = |x + 3|$ • For $-3 < x < 0$, as $|x| + 3 > 3$ and $\{0 <\}\ |x + 3| < 3$, then $|x| + 3 > |x + 3|$	M1	3.1a
	• For $x \le -3$, as $|x| + 3 = -x + 3$ and $|x + 3| = -x - 3$, then $|x| + 3 > |x + 3|$	A1	2.4

5.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

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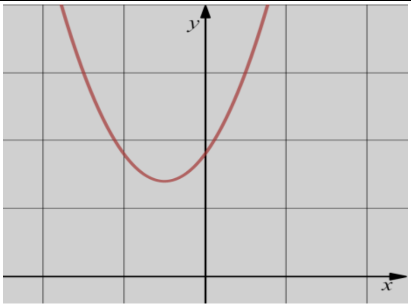
Question	Scheme	Marks	AOs
5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $a=2\ \&b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1	2.2a
	(ii) $\text{h}(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\text{h}(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2\pm....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2\pm...$or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

(b)

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y - intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix}2\\-4\end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $g(x)=f(x-2)-4$ can score M1

For example, possible methods of arriving at this deduction are:

- $f(x)\to g(x)$ is $2x^2+4x+9\to 2(x-2)^2+4(x-2)+5$ So $g(x)=f(x-2)-4$
- $g(x)=2(x-1)^2+3$ New curve has its minimum at $(1,3)$ so $(-1,7)\to(1,3)$
- Using a graphical calculator to sketch $y=g(x)$ and compares to the sketch of $y=f(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations. Eg, stretch and .....

A1: Requires both 'translate' and '$\begin{pmatrix}2\\-4\end{pmatrix}$' Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix}0\\-4\end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix}-2\\4\end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $h(x)=\dfrac{21}{"2(x+1)^2+7"}$ and attempts to find $\dfrac{21}{\text{their "7"}}$
- Attempts to differentiate, sets $4x+4=0\to x=-1$ and substitutes into $h(x)=\dfrac{21}{2x^2+4x+9}$
- Uses a graphical calculator to sketch $y=h(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0<h(x)\leqslant 3$ Allow for $0<h\leqslant 3$ $(0,3]$ and $0<y\leqslant 3$ but not $0<x\leqslant 3$

Follow through on their $a(x+b)^2+c$ so award for $0<h(x)\leqslant \dfrac{21}{c}$

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

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P 5 8 3 5 3 A 0 2 8 4 4

Question 10

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

All $n \in \mathbb{N}$ mod 4	0	1	2	3
All $n^2 \in \mathbb{N}$ mod 4	0	1	0	1
All $n^2+2 \in \mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Question 10 (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up
(i**)**
M1: Awarded for setting up the proof for either the even or odd numbers.
A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either *n* odd or even.
dM1: Awarded for setting up the proof for both even and odd numbers
A1: Fully correct proof with valid explanation and conclusion for all *n*

Example of an algebraic proof

For $n=2m,\quad n^2+2=4m^2+2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n=2m+1,\quad n^2+2=(2m+1)^2+2=\ldots$ FYI $\left(4m^2+4m+3\right)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4\left(m^2+m\right)+3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\dfrac{4m^2+2}{4} = m^2 + \dfrac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m+1$, $\dfrac{n^2+2}{4} = \dfrac{4m^2+4m+3}{4} = m^2 + m + \dfrac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{3}{4}$ ...AND states ...... hence for all n, n^2+2 is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$odd" = odd	M1	2.1
so n^2+2 is odd , so (when n is odd) n^2+2 cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$even" is a multiple of 4	dM1	2.1
Concludes that when n is even n^2+2 cannot be divisible by 4 because n^2 is divisible by 4…..AND STATES ……..trues for all n.	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction ‘Assume that n^2+2 is divisible by 4 $\Rightarrow n^2+2=4k$ ’	M1	2.1
$\Rightarrow n^2 = 4k-2 = 2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then n^2+2 cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

A similar proof exists via contradiction where

A1: $n^2 = 2(2k-1) \Rightarrow n = \sqrt{2} \times \sqrt{2k-1}$

dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that n is irrational

Question 10 (ii)	Scheme	Marks	AOs

(ii)

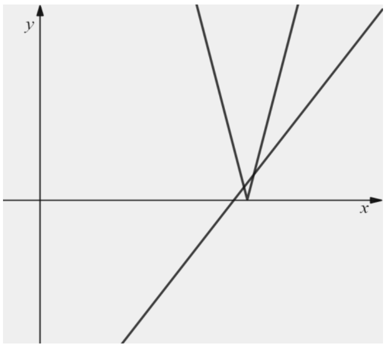
M1: States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.

