Y2P10 XMQs and MS

(Total: 70 marks)

1. P1_Sample Q8 . 6 marks - Y2P10 Numerical methods
2. P2_Specimen Q6 . 9 marks - Y2P9 Differentiation
3. P1_2018 Q4 . 4 marks - Y2P10 Numerical methods
4. P2_2018 Q5 . 6 marks - Y2P10 Numerical methods
5. P2_2019 Q11. 11 marks - Y2P9 Differentiation
6. P2_2020 Q7 . 10 marks - Y2P9 Differentiation
7. P1_2021 Q4 . 9 marks - Y2P9 Differentiation
8. P1_2022 Q8 . 8 marks - Y2P9 Differentiation
9. P2_2022 Q6 . 7 marks - Y2P10 Numerical methods

8. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval $[3.5, 4]$

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

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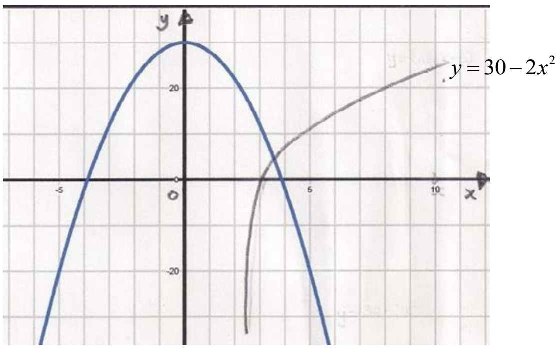
Question	Scheme	Marks	AOs
8 (a)	$\mathrm{f}(3.5) = -4.8,\ \mathrm{f}(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval $[3.5, 4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} \Rightarrow \quad x_1 = 4 - \dfrac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x - 5)$	**(2)**	
(c)	 Attempts to sketch both $y = \ln(2x - 5)$ and $y = 30 - 2x^2$	M1	3.1a
	States that $y = \ln(2x - 5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x - 5) = 30 - 2x$ has just one root $\Rightarrow \mathrm{f}(x) = 0$ has just one root	A1	2.4
		(2)	
			(6 marks)

Notes:

(a)

M1: Attempts f(x) at both $x = 3.5$ and $x = 4$ with at least one correct to 1 significant figure

A1*: f (3.5) and f(4) correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or $\mathrm{f}(3.5)\times\mathrm{f}(4) < 0$ or similar with f(x) being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval'

(b)

M1: Attempts $x_1 = x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)}$ evidenced by $x_1 = 4 - \dfrac{3.099}{16.67}$

A1: Correct answer only $x_1 = 3.81$

(c)

M1: For a valid attempt at showing that there is only one root. This can be achieved by

- Sketching graphs of $y = \ln(2x - 5)$ and $y = 30 - 2x^2$ on the same axes
- Showing that $\mathrm{f}(x) = \ln(2x - 5) + 2x^2 - 30$ has no turning points
- Sketching a graph of $\mathrm{f}(x) = \ln(2x - 5) + 2x^2 - 30$

A1: Scored for correct conclusion

6.

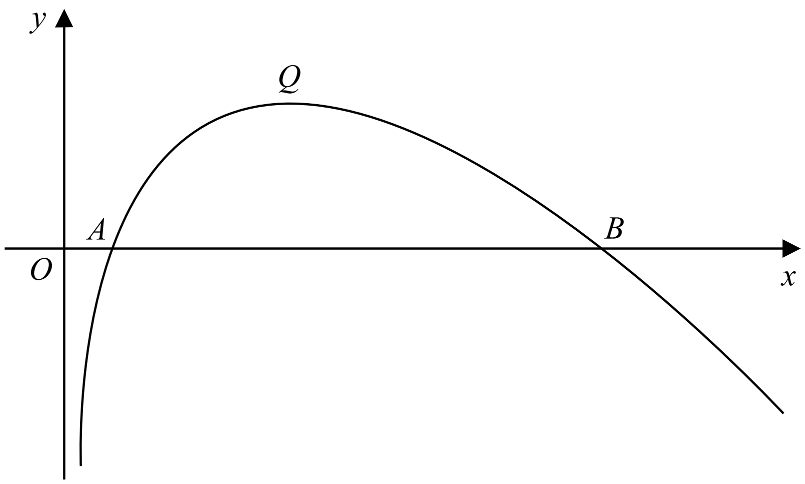


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \qquad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 2.

