Y2M8 XMQs and MS

(Total: 104 marks)

1. P3_2018 Q6 . 6 marks - Y2M8 Further kinematics
2. P3_2018 Q8 . 8 marks - Y2M8 Further kinematics
3. P3_Sample Q8 . 10 marks - Y2M8 Further kinematics
4. P3_Specimen Q6 . 4 marks - Y2M8 Further kinematics
5. P3_Specimen Q7 . 7 marks - Y2M8 Further kinematics
6. P32_2019 Q1 . 6 marks - Y2M8 Further kinematics
7. P32_2019 Q2 . 8 marks - Y2M8 Further kinematics
8. P32_2020 Q2 . 8 marks - Y2M8 Further kinematics
9. P32_2020 Q3 . 12 marks - Y2M8 Further kinematics
10. P32_2021 Q1 . 4 marks - Y2M8 Further kinematics
11. P32_2021 Q5 . 14 marks - Y2M8 Further kinematics
12. P32_2022 Q1 . 8 marks - Y2M8 Further kinematics
13. P32_2022 Q3 . 9 marks - Y2M8 Further kinematics

SECTION B: MECHANICS

Unless otherwise stated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Answer ALL questions. Write your answers in the spaces provided.

6. At time t seconds, where $t \geqslant 0$, a particle P moves in the x-y plane in such a way that its velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$ is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\mathbf{i} - 4t\mathbf{j}$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B.

Find the exact distance AB.

(6)

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Section B: MECHANICS

Question	Scheme	Marks	AOs
6.	Integrate **v** w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j}\ \ (+\ \mathbf{C})$	A1	1.1b
	Substitute t = 4 and t = 1 into their **r**	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C});\ t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4, -32); (2, -2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
		(6 marks)	

Notes: Allow column vectors throughout

M1: At least one power increasing by 1.

A1: Any correct (unsimplified) expression

M1: Must have attempted to integrate **v**. Substitute t = 4 and t = 1 into their **r** to produce 2 vectors (or 2 points if just working with coordinates).

A1: $4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C})$ and $2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4, -32)$ and $(2, -2)$. These can be seen or implied.

M1: Attempt at distance of form $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for their points. Must have 2 non zero terms.

A1: $\sqrt{904} = 2\sqrt{226}$ or any equivalent surd (exact answer needed)

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O*.]

A particle P moves with constant acceleration.
At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$
At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})\,\text{m}$.

(a) Show that the magnitude of the acceleration of P is $2.5\,\text{m s}^{-2}$

(4)

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\,\text{m s}^{-2}$

At the instant when P reaches the point B, the direction of motion of P is north east.

(b) Find the time it takes for P to travel from A to B.

(4)

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	$= 2.5$ m s^{-2} * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$= (5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	$t = 2.5$ (s)	A1	1.1b
		(4)	
		(8 marks)	

Notes: Allow column vectors throughout

(a)

No credit for individual component calculations

M1: Using a complete method to obtain the acceleration. **N.B.** Equation, in **a** only, could be obtained by two integrations

ALTERNATIVE

M1: Use velocity at half-time (t = 1) = Average velocity over time period

So at t = 1, $\mathbf{v} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j})$ so $\mathbf{a} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j}) - (2\mathbf{i} - 3\mathbf{j})$

N.B. could see $(7\mathbf{i} - 10\mathbf{j}) = (4\mathbf{i} - 6\mathbf{j}) + 2\mathbf{a}$ as first line of working

A1: Correct **a** vector

M1: Attempt to find magnitude of their **a** using form $\sqrt{a^2 + b^2}$

A1*: Correct GIVEN ANSWER obtained correctly

(b)

M1: Using a complete method to obtain the velocity at A e.g.by use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with t = 2 and $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$ and their **a**

OR: by use of $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

OR: by integrating their **a,** with addition of $\mathbf{C} = 2\mathbf{i} - 3\mathbf{j}$, and putting t = 2

A1: correct vector

M1: Complete method to find equation in t only e.g. by using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, with their **u** and equating **i** and **j** components

OR: by integrating (4**i** + 8.8**j**), with addition of a constant, and equating **i** and **j** components.