A1: and gives an example where it is true,

Proof using numerical values

SOMETIMES TRUE and chooses any number $x: 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $\|3x-28\| = 0.2$ and $x - 9 = 0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x = 12$ $\|3x-28\| = 8$ $x - 9 = 3$ ✓	A1	2.4
	(2)	

Graphical Proof

 States or implies ''sometimes true'' Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x -axis. Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $\|3x-28\| < x - 9$ and points where $\|3x-28\| > x - 9$ oe in words like 'above' and 'below' or 'dips below at one point'	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x - 28 < x - 9$ and $-3x + 28 < x - 9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true. Solves both $3x - 28 \geqslant x - 9$ and $-3x + 28 \geqslant x - 9$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

6.

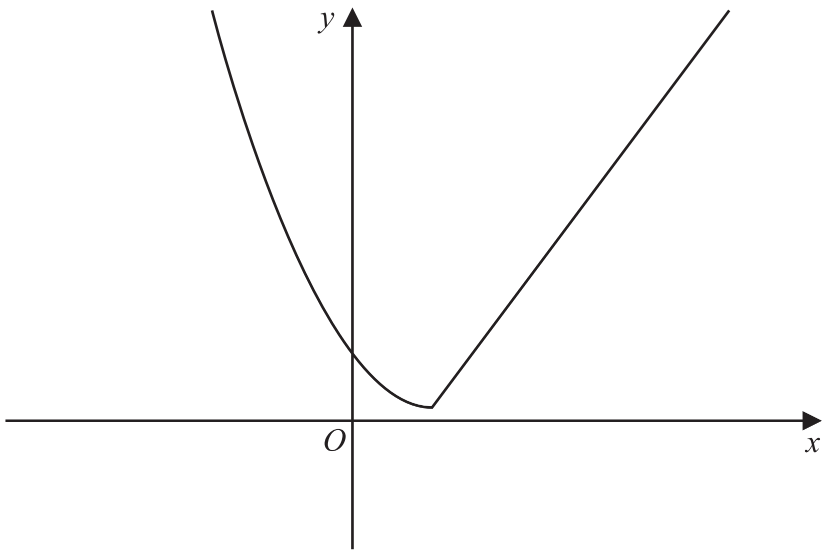


Figure 4

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which

$$g(x) > 28$$

(4)

The function h is defined by

$$h(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

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Question	Scheme	Marks	AOs
6 (a)	$\text{gg}(0) = \text{g}((0-2)^2+1) = \text{g}(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x=...$ or $4x-7=28 \Rightarrow x=...$	M1	1.1b
	At least one critical value $x=2-3\sqrt{3}$ or $x=\frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x=...$ and $4x-7=28 \Rightarrow x=...$	M1	1.1b
	Correct final answer of '$x<2-3\sqrt{3}$, $x>\frac{35}{4}$'	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	**(4)**	
(c)	<u>h is a one-one</u> {function (or mapping) so has an inverse} <u>g is a many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{\text{h}^{-1}(x) = -\frac{1}{2} \Rightarrow\right\}$ $x = \text{h}\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x=\left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x=\left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
(d) Way 2	{their $\text{h}^{-1}(x)$ } $= \pm 2 \pm \sqrt{x \pm 1}$	M1	1.1b
	Attempts to solve $\pm 2 \pm \sqrt{x \pm 1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x \pm 1} = ...$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
			(10 marks)

Notes for Question 6

(a)	
M1:	Uses a complete method to find $\text{gg}(0)$. E.g. • Substituting $x=0$ into $(0-2)^2+1$ and the result of this into the relevant part of $\text{g}(x)$ • Attempts to substitute $x=0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$\text{gg}(0)=13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x=-1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x \in \mathbb{R}: x<2-3\sqrt{3} \cup x>8.75\}$ is fine for the final A mark