(a) Find the x coordinate of A and the x coordinate of B.

(1)

(b) Show that the x coordinate of Q satisfies

$$x = \frac{8}{1 + \ln x}$$

(4)

(c) Show that the x coordinate of Q lies between 3.5 and 3.6

(2)

(d) Use the iterative formula

$$x_{n+1} = \frac{8}{1 + \ln x_n} \qquad n \in \mathbb{N}$$

with $x_1 = 3.5$ to

(i) find the value of x_5 to 4 decimal places,

(ii) find the x coordinate of Q accurate to 2 decimal places.

(2)

Question	Scheme	Marks	AOs
6(a)	$\mathrm{f}(x) = (8 - x)\ln x,\ x > 0$		
	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	x coordinates are 1 and 8	B1	1.1b
		(1)	
(b)	Complete strategy of setting $\mathrm{f}'(x) = 0$ and rearranges to make $x = ...$	M1	3.1a
	$\left\{\begin{matrix} u = (8 - x) & v = \ln x \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = -1 & \dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{1}{x} \end{matrix}\right\}$		
	$\mathrm{f}'(x) = -\ln x + \dfrac{8-x}{x}$	M1	1.1b
		A1	1.1b
	$-\ln x + \dfrac{8-x}{x} = 0 \Rightarrow -\ln x + \dfrac{8}{x} - 1 = 0$ $\Rightarrow \dfrac{8}{x} = 1 + \ln x \Rightarrow x = \dfrac{8}{1 + \ln x}$ *	A1*	2.1
		(4)	
(c)	Evaluates both $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$	M1	1.1b
	$\mathrm{f}'(3.5) = 0.032951317...$ and $\mathrm{f}'(3.6) = -0.058711623...$ Sign change and as $\mathrm{f}'(x)$ is continuous, the x coordinate of Q lies between $x = 3.5$ and $x = 3.6$	A1	2.4
		(2)	
(d)(i)	$\{x_5 =\}$ 3.5340	B1	1.1b
(d)(ii)	$\{x_Q =\}$ 3.54 (2 dp)	B1	2.2a
		(2)	
			(9 marks)

Question 6 Notes:	
(a)	
B1:	Either • 1 and 8 • on Figure 2, marks 1 next to A and 8 next to B
(b)	
M1:	Recognises that Q is a stationary point (and not a root) and applies a complete strategy of setting $\mathrm{f}'(x) = 0$ and rearranges to make $x = ...$
M1:	Applies $vu' + uv'$, where $u = 8 - x,\ v = \ln x$ **Note:** This mark can be recovered for work in part (c)
A1:	$(8 - x)\ln x \rightarrow\ -\ln x + \dfrac{8-x}{x}$, or equivalent **Note:** This mark can be recovered for work in part (c)
A1*:	Correct proof with no errors seen in working.
(c)	
M1:	Evaluates both $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$
A1:	$\mathrm{f}'(3.5) = \text{awrt } 0.03$ and $\mathrm{f}'(3.6) = \text{awrt } -0.06$ or $\mathrm{f}'(3.6) = -0.05$ (truncated) **and** a correct conclusion
(d)(i)	
B1:	See scheme
(d)(ii)	
B1:	Deduces (e.g. by the use of further iterations) that the x coordinate of Q is 3.54 accurate to 2 dp **Note:** $3.5 \rightarrow 3.55119 \rightarrow 3.52845 \rightarrow 3.53848 \rightarrow 3.53404 \rightarrow 3.53600 \rightarrow 3.53514\ (\rightarrow 3.535518...)$