N.B. Must be equating **i** and **j** components of a velocity vector and must be their velocity at *A*, to give an equation in *t* only for this M mark

A1: 2.5 (s)

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}. **(2)**

(b) Find **r** in terms of t. **(2)**

(c) Find the value of t when the boat is north-east of O. **(3)**

(d) Find the value of t when the boat is moving in a north-east direction. **(3)**

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the **i** and **j** components of **r**	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7\,t^2 = 0.6\,t - \frac{1}{2} \leftarrow 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the **i** and **j** components of **v**	M1	3.1b
	$t = 0.75$	A1 **ft**	1.1b
		(3)	
			(10 marks)

Notes:

(a)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

A1: for given answer correctly obtained

(b)

M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

A1: for a correct expression for **r** in terms of t

(c)

M1: for equating the **i** and **j** components of their **r**

A1ft: for a correct equation following their **r**

A1: for $t = 1.5$

(d)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general t

M1: for equating the **i** and **j** components of their **v**

A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

[In this question position vectors are given relative to a fixed origin O.]

6. A particle, P, moves with constant acceleration $(\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-2}$.

At time t = 0 seconds, the particle is at the point A with position vector $(2\mathbf{i} + 5\mathbf{j})$ m and is moving with velocity $\mathbf{u} \text{ m s}^{-1}$.

At time t = 3 seconds, P is at the point B with position vector $(-2.5\mathbf{i} + 8\mathbf{j})$ m.

Find $\mathbf{u}$.

(4)

9MA0/03 Mock Paper: Part B Mechanics Mark scheme

Question	Scheme	Marks	AOs
1	$\mathbf{r} = (-4.5\mathbf{i} + 3\mathbf{j})$	B1	1.1b
	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$(-4.5\mathbf{i} + 3\mathbf{j}) = 3\mathbf{u} + 0.5(\mathbf{i} - 2\mathbf{j})\ 3^2$	A1**ft**	1.1b
	$\mathbf{u} = (-3\mathbf{i} + 4\mathbf{j})$	A1	1.1b
		(4)	
			(4 marks)

Notes:

B1: Correct displacement vector

M1: Use of correct strategy and/or formula to give equation in **u** only (could be obtained by two integrations)

A1ft: Correct equation in **u** only, following their displacement vector

A1: Correct answer

7. A particle, P, moves under the action of a single force in such a way that at time t seconds, where $t \geq 0$, its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = (t^2 - 3t)\,\mathbf{i} - 12t\,\mathbf{j}$$

The mass of P is 0.5 kg.

Find the time at which the magnitude of the force acting on P is 6.5 N.

(7)

Question	Scheme	Marks	AOs
2	Differentiate wrt t	M1	1.1a
	$\mathbf{a} = (2t - 3)\,\mathbf{i} - 12\,\mathbf{j}$	A1	1.1b
	$(2t - 3)^2 + (-12)^2$	M1	1.1b
	$(2t - 3)^2 + (-12)^2 = (6.5 / 0.5)^2$ oe	M1	2.1
	$4t^2 - 12t - 16 = 0$	A1	1.1b
	$(t - 4)(t + 1) = 0$	M1	1.1b
	$t = 4$	A1	1.1b
		(7)	
			(7 marks)

Notes:

M1: At least one power going down
A1: A correct expression
M1: Sum of squares of components (with or without square root) of **a** or **F**
M1: Equating magnitude to 6.5/0.5 or 6.5 as appropriate and squaring both sides
A1: Correct quadratic = 0 in any form
M1: Attempt to solve a 3 term quadratic
A1: 4

Answer ALL questions. Write your answers in the spaces provided.

1. [*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geqslant 0$, a particle, P, moves so that its velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})\,\mathrm{m}$.

