Y2P12 XMQs and MS

(Total: 37 marks)

1. P1_Sample   Q7 . 5 marks - Y2P12 Vectors
2. P1_Specimen Q4 . 5 marks - Y2P12 Vectors
3. P2_2019   Q10. 6 marks - Y2P12 Vectors
4. P1_2020   Q3 . 4 marks - Y2P12 Vectors
5. P1_2021   Q6 . 5 marks - Y2P12 Vectors
6. P1_2022   Q9 . 6 marks - Y2P12 Vectors
7. P2_2022   Q13. 6 marks - Y2P12 Vectors

7.

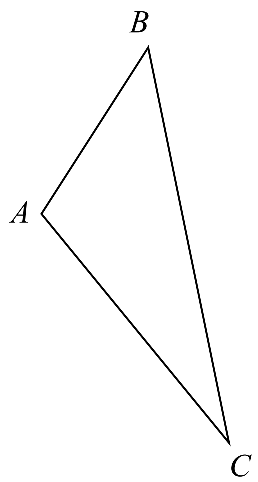

Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

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Question	Scheme	Marks	AOs
7	Attempts $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + \mathbf{i} - 9\mathbf{j} + 3\mathbf{k} = 3\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$	M1	3.1a
	Attempts to find any one length using 3-d Pythagoras	M1	2.1
	Finds all of $\lvert AB\rvert = \sqrt{14}$, $\lvert AC\rvert = \sqrt{61}$, $\lvert BC\rvert = \sqrt{91}$	A1ft	1.1b
	$\cos BAC = \dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$	M1	2.1
	angle $BAC = 105.9°$ *	A1*	1.1b
		(5)	
		(5 marks)	

Notes:

M1: Attempts to find $\overrightarrow{AC}$ by using $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

M1: Attempts to find any one length by use of Pythagoras' Theorem

A1ft: Finds all three lengths in the triangle. Follow through on their $\lvert AC\rvert$

M1: Attempts to find BAC using $\cos BAC = \dfrac{\lvert AB\rvert^2 + \lvert AC\rvert^2 - \lvert BC\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}$

Allow this to be scored for other methods such as $\cos BAC = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert AB\rvert\lvert AC\rvert}$

A1*: This is a show that and all aspects must be correct. Angle $BAC =$ 105.9 °

4. Relative to a fixed origin O,

the point A has position vector $\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$,
the point B has position vector $4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$,
and the point C has position vector $2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$.

Given that $ABCD$ is a parallelogram,

(a) find the position vector of point D.

(2)

The vector $\overrightarrow{AX}$ has the same direction as $\overrightarrow{AB}$.

Given that $|\overrightarrow{AX}| = 10\sqrt{2}$,

(b) find the position vector of X.

(3)

Question	Scheme	Marks	AOs
4 (a)	$\overrightarrow{OA} = \mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = 2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$		
	$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA} = (2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}) + (-3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ or $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + (-2\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OD} = -\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$	A1	1.1b
		(2)	
(b)	$\left\{\overrightarrow{AB} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} \Rightarrow\right\}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{(3)^2 + (-4)^2 + (5)^2}$ $\left\{= \sqrt{50} = 5\sqrt{2}\right\}$	M1	1.1b
	As $\lvert\overrightarrow{AX}\rvert = 10\sqrt{2}$ then $\lvert\overrightarrow{AX}\rvert = 2\lvert\overrightarrow{AB}\rvert \Rightarrow \overrightarrow{AX} = 2\overrightarrow{AB}$		
	$\overrightarrow{OX} = \overrightarrow{OA} + 2\overrightarrow{AB} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + 2(3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ or $\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{AB} = (4 + 3\mathbf{j} + 3\mathbf{k}) + (3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OX} = 7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ only	A1	1.1b
		(3)	
		(5 marks)	

Question 4 Notes:

(a)	
M1:	A complete method for finding the position vector of D
A1:	$-\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} -1 \\ 14 \\ 4 \end{pmatrix}$
(b)	
M1:	A complete attempt to find $\lvert\overrightarrow{AB}\rvert$ or $\lvert\overrightarrow{BA}\rvert$
M1:	A complete process for finding the position vector of X
A1:	$7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ or $\begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix}$