4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

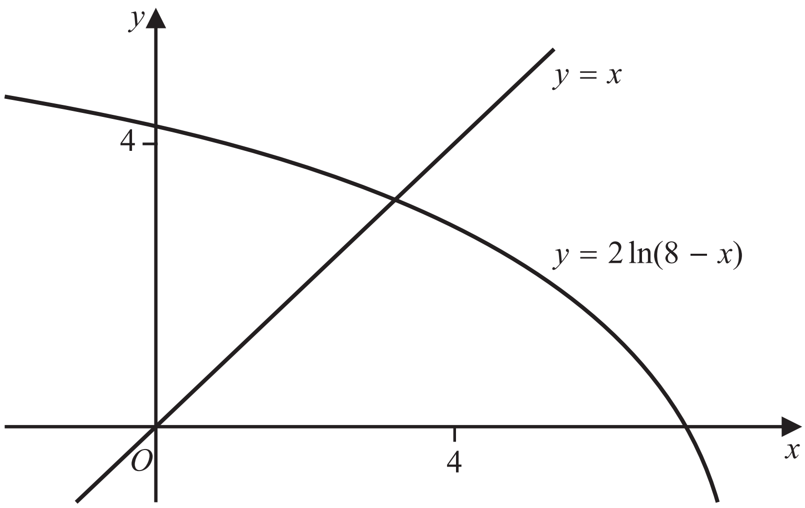


Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

Question	Scheme	Marks	AOs
4 (a)	Attempts $\mathrm{f}(3)=$ and $\mathrm{f}(4)=$ where $\mathrm{f}(x)=\pm\left(2\ln(8-x)-x\right)$	M1	2.1
	$\mathrm{f}(3)=\left(2\ln(5)-x\right)=(+)0.22$ and $\mathrm{f}(4)=\left(2\ln(4)-4\right)=-1.23$ Change of sign and function continuous in interval $[3,4]\Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1=4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)

Notes:

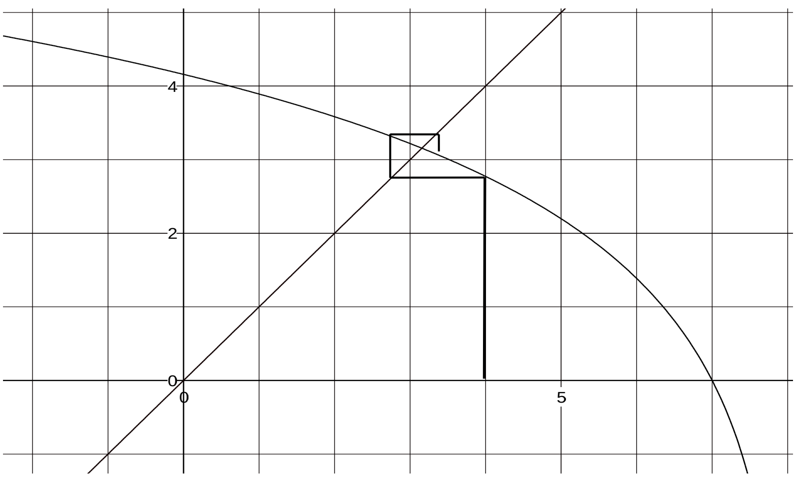
(a)

M1: Attempts $\mathrm{f}(3)=$ and $\mathrm{f}(4)=$ where $\mathrm{f}(x)=\pm\left(2\ln(8-x)-x\right)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)
When comparing terms, allow reasons to be $2ln8=3.21>3,\ 2\ln4=2.77<4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**
A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be " it spirals inwards", it gets closer to the root", it converges "



5. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

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Question	Scheme	Marks	AOs
5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\left\{\mathrm{f}(x) = 2x^3 + x^2 - 1 \Rightarrow\right\}\ \mathrm{f}'(x) = 6x^2 + 2x$	B1	1.1b
	$\left\{x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)} \Rightarrow\right\}\ \{x_{n+1}\} = x_n - \dfrac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$	M1	1.1b
	$= \dfrac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \dfrac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$ *	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\}\ x_2 = \dfrac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \dfrac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \dfrac{3}{4},\ x_3 = \dfrac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1 = 0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the *x*-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
			(6 marks)

Notes for Question 5

(a)	
B1:	States that $\mathrm{f}'(x) = 6x^2 + 2x$ or states that $\mathrm{f}'(x_n) = 6x_n^2 + 2x_n$ (Condone $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 + 2x$)
M1:	Substitutes $\mathrm{f}(x_n) = 2x_n^3 + x_n^2 - 1$ and their $\mathrm{f}'(x_n)$ into $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $\mathrm{f}'(x) = 6x^2 + 2x$ is applied as $\mathrm{f}'(x_n)$ (or $\mathrm{f}'(x)$) in the NR formula $\{x_{n+1}\} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$
Note:	Allow M1A1 for • $x_{n+1} = x - \dfrac{2x^3 + x^2 - 1}{6x^2 + 2x} = \dfrac{x(6x^2 + 2x) - (2x^3 + x^2 - 1)}{6x^2 + 2x} \Rightarrow x_{n+1} = \dfrac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$
Note	Condone $x = x - \dfrac{2x^3 + x^2 - 1}{"6x^2 + 2x"}$ for M1
Note	Condone $x_n - \dfrac{2x_n^3 + x_n^2 - 1}{"6x_n^2 + 2x_n"}$ or $x - \dfrac{2x^3 + x^2 - 1}{"6x^2 + 2x"}$ (i.e. no $x_{n+1} = \ldots$) for M1
Note:	Give M0 for $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$ followed by $x_{n+1} = 2x_n^3 + x_n^2 - 1 - \dfrac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*

Notes for Question 5 Continued	
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1=1$
A1:	$x_2=\frac{3}{4}$ and $x_3=\frac{2}{3}$
Note:	Condone $x_2=\frac{3}{4}$ and $x_3=$ awrt 0.667 for A1
Note:	Condone $\frac{3}{4},\frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. • You cannot divide by 0 • The fraction (or the NR formula) is undefined at $x=0$ • At $x=0$, $f'(x_1)=0$ • x_1 cannot be 0 • $6x^2+2x$ cannot be 0 • the denominator is 0 which cannot happen • if $x_1=0$, $6x^2+2x=0$

11.

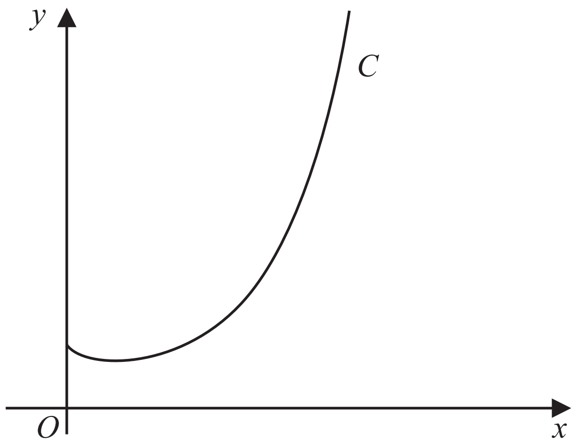

Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)

Question	Scheme	Marks	AOs
11 (a) **Way 1**	$\{y = x^x \Rightarrow\}\;\; \ln y = x\ln x$	B1	1.1a
	$\dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\;\; \dfrac{x}{x} + \ln x = 0 \;\text{ or }\; 1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(a) **Way 2**	$\{y = x^x \Rightarrow\}\;\; y = \mathrm{e}^{x\ln x}$	B1	1.1a
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \left(\dfrac{x}{x} + \ln x\right)\mathrm{e}^{x\ln x}$	M1	1.1b
		A1	2.1
	$\left\{\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\;\; \dfrac{x}{x} + \ln x = 0 \;\text{ or }\; 1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(b) **Way 1**	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}\;\; x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone) Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
	Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
		(2)	
		(11 marks)	

