Y2M6 XMQs and MS

(Total: 88 marks)

1. P3_2018 Q10. 15 marks - Y2M6 Projectiles
2. P3_Sample Q10. 13 marks - Y2M6 Projectiles
3. P3_Specimen Q10. 14 marks - Y2M6 Projectiles
4. P32_2019 Q5 . 13 marks - Y2M6 Projectiles
5. P32_2020 Q5 . 11 marks - Y2M6 Projectiles
6. P32_2021 Q4 . 10 marks - Y2M6 Projectiles
7. P32_2022 Q5 . 12 marks - Y2M6 Projectiles

10.

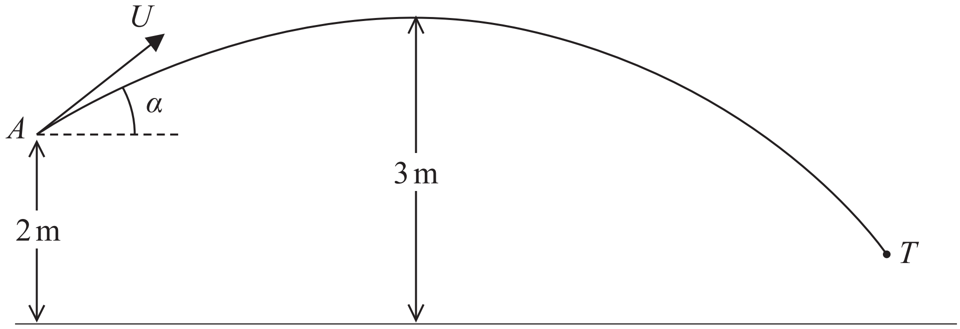

Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground.
The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

Question	Scheme	Marks	AOs
10(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g\times(3-2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\frac{5}{4} = U\sin\alpha\left(\frac{20}{U\cos\alpha}\right) - \frac{1}{2}g\left(\frac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \frac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
	Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
	$-\frac{5}{4} = \sqrt{2g}t - \frac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut\cos\alpha$	M1(I)	
	$\alpha = 14^0$ or better	A1(II)	
(b)	**ALTERNATIVE**		

	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos a$	A1	1.1b
	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$		
	$1 = \frac{1}{2}gt_1^2$ => $t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2$ => $t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $\left(= \frac{5}{\sqrt{2g}}\right)$	A1	1.1b
	$20 = U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
	$20 = \sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b
	$\tan a = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	Þ $a = 14^0$ or better	A1	2.2a
		(9)	
(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\frac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20 = Ut\cos a$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	
(d)	**ALTERNATIVE**		

	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}g{t_1}^2$ => $t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}g{t_2}^2$ => $t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 **M1**	1.1b
	= = $\sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $\left(= \frac{5}{\sqrt{2g}}\right)$ = 1.1 or 1.13 (s)	B1 **A1**	1.1b
		(3)	
			(15 marks)

Notes:

(a)

M1: Or any other complete method to obtain an equation in U, g and α **only**

A1*: Correct GIVEN ANSWER

(b)

M1: Using horizontal motion

A1: Correct equation

M1: Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3, but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75

A1: Correct equation

M1: Using $20 = Ut\cos\alpha$ to sub. for t

M1: Substituting for U^2 using (a)

A1: Correct quadratic equation (in tan α **or cot α)**

M1: Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) **and find α**

A1: $\alpha = 14^\circ$ or better (No restriction on accuracy since g's cancel)

N.B. If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark.

(b) ALTERNATIVE

M1: Using the model with the usual rules applying to the equation

A1: Correct equation

M1: Using the model to obtain the **total** time from A to T

A1: Correct **total** time t

M1: Substitute for t in $20 = Ut\cos\alpha$

M1: Substitute for U in $20 = Ut\cos\alpha$, using part (a)

A1: Correct equation in tan α **only**

M1: Solve equation for α

A1: $\alpha = 14^\circ$ or better (No restriction on accuracy since g's cancel)

N.B. If they quote the equation of the trajectory $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$ oe **AND** put in values for *x* and *y*, could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong *x* value loses first A mark and wrong *y* value loses second A mark

(c)

B1: Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin

N.B. B0 if any incorrect extra(s) but ignore extra consequences.