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

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Question	Scheme	Marks	AO
1(a)	Differentiate **v**	M1	1.1a
	$(\mathbf{a}=)6\mathbf{i}-\frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
	$=6\mathbf{i}-15\mathbf{j}$ (m s^{-2})	A1	1.1b
		(3)	
1(b)	Integrate **v**	M1	1.1a
	$(\mathbf{r}=)(\mathbf{r}_0)+3t^2\mathbf{i}-2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
	$=(-20\mathbf{i}+20\mathbf{j})+(48\mathbf{i}-64\mathbf{j})=28\mathbf{i}-44\mathbf{j}$ (m)	A1	2.2a
		(3)	
		(6)	

Marks		Notes
		N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers
1a	M1	Use of $\mathbf{a}=\frac{d\mathbf{v}}{dt}$ with attempt to differentiate (both powers decreasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct differentiation in any form
	A1	Correct and simplified. Ignore subsequent working (ISW) if they go on and find the magnitude.
1b	M1	Use of $\mathbf{r}=\int \mathbf{v}\,dt$ with attempt to integrate (both powers increasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct integration in any form. Condone $\mathbf{r}_0$ not present
	A1	Correct and simplified.

2. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(b) Find the distance AB.

(4)

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Question	Scheme	Marks	AO
2(a)	$(\mathbf{v}=)\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$	M1	3.1a
	$(\mathbf{v}=)(-\mathbf{i}+4\mathbf{j})+(2\mathbf{i}-3\mathbf{j})t$	A1	1.1b
	$\frac{4-3T}{-1+2T}=\frac{-4}{3}$ oe	M1	3.1a
	$T=8$	A1	1.1b
		(4)	
(b)	$(\mathbf{s}=)\mathbf{C}t+(2\mathbf{i}-3\mathbf{j})\frac{1}{2}t^2$ $(+\mathbf{D})$	M1	3.1a
	$(\mathbf{s}=)(-\mathbf{i}+4\mathbf{j})t+\frac{1}{2}(2\mathbf{i}-3\mathbf{j})t^2$ $(+\mathbf{D})$	A1	1.1b
	$AB=\sqrt{12^2+8^2}$ **N.B. Beware you may see 4(2i – 3j) which leads to** $\sqrt{(8^2+12^2)}$ this is M0A0M0A0.	M1	3.1a
	$=4\sqrt{13}\,(=14.422051....)$ (m)	A1**cso**	1.1b
		(4)	
		(8)	

Marks		Notes
2a	M1	Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ **OR** integration to give an expression of the form $\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$ **, where C is a non-zero constant vector** M0 if **u** and **a** are reversed Condone use of **a** = (2**i** + 3**j**) for this M mark
	A1	Any correct unsimplified expression seen or implied
	M1	Correct use of ratios, using a velocity vector (must be using $\frac{-4}{3}$) to give equation in T only M0 if they equate $4-3T=-4$ and/or $-1+2T=3$ and therefore M0 if they then divide to produce their equation
	A1	Correct only
		N.B. (i) Can score the second M1A1 if they get T = 8, using a calculator to solve two simultaneous equations, but if answer is wrong, and no equation in T only, second M0 (ii) Can score M1A1 M1A1 if they get T = 8, using trial and error, but if they don't get T = 8, can only score max M1A1M0A0

2b	M1	Use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **a = (2i – 3j)** **OR** integration to give an expression of the form $\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2$ **, where C is their non-zero constant <u>vector</u> from (a)** Condone use of **a** = (2**i + 3j)** for this M mark **OR** any other complete method using vector **suva***t* equations
	A1	Correct unsimplified expression seen or implied
	M1	Use of $t = 4$ in their **s** (which must be a **displacement vector**) and then Pythagoras with the root sign **N.B.** This M mark can be implied by a correct answer, otherwise we need to see Pythagoras used, with the root sign, for the M mark.
	A1**cso**	Any surd form or 14 or better