Note	**A common solution** A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x\log x \Rightarrow \dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 + \log x$ $\left\{\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\;\; 1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda\dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x};\; \lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\dfrac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...;\;\; k \neq 0$

Question	Scheme	Marks	AOs
11 (b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 3	For $\ln y = x\ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 4	For $\log y = x\log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question 11

(a)	**Way 1**
B1:	$\ln y = x\ln x$. Condone $\log_x y = x\log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{dy}{dx}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1+\ln x)$ or $\frac{dy}{dx} = x^x(1+\ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x\ln x \to x = 0.368$ with no intermediate working
(a)	**Way 2**
B1:	$y = e^{x\ln x}$
M1:	For either $y = e^{x\ln x} \Rightarrow \frac{dy}{dx} = f(\ln x)e^{x\ln x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x\ln x}$ or $\frac{dy}{dx} = (1+\ln x)e^{x\ln x}$ or $\frac{dy}{dx} = x^x(1+\ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\}$ $\ln y = x\ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1}$ or awrt 0.368

Notes for Question 11 Continued	
(b)	**Way 1**
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} =$ awrt $1.8...$ and $1.6^{1.6} =$ awrt $2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	**Way 2**
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	**Way 3**
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	**Way 4**
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for “Converges in a cob-web pattern” or “Converges up and down to α”

7.

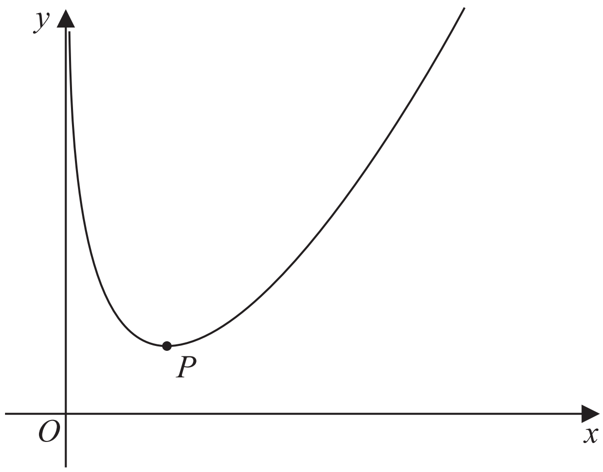

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point P, shown in Figure 1, is the minimum turning point on C.

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

P 6 6 7 8 6 A 0 1 6 5 2

Question	Scheme	Marks	AOs
7(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{dy}{dx}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0 \Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12} \Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	x_2 = awrt 1.13894	A1	1.1b
	x = 1.15650	A1	2.2a
		(3)	
			(10 marks)

Notes:

(a)

B1: Differentiates $\ln x \to \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$:

Look for $\frac{4x^2+x}{2\sqrt{x}} \to ...x^{\frac{3}{2}} + ...x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}} + ...$ **or** $... + Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2+x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{2\sqrt{x}(Ax+B) - (4x^2+x)Cx^{-\frac{1}{2}}}{(2\sqrt{x})^2} (A, B, C > 0)$

But a correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively uses the product rule on $(4x^2+x)(2\sqrt{x})^{-1}$

Condone slips but expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = Ax^{-\frac{1}{2}}(Bx+C) + D(4x^2+x)x^{-\frac{3}{2}} (A, B, C > 0)$

In general condone missing brackets for the M mark. If they quote $u = 4x^2 + x$ and v = 2√*x* and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have *v* rather than v^2 in the denominator.

A1: Correct differentiation of $\frac{4x^2+x}{2\sqrt{x}}$ although may not be simplified.

Examples: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{2\sqrt{x}(8x+1) - (4x^2+x)x^{-\frac{1}{2}}}{(2\sqrt{x})^2}$, $\frac{1}{2}x^{-\frac{1}{2}}(8x+1) - \frac{1}{4}(4x^2+x)x^{-\frac{3}{2}}$, $2 \times \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.