(d)

M1: Using their a to find a value for *U*

A1: Treat as M1: Using their *U* to find a value for *t*

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

(d) ALTERNATIVE

M1: Using their a to find a value for *U*

A1: Treat as M1: Using their *U* to find a value for *t*

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

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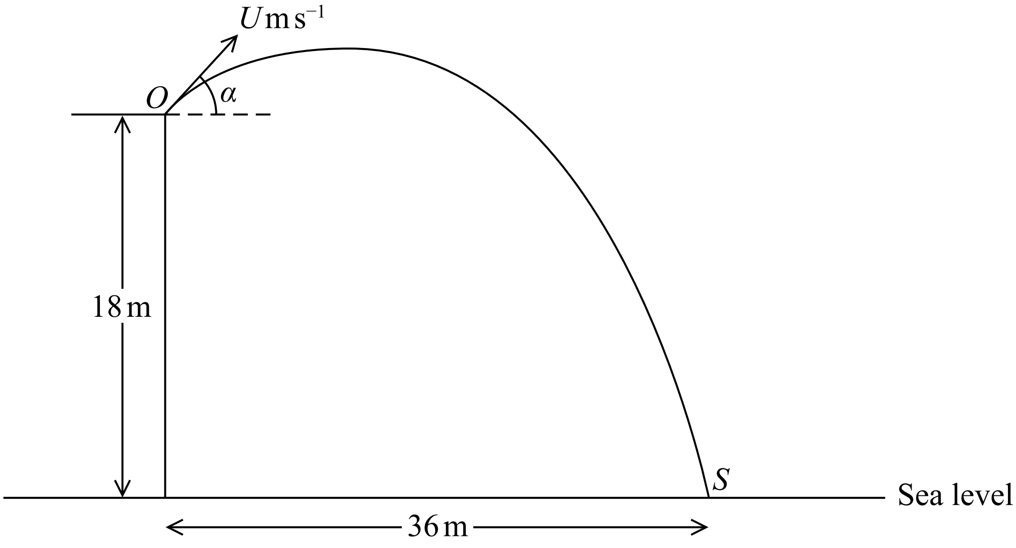

Figure 2

A boy throws a stone with speed $U\,\mathrm{m\,s^{-1}}$ from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10\,\mathrm{m\,s^{-2}}$

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

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Question	Scheme	Marks	AOs
10(a)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-18 = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1	3.1b
	$U = 15$	A1	1.1b
		(6)	
(b)	Using the model and horizontal motion: $U\cos\alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U\sin\alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19$ m s^{-1} (2sf)	A1 **ft**	1.1b
		(5)	
(c)	Possible improvement (see below in notes)	B1	3.5c
	Possible improvement (see below in notes)	B1	3.5c
		(2)	
			(13 marks)

Question 10 continued
Notes:
(a) **1st M1:** for use of $s = ut$ horizontally **1st A1:** for a correct equation **2nd M1:** for use of $s = ut + \frac{1}{2}at^2$ vertically **2nd A1:** for a correct equation **3rd M1:** for correct strategy (need both equations) **2nd A1:** for $U = 15$
(b) **B1:** for $U\cos\alpha$ used as horizontal velocity component **1st M1:** for attempt to find vertical component **1st A1:** for 15 **2nd M1:** for correct strategy (need both components) **2nd A1ft:** for 19 m s $^{-1}$ (2sf) following through on incorrect component(s)
(c) **B1, B1:** for any two of e.g. Include air resistance in the model of the motion e.g. Use a more accurate value for g in the model of the motion e.g. Include wind effects in the model of the motion e.g. Include the dimensions of the stone in the model of the motion

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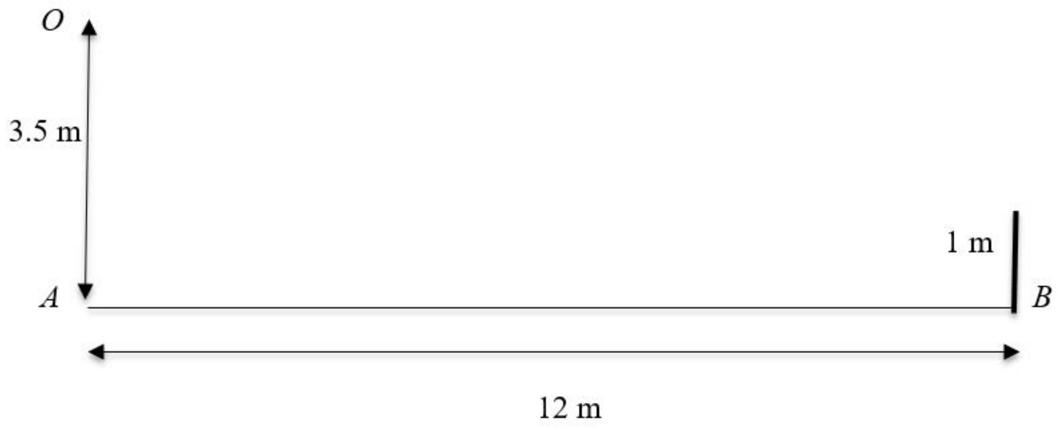

Figure 3

A tennis player serves a ball so as to pass over the net.
The ball is given an initial velocity of 45 m s^{-1} in a direction 10° below the horizontal.