2. A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

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Question	Scheme	Marks	AOs
2(a)	Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ or integrate to give: $\mathbf{v}=(-2\mathbf{i}+2\mathbf{j})+2(4\mathbf{i}-5\mathbf{j})$	M1	3.1a
	$(6\mathbf{i}-8\mathbf{j})\ (\text{m s}^{-1})$	A1	1.1b
		(2)	
2(b)	Solve problem through use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ or integration (M0 if **u** = **0**) **Or** any other complete method e.g use **v** = **u** + **a***T* and $\mathbf{r}=\frac{(\mathbf{u}+\mathbf{v})T}{2}$:	M1	3.1a
	$-4.5\mathbf{j}=2t\mathbf{j}-\frac{1}{2}t^2 5\mathbf{j}$ (**j** terms only)	A1	1.1b
	The first two marks could be implied if they go straight to an algebraic equation.		
	Attempt to equate **j** components to give equation in T only $(-4.5=2T-\frac{5}{2}T^2)$	M1	2.1
	T = 1.8	A1	1.1b
		(4)	
2(c)	Solve problem by substituting their T value (M0 if T < 0) into the **i** component equation to give an equation in λ only: $\lambda=-2T+\frac{1}{2}T^2\times 4$	M1	3.1a
	λ = 2.9 or 2.88 or $\frac{72}{25}$ oe	A1	1.1b
		(2)	

Notes: Accept column vectors throughout **(8 marks)**

2a	M1	For any complete method to give a **v** expression with correct no. of terms with t = 2 used, so if integrating, must see the initial velocity as the constant. Allow sign errors.
	A1	Cao isw if they go on to find the speed.
2b	M1	For any complete method to give a vector expression for **j** component of displacement in t (or T) only, using **a** = (4**i** – 5**j**), so if integrating, RHS of equation must have the correct structure. Allow sign errors.
	A1	Correct **j** vector equation in t or T. Ignore **i** terms.
	M1	Must have earned 1st M mark.

		Equate **j** components to give equation in *T* (allow *t*) only (no **j**'s) which has come from a displacement. Equation must be a 3 term quadratic in *T*.
	A1	cao
2c	M1	Must have earned 1st M mark in (b) Complete method - must have an equation in λ only (no **i**'s) which has come from an appropriate displacement.. (e.g M0 if **a** = **0** has been used) Expression for λ must be a quadratic in *T*
	A1	cao

3. (i) At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i}\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(ii) At time t seconds, where $t \geqslant 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is $5\,\mathrm{m\,s^{-1}}$

(6)

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Question	Scheme	Marks	AOs
3(i)(a)	Integrate **a** wrt t to obtain velocity	M1	3.4
	$\mathbf{v} = (t - 2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j}\ (+\mathbf{C})$	A1	1.1b
	$8\mathbf{i} - \frac{28}{3}\mathbf{j}\ (\text{m s}^{-1})$	A1	1.1b
		(3)	
3(i)(b)	Equate **i** component of **v** to zero	M1	3.1a
	$t - 2t^2 + 36 = 0$	A1**ft**	1.1b
	$t = 4.5$ (ignore an incorrect second solution)	A1	1.1b
		(3)	
3(ii)	Differentiate **r** wrt to t to obtain velocity	M1	3.4
	$\mathbf{v} = (2t - 1)\mathbf{i} + 3\mathbf{j}$	A1	1.1b
	Use magnitude to give an equation in t only	M1	2.1
	$(2t-1)^2 + 3^2 = 5^2$	A1	1.1b
	Solve problem by solving this equation for t	M1	3.1a
	$t = 2.5$	A1	1.1b
		(6)	
		(12 marks)	