Notes for Question 6 Continued	
(b)	***continued***
Note:	Give final A0 for $\{ x \in \mathbb{R}: x < 2 - 3\sqrt{3} \cap x > 8.75\}$
Note:	Give final A0 for $2 - 3\sqrt{3} > x > 8.75$
Note:	Allow final A1 for their writing a final answer of "$x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$"
Note:	Allow final A1 for a final answer of $x < 2 - 3\sqrt{3},\ x > \frac{35}{4}$
Note:	Writing $2 - \sqrt{27}$ in place of $2 - 3\sqrt{3}$ is accepted for any of the A marks
Note:	Allow final A1 for a final answer of $x < -3.20,\ x > 8.75$
Note:	Using 29 instead of 28 is M0 A0 M0 A0
(c)	
B1:	A correct explanation that conveys the underlined points
Note:	A minimal acceptable reason is "h is a one-one and g is a many-one"
Note:	Give B1 for "h^{-1} is one-one and g^{-1} is one-many"
Note:	Give B1 for "h is a one-one and g is not"
Note:	Allow B1 for "g is a many-one and h is not"
(d)	**Way 1**
M1:	Writes $x = \mathrm{h}\left(-\frac{1}{2}\right)$
M1:	See scheme
A1:	Uses $x = \mathrm{h}\left(-\frac{1}{2}\right)$ to deduce that $x = 7.25$ only, **cso**
(d)	**Way 2**
M1:	See scheme
M1:	See scheme
A1:	Use a correct $\mathrm{h}^{-1}(x) = 2 - \sqrt{x-1}$ to deduce that $x = 7.25$ only, **cso**
Note:	Give final A0 cso for $2 + \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A0 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A1 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow -\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Allow final A1 for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$

4. The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \dfrac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

Question	Scheme	Marks	AOs
4 (a)	Either attempts $\frac{3x-7}{x-2}=7 \Rightarrow x=...$ Or attempts $f^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x)=\frac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\frac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\frac{2x-7}{x-3}$	A1	2.1
		(3)	
			(5 marks)
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for *x*.

Or score for substituting in $x=7$ in $f^{-1}(x)$. FYI $f^{-1}(x)=\frac{2x-7}{x-3}$

The method for finding $f^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\frac{3x-7}{x-2}$ into $f(x)$. Condone slips but the expression must have a correct form. E.g. $\frac{3\times\left(\frac{*-*}{*-*}\right)-a}{\left(\frac{*-*}{*-*}\right)-b}$ where *a* and *b* are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\frac{P(x)}{Q(x)}\times\frac{x-2}{x-2}$

A1: Reaches $\frac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

....

Methods involving $\frac{3x-7}{x-2}\equiv a+\frac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\frac{3x-7}{x-2}\equiv 3-\frac{1}{x-2}$

11.

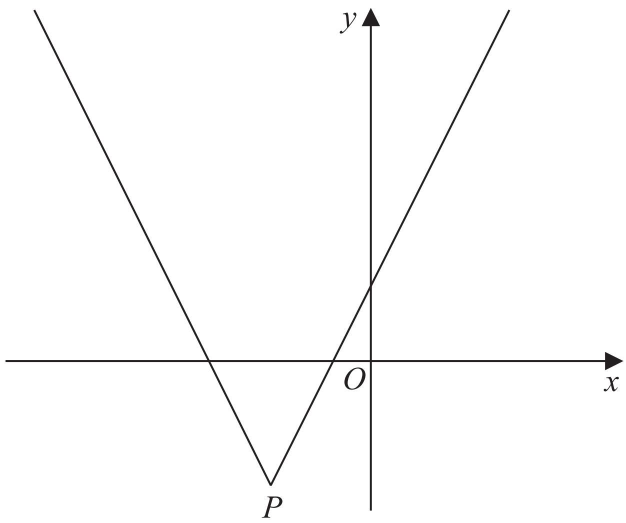

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

Question	Scheme	Marks	AOs
11(a)	$x = -4$ **or** $y = -5$	B1	1.1b
	$P(-4, -5)$	B1	2.2a
		(2)	
(b)	$3x + 40 = -2(x + 4) - 5 \Rightarrow x = \ldots$	M1	1.1b
	$x = -10.6$	A1	2.1
		(2)	
(c)	$a > 2$	B1	2.2a
	$y = ax \Rightarrow -5 = -4a \Rightarrow a = \frac{5}{4}$	M1	3.1a
	$\{a : a \leqslant 1.25\} \cup \{a : a > 2\}$	A1	2.5
		(3)	
			(7 marks)

<u>**Notes:**</u>

(a)

B1: One correct coordinate. Either $x = -4$ or $y = -5$ or $(-4, \ldots)$ or $(\ldots, -5)$ seen.