<u>There must be no errors e.g. missing brackets.</u>

(b)

M1: Sets $12x^2 + x - 16\sqrt{x} = 0$ and divides by $\sqrt{x}$ or equivalent e.g. divides by *x* and multiplies by √*x*

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula

A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}} \Rightarrow x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow 12x^{\frac{3}{2}} = 16 - \sqrt{x} \Rightarrow 12x^2 = 16\sqrt{x} - x \Rightarrow 12x^2 - 16\sqrt{x} + x = 0$$

M1: For raising to power of 3/2 both sides. dM1: Multiplies through by √*x*. A1: Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with $x_1 = 2$. This is implied by sight of $x_2 = \left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: x_2 = awrt 1.13894

A1: Deduces that $x = 1.15650$

<u>Via firstly integrating</u>

4. The curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^{\ 2} - 2x_n^{\ 3})$$

is used to find an approximate value for α.

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
4(a)	$\mathrm{f}'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x(2x^2-4x+5)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$*	A1*	2.1
		(4)	
(b)	(i) $x_2 = \dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$\mathrm{h}(x) = 2x^3-4x^2+7x-2$ $\mathrm{h}(0.3415) = 0.00366...$ $\quad \mathrm{h}(0.3405) = -0.00130...$	M1	3.1a
	States: • there is a change of sign • $\mathrm{f}'(x)$ is continuous • $\alpha = 0.341$ to 3dp	A1	2.4
		(2)	
			(9 marks)

Notes

(a)

M1: Differentiates $\ln\left(2x^2-4x+5\right)$ to obtain $\frac{g(x)}{2x^2-4x+5}$ where g(x) could be 1

A1: For $f'(x)=2x+\frac{4x-4}{2x^2-4x+5}$

dM1: Sets their f′(x) $=ax+\frac{g(x)}{2x^2-4x+5}$ = 0 and uses "**correct''** algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax\left(2x^2-4x+5\right)\pm g(x)=0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3-4x^2+7x-2=0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1=0.3$. If no method is shown award for $x_2=\text{awrt } 0.33$

A1: $x_2=\text{awrt } 0.3294$ Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: $x_4=\text{awrt } 0.3398$ Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute x = 0.3415 and x = 0.3405 into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3-4x^2+7x-2,\ x-\frac{1}{7}\left(4x^2-2x^3+2\right)$ and f '(x) as this has been found in part (a) with f '(0.3405)= - 0.00067.., f '(0.3415)= (+) 0.0018

There must be sufficient evidence for the function, which would be for example, a statement such as $h(x)=2x^3-4x^2+7x-2$ or sight of embedded values that imply the function, not just a value or values even if both are correct. Condone $h(x)$ being mislabelled as f

$h(0.3415)=2\times0.3415^3-4\times0.3415^2+7\times0.3415-2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. ✓, proven, α = 0.341, root

8.

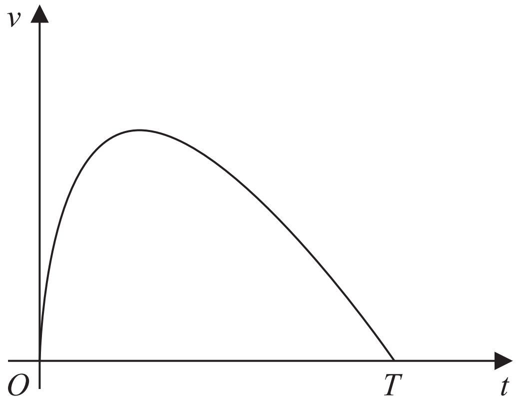

Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v\,\text{ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t)\ln(t + 1) \qquad 0 \leqslant t \leqslant T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