Notes: Accept column vectors throughout

3(i)(a)	M1	At least 3 terms with powers increasing by 1 (but M0 if clearly just multiplying by t)
	A1	Correct expression
	A1	Accept $8\mathbf{i} - 9.3\mathbf{j}$ or better. Isw if speed found.
3(i)(b)	M1	Must have an equation in t only (Must have integrated to find a velocity vector)
	A1 **ft**	Correct equation follow through on their **v** but must be a 3 term quadratic
	A1	cao
3(ii)	M1	At least 2 terms with powers decreasing by 1 (but M0 if clearly just dividing by t)
	A1	Correct expression
	M1	Use magnitude to give an equation in t only, must have differentiated to find a velocity (M0 if they use $\sqrt{x^2 - y^2}$)

	A1	Correct equation $\sqrt{(2t-1)^2+3^2}=5$
	M1	Solve a 3 term quadratic for t which has come from differentiating and using a magnitude. This M mark can be implied by a correct answer with no working.
	A1	2.5

1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\text{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

Question	Scheme	Marks	AOs
1(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $t = 2$: $\mathbf{v} = 4\mathbf{i} + 2(2\mathbf{i} - 3\mathbf{j})$ **OR integration**: $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})t + 4\mathbf{i}$,with $t = 2$	M1	3.1a
	$\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$	A1	1.1b
		(2)	
1(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ at $t = 3$: $(\mathbf{i}+\mathbf{j}) + \left[3 \times 4\mathbf{i} + \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR**: find **v** at $t = 3$: $4\mathbf{i} + 3(2\mathbf{i} - 3\mathbf{j}) = (10\mathbf{i} - 9\mathbf{j})$ then use $\mathbf{r} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$ $(\mathbf{i}+\mathbf{j}) + \left[\frac{1}{2}[4\mathbf{i} + (10\mathbf{i} - 9\mathbf{j})] \times 3\right]$ or $\mathbf{r} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $(\mathbf{i}+\mathbf{j}) + \left[3 \times (10\mathbf{i} - 9\mathbf{j}) - \frac{1}{2} \times (2\mathbf{i} - 3\mathbf{j}) \times 3^2\right]$ **OR integration:** $\mathbf{r} = (\mathbf{i}+\mathbf{j}) + \left[(2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2 + 4t\mathbf{i}\right]$, with $t = 3$	M1	3.1a
	$\mathbf{r} = 22\mathbf{i} - 12.5\mathbf{j}$	A1	2.2a
		(2)	
		(4 marks)	

Notes: Accept column vectors throughout

1a	M1	Complete method to find **v**, using **ruva***t* or integration (M0 if **i** and/or **j** is missing)
	A1	Apply isw if they also find the speed
1b	M1	Complete method to find the p.v. but this mark can be scored if they omit (**i** + **j**) i.e. the M1 is for the expression in the square bracket If they integrate, the M1 is earned once the expression in the square bracket is seen with $t = 3$ (M0 if **i** and/or **j** is missing)
	A1	cao

5. At time t seconds, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\mathrm{m\,s^{-1}}$

(6)

4c	B1	B0 if incorrect extras

Question	Scheme	Marks	AOs
	Allow column vectors throughout this question		
5(a)	Differentiate **v** wrt t	M1	3.1a
	$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i}-2\mathbf{j}$ isw	A1	1.1b
		(2)	
5(b)	$3t^{\frac{1}{2}}=2t$	M1	2.1
	Solve for t	DM1	1.1b
	$t=\frac{9}{4}$	A1	1.1b
		(3)	
5(c)	Integrate **v** wrt t	M1	3.1a
	$\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}(+\mathbf{C})$	A1	1.1b
	t = 1, **r** = $-\mathbf{j}$ => **C** = $-2\mathbf{i}$ so $\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}-2\mathbf{i}$	A1	2.2a
		(3)	
5(d)	$\sqrt{(3t^{\frac{1}{2}})^2+(2t)^2}=10$ or $(3t^{\frac{1}{2}})^2+(2t)^2=10^2$	M1	2.1
	$9t+4t^2=100$	M(A)1	1.1b
	t = 4	A1	1.1b
	$\mathbf{r}=14\mathbf{i}-16\mathbf{j}$	M1	1.1b
	$\sqrt{14^2+(-16)^2}$	M1	3.1a
	$\sqrt{452}$ $(2\sqrt{113})$ (m)	A1	1.1b
		(6)	
			(14 marks)