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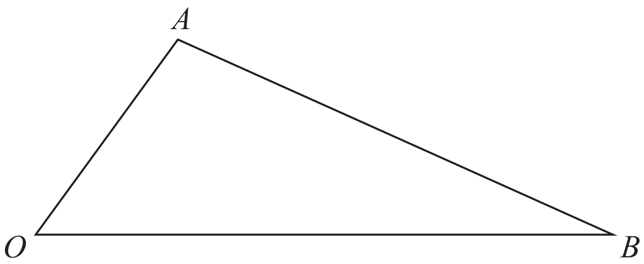


Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

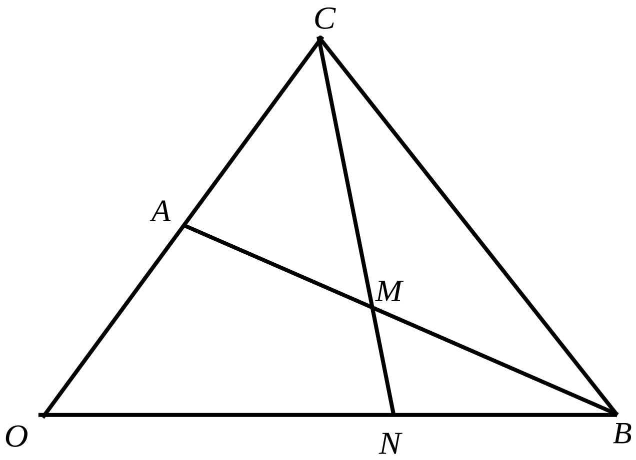
(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON : NB = 2 : 1$

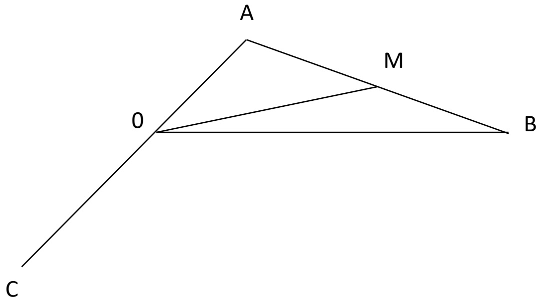
(2)

Question	Scheme	Marks	AOs
10			
	$\overrightarrow{OA}=\mathbf{a},\ \overrightarrow{OB}=\mathbf{b}$		
(a)	$\left\{\overrightarrow{CM}=\overrightarrow{CA}+\overrightarrow{AM}=\overrightarrow{CA}+\frac{1}{2}\overrightarrow{AB}\Rightarrow\right\}\ \overrightarrow{CM}=-\mathbf{a}+\frac{1}{2}(\mathbf{b}-\mathbf{a})$	M1	3.1a
	$\left\{\overrightarrow{CM}=\overrightarrow{CB}+\overrightarrow{BM}=\overrightarrow{CB}+\frac{1}{2}\overrightarrow{BA}\Rightarrow\right\}\ \overrightarrow{CM}=(-2\mathbf{a}+\mathbf{b})+\frac{1}{2}(\mathbf{a}-\mathbf{b})$		
	$\Rightarrow\overrightarrow{CM}=-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}$ (***needs to be simplified and seen in (a) only***)	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON}=\overrightarrow{OC}+\overrightarrow{CN}\Rightarrow\ \overrightarrow{ON}=\overrightarrow{OC}+\lambda\overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON}=2\mathbf{a}+\lambda\left(-\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}\right)\Rightarrow\overrightarrow{ON}=\left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}$ *	A1*	2.1
		(2)	
(c) **Way 1**	$\left(2-\frac{3}{2}\lambda\right)=0\ \Rightarrow\lambda=...$	M1	2.2a
	$\lambda=\frac{4}{3}\ \Rightarrow\overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow\overrightarrow{NB}=\frac{1}{3}\mathbf{b}\right\}\Rightarrow ON:NB=2:1$ *	A1*	2.1
		(2)	
(c) **Way 2**	$\overrightarrow{ON}=\mu\mathbf{b}\ \Rightarrow\ \left(2-\frac{3}{2}\lambda\right)\mathbf{a}+\frac{1}{2}\lambda\mathbf{b}=\mu\mathbf{b}$		
	$\mathbf{a}:\ \left(2-\frac{3}{2}\lambda\right)=0\Rightarrow\lambda=...\ \left\{\mathbf{b}:\ \frac{1}{2}\lambda=\mu\ \ \&\ \lambda=\frac{4}{3}\ \Rightarrow\ \mu=\frac{2}{3}\right\}$	M1	2.2a
	$\lambda=\frac{4}{3}$ or $\mu=\frac{2}{3}\ \Rightarrow\overrightarrow{ON}=\frac{2}{3}\mathbf{b}\ \left\{\Rightarrow\overrightarrow{NB}=\frac{1}{3}\mathbf{b}\right\}\Rightarrow ON:NB=2:1$ *	A1*	2.1
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...$ $\left\{\mathbf{b}: 1 = \frac{1}{2}\lambda + K \text{ \& } \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b}$ & $\overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}$, $\mathbf{b}: \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = ...$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ $\left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	