Question	Scheme	Marks	AOs
8 (a)	25	B1	3.4
		(1)	
(b)	Attempts to differentiate using the product rule $\frac{\mathrm{d}v}{\mathrm{d}t} = \ln(t+1)\times -0.4 + \frac{(10-0.4t)}{t+1}$	M1 A1	3.1b 1.1b
	Sets their $\frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Rightarrow \frac{(10-0.4\boldsymbol{t})}{(\boldsymbol{t}+1)} = 0.4\ln(t+1)$ **and** then makes progress towards making "$\boldsymbol{t}$" the subject (See notes for this)	dM1	1.1b
	$t = \frac{25-\ln(t+1)}{1+\ln(t+1)}$ $t = \frac{26}{1+\ln(t+1)} - 1$ *	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = \frac{26}{1+\ln 8} - 1$	M1	1.1b
	awrt 7.298	A1	1.1b
	(ii) awrt 7.33 seconds	A1	3.2a
		(3)	
			(8 marks)
Notes:			

(a)
B1: 25 but condone 25 seconds. If another value is given (apart from 0) it is B0

(b)
M1: Attempts to use the product rule in an attempt to differentiate $v = (10-0.4t)\ln(t+1)$

Look for $(10-0.4t)\times\frac{1}{(t+1)} \pm k\ln(t+1)$, where k is a constant, condoning slips.

If you see direct evidence of an incorrect rule used e.g. $vu' - uv'$ it is M0

You will see attempts from $v = 10\ln(t+1) - 0.4t\ln(t+1)$ which can be similarly marked.

In this case look for $\frac{a}{t+1} \pm \frac{bt}{t+1} \pm c\ln(t+1)$

A1: Correct differentiation. Condone a missing left hand or it seen as v', $\frac{\mathrm{d}y}{\mathrm{d}x}$ or even $= 0$

$\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right) = \ln(t+1)\times -0.4 + \frac{(10-0.4t)}{t+1}$ or equivalent such as $\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right) = \frac{10}{t+1} - \frac{0.4t}{(t+1)} - 0.4\ln(t+1)$

dM1: Score for setting their dV/dt = 0 (which must be in an appropriate form) and proceeding to an equation where the variable t occurs only once – ignoring ln(t + 1).

See two examples of how this can be achieved below. It is dependent upon the previous M.

Look for the following steps

- An allowable derivative set (or implied) = 0 E.g. $\ln(t+1)\times 0.4 = \dfrac{(10-0.4t)}{t+1}$
- Cross multiplication (or division) and rearrangement to form an equation where the variable t only occurs once.

E.g.1.

$$\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$$
$$\Rightarrow \ln(t+1) = \frac{25-t}{t+1}$$
$$\Rightarrow \ln(t+1) = -1+\frac{26}{t+1}$$

E.g 2

$$\ln(t+1)\times 0.4 = \frac{(10-0.4t)}{t+1}$$
$$\Rightarrow 0.4t\ln(t+1)+0.4\ln(t+1) = 10-0.4t$$
$$\Rightarrow 0.4t\left(1+\ln(t+1)\right) = 10-0.4\ln(t+1)$$

A1*: Correctly proceeds to the given answer of $t = \dfrac{26}{1+\ln(t+1)}-1$ showing all key steps.

The key steps must include

- use of $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ or v' which must be correct
- a correct line preceding the given answer, usually $t = \dfrac{25-\ln(t+1)}{1+\ln(t+1)}$ or $\dfrac{26}{t+1}-1 = \ln(t+1)$

(c) (i)

M1: Attempts to use the iteration formula at least once.

Usually to find $t_2 = \dfrac{26}{1+\ln 8}-1$ which may be implied by awrt 7.44

A1: awrt 7.298. This alone will score both marks as iteration is implied. ISW after sight of this value. As t_3 is the only value that rounds to 7.298 just score the rhs, it does not need to be labelled t_3

(c)(ii)

A1: Uses repeated iteration until value established as awrt 7.33 **seconds**. Allow awrt 7.33 **s**

Requires units. It also requires some evidence of iteration which will be usually be awarded from the award of the M

6.