Notes:

5a	M1	Both powers decreasing by 1 (M0 if vector(s) disappear but allow recovery)
	A1	cao
5b	M1	Complete method, using **v**, to obtain an equation in t only, allow a sign error
	DM1	Dependent on M1,solve for t

	A1	cao
5c	M1	Both powers increasing by 1 (M0 if vectors disappear but allow recovery)
	A1	Correct expression without **C**
	A1	cao
5d	M1	Use of Pythagoras on **v** and 10 to set up equation in *t*
	M(A)1	Correct 3 term quadratic in *t*
	A1	cao
	M1	Substitute their numerical *t* value into their **r**
	M1	Use of Pythagoras to find the magnitude of their **r**
	A1	cso

1. [*In this question, position vectors are given relative to a fixed origin*.]

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})\,\text{m}$.

(c) Find the position vector of P at time $t = 1$ second.

(4)

Question	Scheme	Marks	AOs
1(a)	Put $t = 2$ in **v** and use Pythagoras: $\sqrt{12^2 + (-6\sqrt{2})^2}$	M1	3.1a
	$\sqrt{216}, 6\sqrt{6}$ or 15 or better (m s^{-1})	A1	1.1b
		(2)	
1(b)	Differentiate **v** wrt t to obtain **a**	M1	3.4
	$6t\mathbf{i} - 3t^{-\frac{1}{2}}\mathbf{j}$ oe (m s^{-2}) isw	A1	1.1b
		(2)	
1(c)	Integrate **v** wrt t to obtain **r**	M1	3.4
	$\mathbf{r} = t^3\mathbf{i} - 4t^{\frac{3}{2}}\mathbf{j}\ (+\mathbf{C})$	A1	1.1b
	$(\mathbf{i} - 4\mathbf{j}) = 4^3\mathbf{i} - 4\times 4^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$	M1	3.1a
	$(-62\mathbf{i} + 24\mathbf{j})$ (m) isw e.g. if they go on to find the distance.	A1	1.1b
		(4)	
			(8 marks)

Notes: Accept column vectors throughout apart from the answer to (b).

1a	M1	Need square root but -ve sign not required. Allow **i**'s and/or **j**'s to go missing from their **v** at $t = 2$, provided they have applied Pythagoras correctly.
	A1	cao **N.B**. Correct answer with no working can score 2 marks.
1b	M1	Both powers decreasing by 1. Allow a column vector. M0 if **i** or **j** is missing but allow recovery in (b).
	A1	cao. Do not accept a column vector.
1c	M1	Both powers increasing by 1 M0 if **i** or **j** is missing but allow recovery.
	A1	(**r** =) not required
	M1	Putting $\mathbf{r} = (\mathbf{i} - 4\mathbf{j})$ and $t = 4$ into their displacement **vector** expression which must have **C** (allow C) to give an equation in **C** only, seen or implied. Must have attempted to integrate **v** for this mark to be available. **N.B**. **C** does not need to be found and this is a method mark, so allow slips.
	A1	cao

3. [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})\,\text{N}$ and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})\,\text{N}$, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

(4)