B1: Deduces that $P(-4, -5)$ Accept written separately e.g. $x = -4,\ y = -5$

(b)

M1: Attempts to solve $3x + 40 = -2(x+4) - 5 \Rightarrow x = \ldots$ Must reach a value for x.

You may see the attempt crossed out but you can still take this as an attempt to solve the required equation.

A1: $x = -10.6$ oe e.g. $-\frac{53}{5}$ only. If other values are given, e.g. $x = -37$ they must be rejected or the $-\frac{53}{5}$ clearly chosen as their answer. Ignore any attempts to find y.

Alternative by squaring:

$$3x + 40 = 2|x+4| - 5 \Rightarrow 3x + 45 = 2|x+4| \Rightarrow 9x^2 + 270x + 2025 = 4\left(x^2 + 8x + 16\right)$$

$$\Rightarrow 5x^2 + 238x + 1961 = 0 \Rightarrow x = -37, -\frac{53}{5}$$

M1 for isolating the $|x+4|$, squaring both sides and solving the resulting quadratic

A1 for selecting the $-\frac{53}{5}$

Correct answer with no working scores both marks.

(c)

B1: Deduces that $a > 2$

M1: Attempts to find a value for a using their $P(-4, -5)$

Alternatively attempts to solve $ax = 2(x + 4) - 5$ and $ax = 2(x + 4) - 5$ to obtain a value for a.

A1: Correct range in acceptable set notation.

Examples:

$\{a : a \leqslant 1.25\} \cup \{a : a > 2\}$

$\{a : a \leqslant 1.25\},\ \{a : a > 2\}$

$\{a : a \leqslant 1.25 \text{ or } a > 2\}$

$\{a : a \leqslant 1.25,\ a > 2\}$

$(-\infty, 1.25] \cup (2, \infty)$

$(-\infty, 1.25],\ (2, \infty)$

2. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find gf(1.8)

(2)

(c) Find $g^{-1}(x)$

(2)

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Question	Scheme	Marks	AOs
2(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$\mathrm{f}(1.8) = 7 - 2\times 1.8^2 = 0.52 \Rightarrow \mathrm{gf}(1.8) = \mathrm{g}(0.52) = \dfrac{3\times 0.52}{5\times 0.52 - 1} = \ldots$	M1	1.1b
	$\mathrm{gf}(1.8) = 0.975$ oe e.g. $\dfrac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(\mathrm{g}^{-1}(x) =\right)\dfrac{x}{5x-3}$	A1	2.2a
		(2)	
		(5 marks)	

Notes

(a)

B1: Correct range. Allow f (x) or f for y. Allow e.g. $\{y \in \mathbb{R}: y \leqslant 7\}, -\infty < y \leqslant 7, (-\infty, 7]$

(b)

M1: Full method to find f (1.8) and substitutes the result into g to obtain a value.
Also allow for an attempt to substitute x = 1.8 into an attempt at gf (x).

E.g. $\mathrm{gf}(x) = \dfrac{3(7-2x^2)}{5(7-2x^2)-1} = \dfrac{3\left(7-2(1.8)^2\right)}{5\left(7-2\times(1.8)^2\right)-1} = \ldots$

A1: Correct value

(c)

M1: Correct attempt to cross multiply, followed by an attempt to factorise out x from an xy term and an x term.
If they swap x and y at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out y from an xy term and a y term.

A1: Correct expression. Allow equivalent correct expressions e.g. $\dfrac{-x}{3-5x}, \dfrac{1}{5} + \dfrac{3}{25x-15}$

Ignore any domain if given.

11.