The ball is struck at a point *O* which is 3.5 m vertically above the point *A* which is on horizontal ground.
The bottom of the net is the point *B* which is on the ground and *AB* = 12 m.
The height of the net is 1 m, as shown in Figure 3.

The ball is modelled as a particle moving freely under gravity.
The ball passes over the net at a point which is vertically above B.

Using the model,

(a) find, in centimetres to 2 significant figures, the distance between the ball and the top of the net, as the ball passes over the net,

(8)

(b) find, to 2 significant figures, the speed of the ball as it passes over the net.

(4)

(c) State two limitations of the model that could affect the reliability of your answers.

(2)

Question	Scheme	Marks	AOs
5(a)	Using the model and horizontal motion: $s = ut$	M1	3.3
	$12 = T \times 45\cos 10^{\circ}$	A1	1.1b
	$T = 0.2707..$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$s = 45T\sin 10^{\circ} + 4.9T^2$	A1	1.1b
	Correct strategy: sub for T and find s	M1	3.1b
	$d = 3.5 - 2.4752 - 1$	M1	3.1b
	$= 2.5$ (cm) (2 SF)	A1	2.2a
		(8)	
(b)	Using the model and vertical motion: $v = u + at$	M1	3.3
	$v = 45\sin 10^{\circ} + 9.8T$	A1	1.1b
	Speed $= ((45\cos 10^{\circ})^2 + v^2)^{0.5}$	M1	3.1b
	46 (m s^{-1}) (2 SF)	A1	1.1b
		(4)	
(c)	Model does not take account of air resistance.	B1	3.5b
	Model does not take account of the size of the tennis ball	B1	3.5b
		(2)	
		(14 marks)	

Notes:

(a)
M1: Using the model and correct strategy
A1: Correct equation in T only
A1: 0.271 or better
M1: Using the model and correct strategy
A1: Correct equation
M1: Sub for T and solve for s
M1: Correct method to find d using their s
A1: 2.5 is the only correct answer

(b)
M1: Using the model and correct strategy
A1: Correct equation
M1: Must have found a v and usual rules apply. Square root is needed.

A1: 46 (2 SF) is only correct answer **(c)** **B1:** Other appropriate answer e.g. spin of the ball, wind effect **B1:** Other appropriate answer e.g. spin of the ball, wind effect

5.

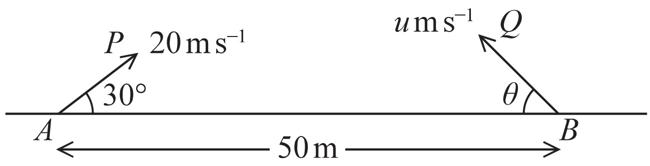

Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed $20\,\text{m s}^{-1}$ at 30° to AB.

Ball Q is projected from B with speed $u\,\text{m s}^{-1}$ at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		
5(a)	Horizontal speed $=20\cos 30°$	B1	3.4
	Vertical velocity at t = 2	M1	3.4
	$=20\sin 30° - 2g$	A1	1.1b
	$\theta = \tan^{-1}\left(\pm\frac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $=\sqrt{100\times 3+9.6^2}$ or e.g. speed = $\frac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 $\left(\text{m s}^{-1}\right)$ at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances $=50$ at t = 2	M1	3.3
	$(u\cos\theta)\times 2 + (20\cos 30°)\times 2 = 50$ $(u\cos\theta = 25 - 20\cos 30°)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow (20\sin 30°)\times 2 - \frac{g}{2}\times 4 = (u\sin\theta)\times 2 - \frac{g}{2}\times 4$ $(20\sin 30° = u\sin\theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	θ = 52° or better (52.47756849….°) u = 13 or better (12.6085128…)	A1	2.2a
		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