At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

Question	Scheme	Marks	AOs
3(a)	$(4\mathbf{i}-\mathbf{j})+(\lambda\mathbf{i}+\mu\mathbf{j})=(4+\lambda)\mathbf{i}+(-1+\mu)\mathbf{j}$	M1	3.4
	Use **ratios** to obtain an equation in λ and μ *only*	M1	2.1
	$\dfrac{(4+\lambda)}{(-1+\mu)}=\dfrac{3}{1}$ or $\dfrac{\frac{1}{4}(4+\lambda)}{\frac{1}{4}(-1+\mu)}=\dfrac{3}{1}$	A1	1.1b
	$\lambda-3\mu+7=0$ * Allow $0=\lambda-3\mu+7$ but nothing else.	A1*	1.1b
		(4)	
(b)	$\lambda=2 \Rightarrow \mu=3$; Resultant force $=(6\mathbf{i}+2\mathbf{j})$ (N)	M1	3.1a
	$(6\mathbf{i}+2\mathbf{j})=4\mathbf{a}$ **OR** $\lvert(6\mathbf{i}+2\mathbf{j})\rvert=4a$	M1	1.1b
	Use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with **u** = **0**, their **a** and $t=4$: **Or** they may integrate their **a** twice with **u** = **0** and put $t=4$: $\mathbf{r}=\frac{1}{2}\times\frac{(6\mathbf{i}+2\mathbf{j})}{4}4^2=(12\mathbf{i}+4\mathbf{j})$	DM1	2.1
	$\sqrt{12^2+4^2}$	M1	1.1b
	ALTERNATIVE 1 for last two M marks: Use of $s=ut+\frac{1}{2}at^2$, with $u=0$, their a and $t=4$: DM1 $s=\frac{1}{2}\times\sqrt{1.5^2+0.5^2}\times4^2$ Use of Pythagoras to find mag of **a** : $a=\sqrt{1.5^2+0.5^2}$ M1		
	ALTERNATIVE 2 for last two M marks: Use of $s=ut+\frac{1}{2}at^2$, with $u=0$, their a and $t=4$: DM1 $s=\frac{1}{2}\times\left(\frac{\sqrt{6^2+2^2}}{4}\right)\times4^2$ Use of Pythagoras to find $\lvert(6\mathbf{i}+2\mathbf{j})\rvert$: $=\sqrt{6^2+2^2}$ M1		
	$\sqrt{160}$, $2\sqrt{40}$, $4\sqrt{10}$ oe or 13 or better (m)	A1	1.1b
		(5)	
		(9 marks)	

Notes: Accept column vectors throughout

3a	M1	Adding the two forces, **i**'s and **j**'s must be collected (or must be a **single** column vector) seen or implied
	M1	Must be using ratios; Ignore an equation e.g. $(4+\lambda)\mathbf{i}+(-1+\mu)\mathbf{j}=3\mathbf{i}+\mathbf{j}$ if they go on to use ratios.

		However, if they write $4+\lambda=3$ and $-1+\mu=1$ then $3(-1+\mu)=3$ so $4+\lambda=3(-1+\mu)$ with no use of a constant, it's M0 They may use the acceleration, with a factor of $\frac{1}{4}$ top and bottom, see alternative **Allow one side of the equation to be inverted**
	A1	Correct equation
	A1*	Given answer correctly obtained. Must see at least one line of working, with the LH fraction 'removed'**.**
3b	M1	Adding $\mathbf{F}_1$ and $\mathbf{F}_2$ to find the resultant force, λ and μ must be substituted **N.B.** M0 if they use $\mu=2$ coming from $-1+\mu=1$ in part (a).
	M1	Use of $\mathbf{F}=4\mathbf{a}$ Or $\lvert\mathbf{F}\rvert=4a$, where **F** is <u>their</u> resultant. (including 3**i** + **j**) This is an independent mark, so could be earned, for example, if they have subtracted the forces to find the 'resultant' **N.B.** M0 if only using $\mathbf{F}_1$ or $\mathbf{F}_2$
	DM 1	Dependent on previous M mark for **Either**: use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with **u** = **0**, their **a** and t = 4 to produce a displacement vector **Or** : integrate twice, with **u** = **0**, their **a** and t = 4 to produce a displacement Vector **Or**: use of $s=ut+\frac{1}{2}at^2$ with u = 0, their a and t = 4 to produce a length
	M1	Use of Pythagoras, with square root, to find the magnitude of their displacement vector, **a** or **F** **(**M0 if only using $\mathbf{F}_1$ or $\mathbf{F}_2$) depending on which method they have used.
	A1	cao