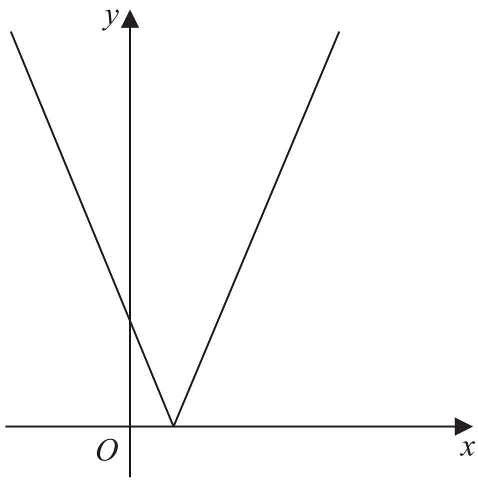


Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

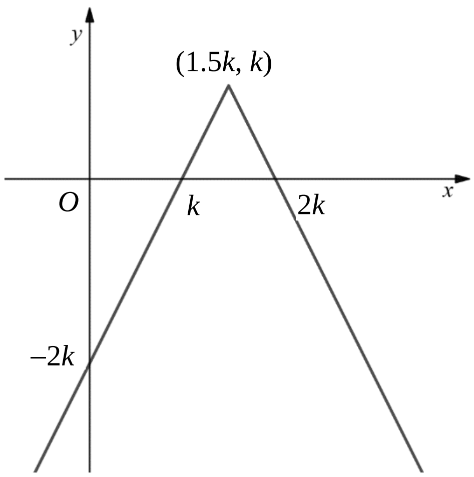
giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5\mathrm{f}\left(\frac{1}{2}x\right)$$

(2)

Question	Scheme	Marks	AOs
11(a)			
	∧ shape in any position	B1	1.1b
	Correct x-intercepts or coordinates	B1	1.1b
	Correct y-intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a ∧ shape	B1	1.1b
		(4)	
(b)	$x = k$	B1	2.2a
	$k-(2x-3k)=x-k \Rightarrow x=...$	M1	3.1a
	$x=\frac{5k}{3}$	A1	1.1b
	Set notation is required here for this mark $\left\{x : x<\frac{5k}{3}\right\}\cap\{x : x>k\}$	A1	2.5
		(4)	
(c)	$x = 3k$ **or** $y = 3 - 5k$	B1ft	2.2a
	$x = 3k$ **and** $y = 3 - 5k$	B1ft	2.2a
		(2)	
			(10 marks)

Notes

(a) **Note that the sketch may be seen on Figure 4**
B1: See scheme
B1: Correct x-intercepts. Allow as shown or written as $(k, 0)$ and $(2k, 0)$ and condone coordinates written as $(0, k)$ and $(0, 2k)$ as long as they are in the correct places.
B1: Correct y-intercept. Allow as shown or written as $(0, -2k)$ or $(-2k, 0)$ as long as it is in the correct place. Condone $k - 3k$ for $-2k$.
B1: Correct coordinates as shown
Note that the marks for the intercepts and the maximum can be seen away from the sketch but the coordinates must be the right way round or e.g. as $y = 0$, $x = k$ etc. These marks can be awarded without a sketch but if there is a sketch, such points must not contradict the sketch.
(b)
B1: Deduces the correct critical value of $x = k$. May be implied by e.g. $x > k$ or $x < k$
M1: Attempts to solve $k - (2x - 3k) = x - k$ or an equivalent equation/inequality to find the other critical value. Allow this mark for reaching $k = …$ or $x = …$ as long as they are solving the required equation.
A1: Correct value
A1: Correct answer using the correct set notation.

Allow e.g. $\left\{x : x \in \mathbb{R}, k < x < \frac{5k}{3}\right\}$, $\left\{x :\ k < x < \frac{5k}{3}\right\}$, $x \in \left(k, \frac{5k}{3}\right)$ and allow "|" for ":"

But $\left\{x : x < \frac{5k}{3}\right\} \cup \{x : x > k\}$ scores A0 $\left\{x :\ k < x,\ x < \frac{5k}{3}\right\}$ scores A0

(c)

B1ft: Deduces one correct coordinate. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" or $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = ..$, $y = \ldots$

B1ft: Deduces both correct coordinates. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" and $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = ..$, $y = \ldots$

If coordinates are given the wrong way round and not seen correctly as $x = \ldots$, $y = \ldots$

e.g. $(3 - 5k, 3k)$ this is B0B0

1. The point $P(-2, -5)$ lies on the curve with equation $y = \mathrm{f}(x), \quad x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(a) $y = \mathrm{f}(x) + 2$

(1)

(b) $y = |\mathrm{f}(x)|$

(1)

(c) $y = 3\mathrm{f}(x - 2) + 2$

(2)

Question	Scheme	Marks	AOs
1 (a)	$(-2, -3)$	B1	1.1b
		(1)	
(b)	$(-2, 5)$	B1	1.1b
		(1)	
(c)	Either $x = 0$ or $y = -13$	M1	1.1b
	$(0, -13)$	A1	1.1b
		(2)	
			(4 marks)
Notes:			

Watch for answers in the body of the question and on sketch graphs. This is acceptable.
If coordinates are written by the question and in the main answer section the answer section takes precedence.