Marks		Notes
5a	B1	Seen or implied, possibly on a diagram
	M1	Use of $v = u + at$ or any other complete method <u>using t = 2</u> Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation in v or v^2
	M1	Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component
	M1	Use Pythagoras or trig to find the magnitude Must have found a horizontal and a vertical velocity component
	A1	Or equivalent. Need magnitude **and** direction stated or implied in a diagram. (0.506 or 0.51 rads)
5b	M1	First equation,in terms of u and θ (could be implied by subsequent working), using the horizontal motion with t = 2 used Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Second equation, in terms of u and θ (could be implied by subsequent working), using the vertical motion – equating distances or just vertical components of velocities. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Complete strategy: all necessary equations formed and solve for u and θ **N.B.** This is an independent method mark but can only be earned if 50 m has been used in their solution.
	A1	Both values correct. (Here we accept 2SF or better, since the g's cancel) Allow radians for θ: 0.92 or better (0.915906..) rads.
5c	B1	Any factor **related to the model** as stated in the question. Penalise incorrect extras but ignore consequences e.g. 'AB (or the ground) is not horizontal' should be penalised or 'they do not move in a vertical plane' should be penalised

5.

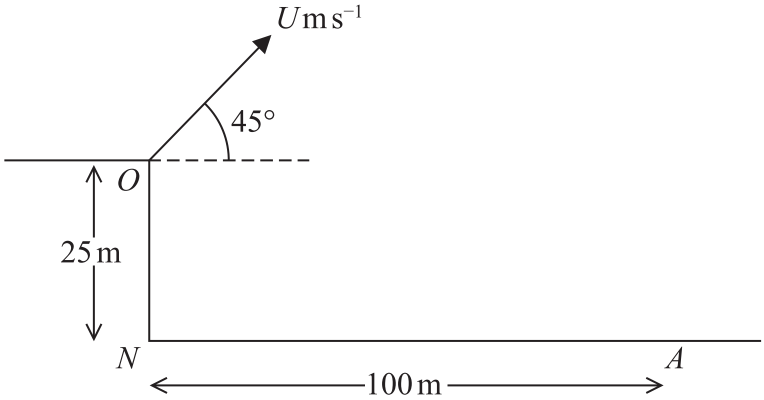

Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

Question	Scheme	Marks	AOs
5(a)	Using horizontal motion	M1	3.3
	$U\cos 45^\circ t = 100$	A1	1.1b
	Using vertical motion	M1	3.4
	$U\sin 45^\circ t - \frac{1}{2}gt^2 = -25$	A1	1.1b
	Solve problem by eliminating *t* and solving for *U*	M1	3.1b
	U = 28*	A1*	1.1b
		(6)	
5(b)	Using vertical motion	M1	3.4
	$0^2 = (28\sin 45^\circ)^2 - 2gh$	A1	1.1b
	Greatest height = 45 m	A1	1.1b
		(3)	
5(c)	New value > 28	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of *g*, spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
		(1)	
			(11 marks)

Notes:

5a	M1	Complete method to give equation in *U* and *t* only, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Complete method to give equation in *U* and *t* only, condone sin/cos confusion and sign errors
	A1	Correct equation (*g* does not need to be substituted)
	M1	Must have earned the previous two M marks. Eliminate *t* and solve for *U*. **N.B**. They may solve for *t* first ($100 - \frac{1}{2}gt^2 = -25$) and then use it to find *U*.
	A1*	Exact given answer correctly obtained with no wrong working (e.g. *g* = 9.81 used) or approximation seen.
5b	M1	Complete method to give equation in *h* only (allow if *U* not substituted), condone sin/cos confusion and sign errors

	A1	Correct equation (*g* does not need to be substituted) (A0 if *U* is used instead of 28)
	A1	cao
5c	B1	Clear statement
5d	B1	Penalise incorrect extras i.e. B0 if there are incorrect extras. The ground being horizontal, the cliff being vertical, .. are not part of the model so B0 Include weight/mass of the ball B0

4.