Notes for Question 10

(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors **a** and **b**
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow\right\}$ $\overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ **only o.e.**
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	**Special Case**
	Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$
	$\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ $\left\{= \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b}\right\}$
Note:	**Alternative 1:** Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	**Way 1, Way 2 and Way 3**
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	**Way 4**
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

Notes for Question 10 Continued	
Note:	Part (b) and part (c) can be marked together.
(a) Special Case	**<u>Special Case where the point *C* is believed to be below the origin *O*</u>** 
	Give Special Case M1 A0 in part (a) for $\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\right\}$ $\overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$
	$\left\{\text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right\}$

3. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

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Question	Scheme	Marks	AOs
3 (a)	$\overrightarrow{AB} = (3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$= \mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as *OC* is parallel to *AB*, so *OABC* is a trapezium.	A1	2.4
		(2)	
			(4 marks)
Notes:			

(a)

M1: Attempts to subtract either way around. If no method is seen it is implied by two of $\pm 1\mathbf{i}\pm 8\mathbf{j}\pm 2\mathbf{k}$.

A1: $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ or $\begin{pmatrix}1\\-8\\2\end{pmatrix}$ but not $(1,-8,2)$

(b)

M1: Compares their $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ with $2\mathbf{i}-16\mathbf{j}+4\mathbf{k}$ by stating **any one of**

- $\overrightarrow{OC} = 2\times\overrightarrow{AB}$
- $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$
- $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$ or vice versa

This may be awarded if *AB* was subtracted "the wrong way around" or if there was one numerical slip

A1: A full explanation as to why *OABC* is a trapezium.

Requires fully correct calculations, so part (a) must be $\overrightarrow{AB} = (\mathbf{i}-8\mathbf{j}+2\mathbf{k})$

It requires a reason and minimal conclusion.

Example 1:

$\overrightarrow{OC} = 2\times\overrightarrow{AB}$, therefore *OC* is parallel to *AB* so *OABC* is a trapezium

Example 2:

A trapezium has one pair of parallel sides. As $\overrightarrow{OC} = 2\times\overrightarrow{AB}$, they are parallel, so ✓.

Example 3

As $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$, *OC* and *AB* are parallel, so proven

Example 4

Accept as $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides *OA* and *CB* in this question may be ignored, even if incorrect.

6.

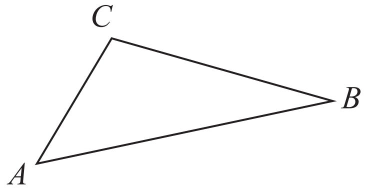


Figure 1

Figure 1 shows a sketch of triangle *ABC*.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that $\cos ABC = \dfrac{9}{10}$

(3)

Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=-3\mathbf{i}-4\mathbf{j}-5\mathbf{k}+\mathbf{i}+\mathbf{j}+4\mathbf{k}=...$	M1	1.1b
	$=-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $\left(AC^2\right)="2^2+3^2+1^2",\left(AB^2\right)=3^2+4^2+5^2,\ \left(BC^2\right)=1^2+1^2+4^2$	M1	1.1b
	$2^2+3^2+1^2=3^2+4^2+5^2+1^2+1^2+4^2-2\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC=\dfrac{50+18-14}{2\sqrt{50}\sqrt{18}}=\dfrac{9}{10}$ *	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2=3^2+4^2+5^2,\ BC^2=1^2+1^2+4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC}=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\bullet(\mathbf{i}+\mathbf{j}+4\mathbf{k})=27=\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$27=\sqrt{50}\sqrt{18}\cos ABC\Rightarrow\cos ABC=\dfrac{27}{\sqrt{50}\sqrt{18}}==\dfrac{9}{10}$ *	A1*	2.1
			(5 marks)

Notes

(a)

M1: Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ and $\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ but not $\begin{pmatrix}-2\mathbf{i}\\-3\mathbf{j}\\-1\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2+b^2+c^2$ or $\sqrt{a^2+b^2+c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC\leftrightarrow\theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$

via $\cos ABC=\dfrac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC=\dfrac{\left(5\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2-\left(\sqrt{14}\right)^2}{2\times5\sqrt{2}\times3\sqrt{2}}$ to $\cos ABC=\dfrac{9}{10}$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result

9.

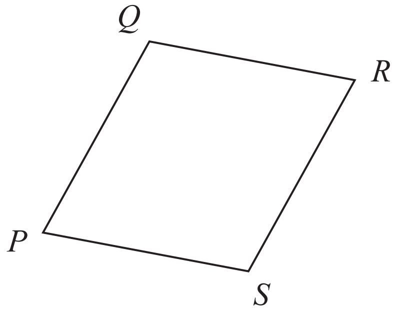


Figure 3

Figure 3 shows a sketch of a parallelogram *PQRS*.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram *PQRS* is a rhombus.

(2)

(b) Find the exact area of the rhombus *PQRS*.

(4)

Question	Scheme	Marks	AOs
9(a)	Attempts both $\|\overrightarrow{PQ}\|=\sqrt{2^2+3^2+(-4)^2}$ and $\|\overrightarrow{QR}\|=\sqrt{5^2+(-2)^2}$	M1	3.1a
	States that $\|\overrightarrow{PQ}\|=\|\overrightarrow{QR}\|=\sqrt{29}$ so $PQRS$ is a rhombus	A1	2.4
		(2)	
(b)	Attempts BOTH $\overrightarrow{PR}=\overrightarrow{PQ}+\overrightarrow{QR}=7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ AND $\overrightarrow{QS}=-\overrightarrow{PQ}+\overrightarrow{PS}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$	M1	3.1a
	Correct $\overrightarrow{PR}=7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$	A1	1.1b
	Correct method for area $PQRS$. E.g. $\frac{1}{2}\times\|\overrightarrow{PR}\|\times\|\overrightarrow{QS}\|$	dM1	2.1
	$=\sqrt{517}$	A1	1.1b
		(4)	
			(6 marks)
Alt (b) Example using the cosine rule	Attempts $\|\overrightarrow{QS}\|=\sqrt{3^2+(-3)^2+2^2}=\sqrt{22}$ and so $22=29+29-2\sqrt{29}\sqrt{29}\cos SPQ$	M1	3.1a
	$\cos PQR=-\frac{18}{29}$ or $\cos SPQ=\frac{18}{29}$ Condone angles in degrees 51.6, 128.4 (1dp) or radians 2.24, 0.901 (3sf) here	A1	1.1b
	Correct method for area $PQRS$. E.g. $PQ\times QR\sin PQR=\sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\times\frac{\sqrt{517}}{29}$	dM1	2.1
	$=\sqrt{517}$	A1	1.1b
		(4)	