(a)
B1: Accept without brackets. May be written $x = -2, y = -3$
(b)
B1: Accept without brackets. May be written $x = -2, y = 5$
(c)
M1: For either coordinate. E.g. $(0, ...)$ or $(..., -13)$
If they are building up their solution in stages e.g. $(-2, -5) \to (0, -5) \to (0, -15) \to (0, -13)$
only mark their final coordinate pair
A1: Correct coordinates. See above for building up solution in stages
Accept without brackets. May be written $x = 0, y = -13$
SC 10 for candidates who write $(-13, 0)$

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

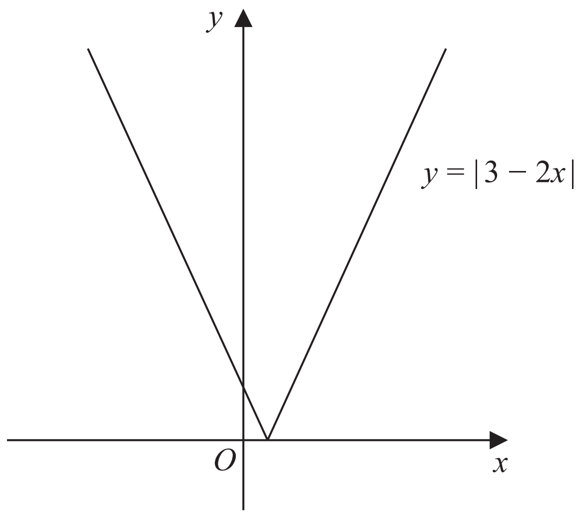


Figure 1

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

(4)

Question	Scheme	Marks	AOs
1	For an attempt to solve **Either** $3-2x=7+x \Rightarrow x=...$ **or** $2x-3=7+x \Rightarrow x=...$	M1	1.1b
	Either $x=-\frac{4}{3}$ **or** $x=10$	A1	1.1b
	For an attempt to solve **Both** $3-2x=7+x \Rightarrow x=...$ **and** $2x-3=7+x \Rightarrow x=...$	dM1	1.1b
	For **both** $x=-\frac{4}{3}$ **and** $x=10$ with no extra solutions	A1	1.1b
		(4)	
ALT	**Alternative by squaring:**		
	$(3-2x)^2=(7+x)^2 \Rightarrow 9-12x+4x^2=49+14x+x^2$	M1	1.1b
	$3x^2-26x-40=0$	A1	1.1b
	$3x^2-26x-40=0 \Rightarrow x=...$	dM1	1.1b
	For **both** $x=-\frac{4}{3}$ **and** $x=10$ with no extra solutions	A1	1.1b
			(4 marks)

Notes:

Note this question requires working to be shown not <u>just answers written down</u>. But correct equations seen followed by the correct answers can score full marks.

M1: Attempts to solve either correct equation.
Allow equivalent equations e.g. $3-2x=-7-x \Rightarrow x=...$

A1: One correct solution. Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not e.g. -1.33

dM1: Attempts to solve both correct equations.
Allow equivalent equations e.g. $3-2x=-7-x \Rightarrow x=...$ **Depends on the first method mark.**

A1: For both $x=-\frac{4}{3}$ and $x=10$ with no extra solutions and neither clearly rejected but ignore any attempts to find the y coordinates whether correct or otherwise and ignore reference to e.g. $x = -7$ (from where $y = 7 + x$ intersects the x-axis) or $x=1.5$ (from finding the value of x at the vertex) as "extras". Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not rounded e.g. -1.33

Isw if necessary e.g. ignore subsequent attempts to put the values in an inequality e.g. $-\frac{4}{3}<x<10$

But if e.g. $x=-\frac{4}{3}$ is obtained and a candidate states $x=\left|-\frac{4}{3}\right|$ then score A0

Alternative solution via squaring

M1: Attempts to square both sides. Condone poor squaring e.g. $(3-2x)^2 = 9 \pm 4x^2$ or $9 \pm 2x^2$

A1: Correct quadratic equation $3x^2 - 26x - 40 = 0$. The "= 0" may be implied by their attempt to solve. Terms must be collected but not necessarily all on one side so allow e.g. $3x^2 - 26x = 40$

dM1: Correct attempt to solve a **3 term** quadratic. See general guidance for solving a quadratic equation. The roots can be written down from a calculator so the method may be implied by their values. **Depends on the first method mark**.