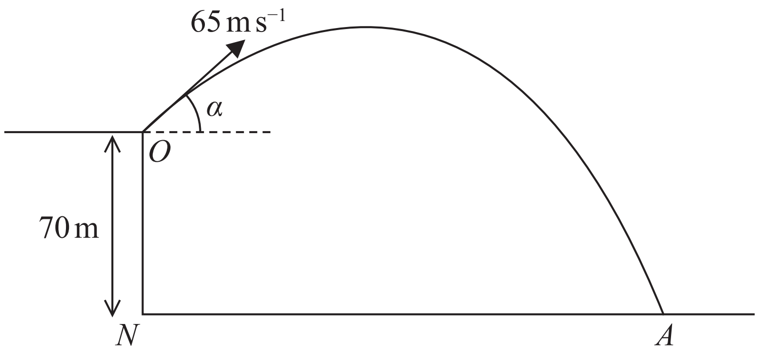

Figure 3

A small stone is projected with speed $65\,\text{m}\,\text{s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m}\,\text{s}^{-2}$

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

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Question	Scheme	Marks	AOs
	Note that g = 10; penalise once for whole question if g = 9.8		
4(a)	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in t only	M1	3.4
	$-70 = 65\sin\alpha \times t - \frac{1}{2}\times g \times t^2$	A1	1.1b
		M (A)1	1.1b
	$t = 7$ (s)	A1	1.1b
		(4)	
4(b)	Horizontal velocity component at A = $65\cos\alpha$ (60)	B1	3.4
	Complete method to find vertical velocity component at A	M1	3.4
	$65\sin\alpha - g\times 7$ **OR** $\sqrt{(-25)^2 + 2g\times 70}$ (45)	A1**ft**	1.1b
	Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$	M1	3.1b
	75 Accept 80 (m s^{-1})	A1	1.1b
		(5)	
4(c)	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, g = 10 instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone	B1	3.5b
		(1)	
		(10 marks)	

Notes:

4a	M1	Complete method, correct no. of terms, condone sign errors and sin/cos confusion
	A1	Correct equation in t only with at most one error
	M(A)1	Correct equation in t only
		N.B. For 'up and down' methods etc, the two A marks are for all the equations that they use, lose a mark for each error.
	A1	Cao (g = 9.8 , 7.1 or 7.11) (g = 9.81 , 7.1 or 7.12)
4b	B1	Seen, including on a diagram.
	M1	Condone sign errors and sin/cos confusion
	A1**ft**	Correct expression; accept negative of this, follow their t
	M1	Sub for trig and use Pythagoras
	A1	Cao (g = 9.8 or 9.81 , 75 or 74.8)

4c	B1	B0 if incorrect extras

Question	Scheme	Marks	AOs
	Allow column vectors throughout this question		
5(a)	Differentiate **v** wrt *t*	M1	3.1a
	$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i}-2\mathbf{j}$ isw	A1	1.1b
		(2)	
5(b)	$3t^{\frac{1}{2}}=2t$	M1	2.1
	Solve for *t*	DM1	1.1b
	$t=\frac{9}{4}$	A1	1.1b
		(3)	
5(c)	Integrate **v** wrt *t*	M1	3.1a
	$\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}(+\mathbf{C})$	A1	1.1b
	$t = 1$, $\mathbf{r} = -\mathbf{j}$ => $\mathbf{C} = -2\mathbf{i}$ so $\mathbf{r}=2t^{\frac{3}{2}}\mathbf{i}-t^2\mathbf{j}-2\mathbf{i}$	A1	2.2a
		(3)	
5(d)	$\sqrt{(3t^{\frac{1}{2}})^2+(2t)^2}=10$ or $(3t^{\frac{1}{2}})^2+(2t)^2=10^2$	M1	2.1
	$9t+4t^2=100$	M(A)1	1.1b
	$t = 4$	A1	1.1b
	$\mathbf{r}=14\mathbf{i}-16\mathbf{j}$	M1	1.1b
	$\sqrt{14^2+(-16)^2}$	M1	3.1a
	$\sqrt{452}$ $(2\sqrt{113})$ (m)	A1	1.1b
		(6)	
			(14 marks)

Notes:

5a	M1	Both powers decreasing by 1 (M0 if vector(s) disappear but allow recovery)
	A1	cao
5b	M1	Complete method, using **v**, to obtain an equation in *t* only, allow a sign error
	DM1	Dependent on M1,solve for *t*

5.

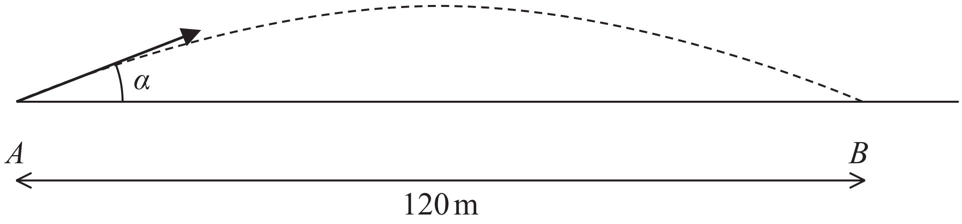

Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120$ m, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is $U\,\text{m s}^{-1}$

Using this model,

(a) show that $U^2 \sin\alpha\cos\alpha = 588$

(6)

The ball reaches a maximum height of 10 m above the ground.