FYI

$\overrightarrow{QR}=5\mathbf{i}+0\mathbf{j}-2\mathbf{k}$

$\overrightarrow{PQ}=2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$

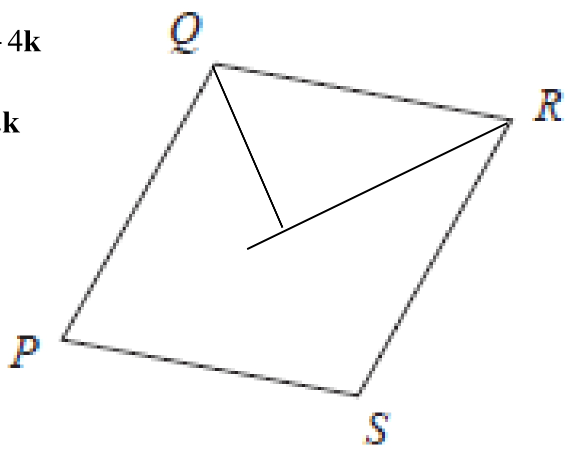
$\overrightarrow{SQ}=-3\mathbf{i}+3\mathbf{j}-2\mathbf{k}$

$\overrightarrow{MQ}=-1.5\mathbf{i}+1.5\mathbf{j}-1\mathbf{k}$

M

$\overrightarrow{PM}=3.5\mathbf{i}+1.5\mathbf{j}-3\mathbf{k}$

$\overrightarrow{PR}=7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$



(a) Do not award marks in part (a) from work in part (b).

M1: Attempts both $|\overrightarrow{PQ}|=\sqrt{2^2+3^2+(\pm4)^2}$ and $|\overrightarrow{QR}|=\sqrt{5^2+(\pm2)^2}$ or PQ^2 and QR^2. For this mark only, condone just the correct answers $|\overrightarrow{PQ}|=\sqrt{29}$ and $|\overrightarrow{QR}|=\sqrt{29}$. Alternatively attempts $\overrightarrow{PR}\bullet\overrightarrow{QS}$ or PM^2, MQ^2 and PQ^2 where M is the mid point of PR

A1: Shows that $|\overrightarrow{PQ}|=|\overrightarrow{QR}|=\sqrt{29}$ (with calculations) and states $PQRS$ is a rhombus.

Condone poor notation such as $\overrightarrow{PQ}=\sqrt{29}$ here, So $\overrightarrow{PQ}=\overrightarrow{QR}=\sqrt{29}$ hence rhombus.

Requires both a reason and a conclusion. The reason may be given at the start of their solution.

In the alternatives $\overrightarrow{PR}\bullet\overrightarrow{QS}=(7\mathbf{i}+3\mathbf{j}-6\mathbf{k})\bullet(3\mathbf{i}-3\mathbf{j}+2\mathbf{k})=21-9-12=0$ so diagonals cross at $90°$ so $PQRS$ is a rhombus or $PM^2+MQ^2=PQ^2=23.5+5.5=29\Rightarrow\angle PMQ=90°\Rightarrow$ Rhombus

(b) **Candidates can transfer answers from (a) to use in part (b) to find the area**
Look through their complete solution first. The first two marks are for finding the elements that are required to calculate the area. The second set of two marks is for combining these elements correctly. If the method is NOT shown on how to find vector it can be implied by two correct components. Allow as column vectors.

M1: For a key step in solving the problem. It is scored for attempting to find both key vectors.

Attempts both $\overrightarrow{PR}=\overrightarrow{PQ}+\overrightarrow{QR}=(7\mathbf{i}+3\mathbf{j}-6\mathbf{k})$ **AND** $\overrightarrow{QS}=-\overrightarrow{PQ}+\overrightarrow{PS}=(3\mathbf{i}-3\mathbf{j}+2\mathbf{k})$

You may see $\overrightarrow{PM}=\frac{1}{2}\overrightarrow{PQ}+\frac{1}{2}\overrightarrow{QR}=\left(\frac{7}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}\right)$ **AND** $\overrightarrow{QM}=-\frac{1}{2}\overrightarrow{PQ}+\frac{1}{2}\overrightarrow{PS}=\left(\frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}+1\mathbf{k}\right)$

A1: Accurately finds both key vectors whose lengths are required to solve the problem.

Score for both $\overrightarrow{PR}=7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ (Allow either way around.)

or both $\overrightarrow{PM}=\frac{7}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}$ and $\overrightarrow{QM}=\frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}+1\mathbf{k}$ (Allow either way around.)

dM1: Constructs a rigorous method leading to the area $PQRS$. Dependent upon previous M.