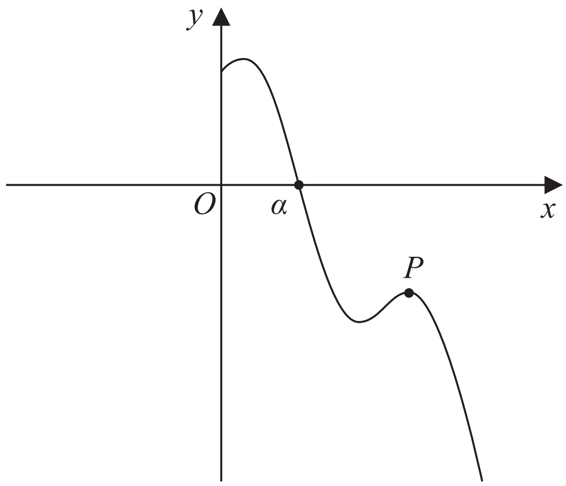

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $\mathrm{f}(4) = 4.274$ and $\mathrm{f}(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to $\mathrm{f}(x)$ to obtain a second approximation to α.

Show your method and give your answer to 3 significant figures.

(2)

Question	Scheme	Marks	AOs
6(a)	$\left(\mathrm{f}'(x)=\right)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1	1.1b 1.1b
	Sets $\mathrm{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=$	dM1	3.1a
	$x=14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $\mathrm{f}(4)>0$, $\mathrm{f}(5)<0$ **and** the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1=5-\frac{8\sin 2.5-15+9}{\text{"}4\cos 2.5-3\text{"}}$ (NB $\mathrm{f}(5)=-1.212\ldots$ and $\mathrm{f}'(5)=-6.204\ldots$)	M1	1.1b
	$x_1=$ awrt 4.80	A1	1.1b
		(2)	
			(7 marks)
Notes:			

(a)

M1: Differentiates to obtain $k\cos\left(\frac{1}{2}x\right)\pm\alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $\mathrm{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3$. Allow unsimplified e.g. $\mathrm{f}'(x)=\frac{1}{2}\times 8\cos\left(\frac{1}{2}x\right)-3x^0$

There is no need for $\mathrm{f}'(x)=\ldots$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for x.

Look for

- $\mathrm{f}'(x)=a\cos\left(\frac{1}{2}x\right)+b=0,\quad a,b\neq 0$
- Correct method of finding a valid solution to $a\cos\left(\frac{1}{2}x\right)+b=0$

Allow for $a\cos\left(\frac{1}{2}x\right)+b=0\Rightarrow\cos\left(\frac{1}{2}x\right)=\pm k\Rightarrow x=2\cos^{-1}(\pm k)$ where $|k|<1$

If this working is not shown then you may need to check their value(s).

For example $4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=1.4\ldots$ or 11.1… (or 82.8… or 637…. or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x=14.0$ and not just 14 or unrounded e.g. 14.011…

Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the y coordinate.

Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)

Accept equivalent statements for $\mathrm{f}(4)>0$, $\mathrm{f}(5)<0$ e.g. $\mathrm{f}(4)\times\mathrm{f}(5)<0$, "there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"

(c)

M1: Attempts $x_1 = 5 - \frac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a "changed" function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees f (5) = –5.65… and f ′(5) = 0.996… and gives $x_1 = 10.67...$

There must be clear evidence that $5 - \frac{f(5)}{f'(5)}$ is being attempted.

so e.g. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80$ scores M0 as does e.g. $x_1 = x - \dfrac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 =$ awrt 4.80 not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$5 - \frac{f(5)}{f'(5)} =$ awrt 4.80 following a correct derivative scores M1A1

$5 - \frac{f(5)}{f'(5)} \neq$ awrt 4.80 with no evidence that $5 - \frac{f(5)}{f'(5)}$ was attempted scores M0