(b) Show that $U^2 = 1960$

(4)

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is $V\,\text{m s}^{-1}$

This refined model is used to calculate a value for V

(c) State which is greater, U or V, giving a reason for your answer.

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

P 7 2 1 3 1 A 0 1 6 2 0

Question	Scheme	Marks	AOs
5(a)	Using horizontal motion	M1	3.3
	Whole Motion **Half way**		
	$U\cos\alpha \times t = 120$ $U\cos\alpha \times t = 60$	A1	1.1b
	Using vertical motion **OR**	M1	3.4
	$U\sin\alpha \times t - \frac{1}{2}gt^2 = 0$ $0 = U\sin\alpha - gt$	A1	1.1b
	Attempt to solve problem by eliminating t	DM1	3.1b
	$U^2\sin\alpha\cos\alpha = 588$ *	A1*	2.2a
		(6)	
	N.B. No credit given if they use the given answer from (b).		
5(b)	Using vertical motion **OR** conservation of energy	M1	3.4
	$0^2 = (U\sin\alpha)^2 - 2g\times 10$ $\frac{1}{2}mU^2 - \frac{1}{2}m(U\cos\alpha)^2 = mg\times 10$	A1	1.1b
	ALTERNATIVE 1: If t is time to top: use of $10 = \frac{1}{2}gt^2$ oe ($t = \frac{10}{7}$) to obtain an equation in U and α only M1 $U\sin\alpha = 14$ or $U\cos\alpha = 42$ A1 **ALTERNATIVE 2:** If t is time to top: use of : $10 = U\sin\alpha\, t - \frac{1}{2}gt^2$ with $t = \frac{60}{U\cos\alpha}$ substituted to obtain an equation in U and α only : M1 $10 = U\sin\alpha \times \frac{60}{U\cos\alpha} - \frac{1}{2}g\left(\frac{60}{U\cos\alpha}\right)^2$ A1		
	Attempt to solve problem by eliminating α : e.g. $U\sin\alpha = 14$ => $U\cos\alpha = 42$, from part (a) or from using $t = \frac{10}{7}$, then square and add to give result **OR:** $U^2\sin^2\alpha = 20g = 196$ and $U^2\sin\alpha\cos\alpha = 588$, divide to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result **OR in ALTERNATIVE 2**: sub for U^2 using part (a), to give $\tan\alpha = \frac{1}{3}$ then $\sin^2\alpha = \frac{1}{10}$, hence result	DM1	3.1b

	N.B. Just stating that $\sin^2\alpha = \frac{1}{10}$, with no working is DM0A0.		
	$U^2 = 1960$ *	A1*	2.2a
	N.B. Verification (i.e. starting with $U^2 = 1960$ and trying to work backwards) is not an acceptable method for this question.		
		(4)	
5(c)	V, since air resistance has to be overcome, or just 'because of <u>air resistance</u>' <u>isw</u>	B1	3.5a
		(1)	
5(d)	e.g. wind effects, more accurate value of *g,* spin of ball, size of ball, shape of ball, dimensions of ball, not a particle, variable acceleration, surface area of ball, humidity. Allow wind resistance and rotational resistance (Ignore any mention of air resistance or drag)	B1	3.5c
		(1)	
			(12 marks)

Notes:

5a		**N.B**. Could score 2/6 for any one of the 4 given equations if there is no corresponding second equation or there is an attempt but it's incorrect.
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	M1	Complete method to give equation in U, α and t only, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate t, dependent on first and second M1's
	A1*	Given answer correctly obtained, <u>with no wrong working seen.</u> Allow $588 = U^2 \sin\alpha\cos\alpha$ but nothing else
5b	M1	Complete method to give equation in U and α only with correct no. of terms, condone sin/cos confusion and sign errors, each term that needs to be resolved must be resolved
	A1	Correct equation
	DM 1	Eliminate α and rearrange, dependent on first M1
	A1*	Given answer correctly obtained with <u>no wrong working seen</u> (**N.B**. If they use a value for α (18.43.°.) they lose the final A1*)
5c	B1	Clear statement isw
5d	B1	B0 if there is an incorrect extra e.g. mass or weight