A1: For both $x = -\frac{4}{3}$ and $x = 10$ with no extra solutions and neither clearly rejected but ignore any attempts to find the *y* coordinates and do not count e.g. $x = -7$ (from where $y = 7 + x$ intersects the *x*-axis) or $x = 1.5$ (from finding the value of *x* at the vertex) as "extras". Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not e.g. -1.33

Isw if necessary e.g. ignore subsequent attempts to put the values in an inequality e.g. $-\frac{4}{3} < x < 10$

But if e.g. $x = -\frac{4}{3}$ is obtained and a candidate states $x = \left|-\frac{4}{3}\right|$ then score A0

10. The function f is defined by

$$\mathrm{f}(x) = \frac{8x + 5}{2x + 3} \qquad x > -\frac{3}{2}$$

(a) Find $\mathrm{f}^{-1}\left(\frac{3}{2}\right)$

(2)

(b) Show that

$$\mathrm{f}(x) = A + \frac{B}{2x + 3}$$

where A and B are constants to be found.

(2)

The function g is defined by

$$\mathrm{g}(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of g^{-1}

(1)

(d) Find the range of $\mathrm{f}\,\mathrm{g}^{-1}$

(3)

Question	Scheme	Marks	AOs
10(a)	Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$ Or substitutes $x=\frac{3}{2}$ into $\frac{5-3x}{2x-8}$	M1	3.1a
	$\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$	A1	1.1b
		(2)	
(b)	$\left(\frac{8x+5}{2x+3}=\right)4\pm\frac{...}{2x+3}$	M1	1.1b
	$\left(\frac{8x+5}{2x+3}=\right)4-\frac{7}{2x+3}$	A1	2.1
		(2)	
(c)	$0\leqslant g^{-1}(x)\leqslant 4$	B1	2.2a
		(1)	
(d)	Attempts either boundary $f(0)=\frac{8\times 0+5}{2\times 0+3}$ **or** $f(4)=\frac{8\times 4+5}{2\times 4+3}$	M1	3.1a
	Attempts both boundaries $f(0)=\frac{8\times 0+5}{2\times 0+3}$ **and** $f(4)=\frac{8\times 4+5}{2\times 4+3}$	dM1	1.1b
	Range $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$	A1	2.1
		(3)	
	Alternative by attempting $fg^{-1}(x)$		
	$g^{-1}(x)=\sqrt{16-x}\Rightarrow fg^{-1}(x)=\frac{8\sqrt{16-x}+5}{2\sqrt{16-x}+3}$ $fg^{-1}(0)=\frac{8\times 4+5}{2\times 4+3}$ **or** $fg^{-1}(16)=\frac{8\times 0+5}{2\times 0+3}$	M1	3.1a
	$fg^{-1}(0)=\frac{8\times 4+5}{2\times 4+3}$ **and** $fg^{-1}(16)=\frac{8\times 0+5}{2\times 0+3}$	dM1	1.1b
	Range $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$	A1	2.1
		(3)	
			(8 marks)
Notes:			

(a)

M1: Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$ You can condone poor algebra as long as they reach a value for x.

Alternatively attempt to substitute $x=\frac{3}{2}$ into $f^{-1}(x)=\frac{\pm 5\pm 3x}{\pm 2x\pm 8}$ or equivalent (may be in terms of y). Note that attempts to find e.g. $f'(x)$ or $\frac{1}{f(x)}$ which may be implied by values such as $\frac{6}{17}, \frac{17}{6}, \frac{7}{18}, \frac{18}{7}$ score M0

A1: Achieves $\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$. Do not be concerned what they call it, just look for the value e.g. $x=-\frac{1}{10}$ or just $-\frac{1}{10}$ is fine. Correct answer with no (or minimal) working scores both marks.

(b)

M1: Attempts to divide $8x+5$ by $2x+3$

Look for $4\pm\frac{...}{2x+3}$ where … is a constant or $8x+5 = A(2x+3)+B$ with A or B correct (which may be in a fraction) or in a long division attempt and obtains a quotient of 4 or attempts to express the numerator in terms of the denominator e.g. $\frac{8x+5}{2x+3}=\frac{4(2x+3)+...}{2x+3}$

A1: A full and complete method showing $\frac{8x+5}{2x+3}=4-\frac{7}{2x+3}$ or $\frac{8x+5}{2x+3}=4+\frac{-7}{2x+3}$

Also allow for correct values e.g. $A = 4$, $B = -7$

Do not isw here e.g. if they obtain $A = 4$, $B = -7$ and then write $-7+\frac{4}{2x+3}$ score A0

(c)

B1: Deduces $0\leqslant g^{-1}(x)\leqslant 4$ o.e.