E.g. See scheme. Alt: the sum of the area of four right angled triangles e.g. $4\times\frac{1}{2}\times\left|\overrightarrow{PM}\right|\times\left|\overrightarrow{QM}\right|$,

A1: $\sqrt{517}$

Alternatives for (b). Two such ways are set out below

Alt 1-Examples via cosine rule but you may see use of scalar product via a Further Maths method.

M1: For a key step in solving the problem. In this case it for an attempt at cos PQR or cos SPQ.

Don't be too concerned with the labelling of the angle which may appear as θ.

Attempts $\pm\begin{pmatrix}2\\3\\-4\end{pmatrix}\bullet\pm\begin{pmatrix}5\\0\\-2\end{pmatrix}=\sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\cos PQR$

A1: Finds the cosine of one of the angles in the Figure.

Look for $\cos\ldots=-\frac{18}{29}$ or $\cos\ldots=\frac{18}{29}$ which may have been achieved via the cosine rule.

Accept rounded answers and the angles in degrees 51.6, 128.4 (1dp) or radians 2.24, 0.901 (3sf) here.

dM1: Constructs a rigorous method leading to the area $PQRS$. Implied by awrt 22.7

E.g. $PQ\times QR\sin PQR=\sqrt{2^2+3^2+(-4)^2}\times\sqrt{5^2+(-2)^2}\times\frac{\sqrt{517}}{29}$

A1: $\sqrt{517}$

Alt 2-Example via vector product via a Further Maths method.

M1: For a key step in solving the problem. In this case it for an attempt at $\pm\overrightarrow{PQ}\times\overrightarrow{QR}$

E.g. $\overrightarrow{PQ}\times\overrightarrow{QR}=\begin{pmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&3&-4\\5&0&-2\end{pmatrix}=(3\times-2-0\times-4)\mathbf{i}-(2\times-2-5\times-4)\mathbf{j}+(2\times0-3\times5)\mathbf{k}$

A1: E.g. $\overrightarrow{PQ}\times\overrightarrow{QR}=-6\mathbf{i}-16\mathbf{j}-15\mathbf{k}$

dM1: Constructs a rigorous method leading to the area $PQRS$. In this case $\left|\overrightarrow{PQ}\times\overrightarrow{QR}\right|$

A1: $=\sqrt{(-6)^2+(-16)^2+(-15)^2}=\sqrt{517}$

13. Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p.

(3)

The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

(b) Find $\left|\overrightarrow{OD}\right|$, writing your answer as a fully simplified surd.

(3)

Question	Scheme	Marks	AOs
13(a)	Attempts two of the relevant vectors $\pm\overrightarrow{AB} = \pm(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$ $\pm\overrightarrow{AC} = \pm(-20\mathbf{i} + (p+3)\mathbf{j} + 5\mathbf{k})$ $\pm\overrightarrow{BC} = \pm(-16\mathbf{i} + (p-4)\mathbf{j} + 4\mathbf{k})$	M1	3.1a
	Uses two of the three vectors in such a way as to find the value of p. E.g. $p+3 = 5\times 7$	dM1	2.1
	$p = 32$	A1	1.1b
		(3)	
	(a) Alternative:		
	$r_{AB} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$	M1	3.1a
	$4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow \lambda = 5$ $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow p = 35 - 3$	dM1	2.1
	$p = 32$	A1	1.1b
(b)	Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - "32")\mathbf{j} + (6\lambda - 10)\mathbf{k}$	M1	3.1a
	Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - "32")\mathbf{j} + (6\lambda - 10)\mathbf{k}$ $\overrightarrow{OA} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ $4\lambda - 32 = -12 \Rightarrow \lambda = ...$ OR $6\lambda - 10 = 20 \Rightarrow \lambda = ...$	dM1	1.1b
	$\lvert\overrightarrow{OD}\rvert = 5\times\sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
	(b) Alternative:		
	Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts $\overrightarrow{OD} = \overrightarrow{OC} + \mu\overrightarrow{OA} = -16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	M1	3.1a
	Correct attempt at λ **or** μ using the fact that $\lambda\overrightarrow{OB} = \overrightarrow{OC} + \mu\overrightarrow{OA}$ E.g. $-16 + 4\mu = 0 \Rightarrow \mu = 4$	dM1	1.1b
	$\lvert\overrightarrow{OD}\rvert = 5\times\sqrt{4^2 + 6^2} = 10\sqrt{13}$	A1	1.1b
		(3)	
			(6 marks)

Notes:

(a)

M1: Attempts two of the three relevant vectors by **subtracting** either way around. See scheme.

Allow equivalent work e.g. $\pm\overrightarrow{AB} = \pm(\overrightarrow{OB} + \overrightarrow{AO})$

If no working is shown, method can be implied by 2 correct components.

dM1: For the key step in using the fact that if the vectors are parallel, they will be multiples of each other (where the multiple is something other than 1) to find p.

E.g. $p+3=5\times 7$, $p-4=\frac{4}{5}(p+3)$, $p-4=4\times 7$

A1: $p=32$ (Condone 32**j**)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB},\ \overrightarrow{AC}=5\overrightarrow{AB},\ \overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC},\ \overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find p and if the 3rd component has errors but is not used, full marks can be awarded.

Alternative:

M1: Forms the vector equation using A or B as position and $\pm\overrightarrow{AB}$ as the direction

dM1: For the key step in using the fact that C lies on the line to find p

A1: $p=32$ (Condone 32**j**)

For reference, $\overrightarrow{BC}=4\overrightarrow{AB},\ \overrightarrow{AC}=5\overrightarrow{AB},\ \overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC},\ \overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

Note that candidates generally only need to use 2 components to find p and if the 3rd component has errors but is not used, full marks can be awarded.

There will be other approaches e.g. using "gradients" and "ratios" and the method marks can be implied – if you are unsure if such attempts deserve credit use Review

(b) Vector approach

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}$

dM1: Correct attempt at finding λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

E.g. $16\mathbf{i}+(4\lambda-"32")\mathbf{j}+(6\lambda-10)\mathbf{k}=4\alpha\mathbf{i}-3\alpha\mathbf{j}+5\alpha\mathbf{k}\Rightarrow\alpha=4\Rightarrow 4\lambda-"32"=-3\times"4"\Rightarrow\lambda=...$

A1: $\left|\overrightarrow{OD}\right|=10\sqrt{13}$

Alternative:

M1: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts

$\overrightarrow{OD}=\overrightarrow{OC}+\mu\overrightarrow{OA}=-16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$

dM1: Correct attempt at finding λ or μ using the fact that $\lambda\overrightarrow{OB}=\overrightarrow{OC}+\mu\overrightarrow{OA}$

E.g. $(-16+4\mu)\mathbf{i}+("32"-3\mu)\mathbf{j}+(10+5\mu)\mathbf{k}=4\lambda\mathbf{j}+6\lambda\mathbf{k}\Rightarrow -16+4\mu=0\Rightarrow\mu=...$

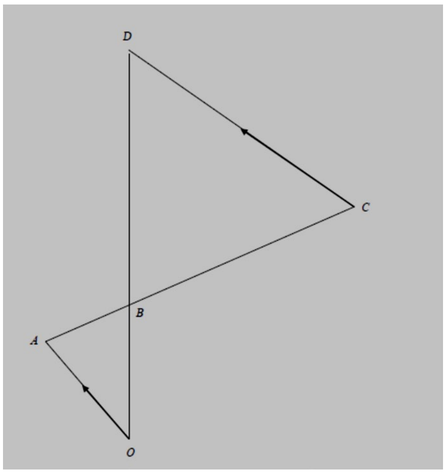
May also solve simultaneously using y and z components to find λ or μ

A1: $\left|\overrightarrow{OD}\right|=10\sqrt{13}$

Note that the correct vector is 20**j** + 30**k**

PTO for similar triangle approach

(b) Similar triangle approach



M1: For the key step in recognising that triangle *BCD* and triangle *BAO* are similar with a ratio of lengths of 4:1

dM1: States or uses the fact that $\left|\overrightarrow{OD}\right| = 5 \times \left|\overrightarrow{OB}\right|$

Stating this will score M1 dM1 provided there is no evidence of incorrect work

Note that they may establish this result using the work from (a) but must be used here to score.

A1: $\left|\overrightarrow{OD}\right| = 10\sqrt{13}$