E.g. $0\leqslant y\leqslant 4$, $0\leqslant \text{range}\leqslant 4$, $g^{-1}(x)\leqslant 4$ and $g^{-1}(x)\geqslant 0$, $0\leqslant g^{-1}\leqslant 4$, $[0, 4]$

but not e.g. $0\leqslant x\leqslant 4$, $0\leqslant g(x)\leqslant 4$, (0, 4)

(d)

M1: Attempts either boundary. Look for either $f(0)=\frac{8\times0+5}{2\times0+3}$ **or** $f(4)=\frac{8\times4+5}{2\times4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times0+3}$ **or** $f(4)=4-\frac{7}{2\times4+3}$

dM1: Attempts both boundaries. Look for $f(0)=\frac{8\times0+5}{2\times0+3}$ **and** $f(4)=\frac{8\times4+5}{2\times4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times0+3}$ **and** $f(4)=4-\frac{7}{2\times4+3}$

A1: Correct answer written in the correct form.

E.g. $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant \text{range}\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant y\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}$ and $fg^{-1}(x)\geqslant\frac{5}{3}$

$\frac{5}{3}\leqslant fg^{-1}\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}\cap fg^{-1}(x)\geqslant\frac{5}{3}$, $\left[\frac{5}{3}, \frac{37}{11}\right]$ but not e.g. $\frac{5}{3}\leqslant x\leqslant\frac{37}{11}$

PTO for an alternative to (d)

(d) **Alternative:**

M1: Attempts $\text{fg}^{-1}(x)$ and either boundary using $x = 0$ or $x = 16$

Look for either $\text{fg}^{-1}(0) = \dfrac{8\times \text{g}^{-1}(0)+5}{2\times \text{g}^{-1}(0)+3}$ **or** $\text{fg}^{-1}(16) = \dfrac{8\times \text{g}^{-1}(16)+5}{2\times \text{g}^{-1}(16)+3}$

Or uses (b) e.g. $\text{fg}^{-1}(0) = 4 - \dfrac{7}{2\times \text{g}^{-1}(0)+3}$ **or** $\text{fg}^{-1}(16) = 4 - \dfrac{7}{2\times \text{g}^{-1}(16)+3}$

The attempt at $\text{fg}^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f

dM1: Attempts both boundaries. Look for $\text{fg}^{-1}(0) = \dfrac{8\times \text{g}^{-1}(0)+5}{2\times \text{g}^{-1}(0)+3}$ **and** $\text{fg}^{-1}(16) = \dfrac{8\times \text{g}^{-1}(16)+5}{2\times \text{g}^{-1}(16)+3}$

Or uses (b) e.g. $\text{fg}^{-1}(0) = 4 - \dfrac{7}{2\times \text{g}^{-1}(0)+3}$ **and** $\text{fg}^{-1}(16) = 4 - \dfrac{7}{2\times \text{g}^{-1}(16)+3}$

The attempt at $\text{fg}^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$) into f

A1: Correct answer written in the correct form with exact values.

E.g. $\dfrac{5}{3} \leqslant \text{fg}^{-1}(x) \leqslant \dfrac{37}{11}$, $\dfrac{5}{3} \leqslant \text{range} \leqslant \dfrac{37}{11}$, $\dfrac{5}{3} \leqslant y \leqslant \dfrac{37}{11}$, $\text{fg}^{-1}(x) \leqslant \dfrac{37}{11}$ and $\text{fg}^{-1}(x) \geqslant \dfrac{5}{3}$

$\dfrac{5}{3} \leqslant \text{fg}^{-1} \leqslant \dfrac{37}{11}$, $\text{fg}^{-1}(x) \leqslant \dfrac{37}{11} \cap \text{fg}^{-1}(x) \geqslant \dfrac{5}{3}$, $\left[\dfrac{5}{3}, \dfrac{37}{11}\right]$ but not e.g. $\dfrac{5}{3} \leqslant x \leqslant \dfrac{37}{11}$

Note that the $\dfrac{37}{11}$ is sometimes obtained fortuitously from incorrect working so check working carefully.