Y1P8 XMQs and MS

(Total: 35 marks)

1. P2_2020 Q4 . 3 marks - Y1P8 The binomial expansion
2. P1(AS)_2018 Q11. 8 marks - Y1P8 The binomial expansion
3. P1(AS)_2019 Q8 . 5 marks - Y1P8 The binomial expansion
4. P1(AS)_2020 Q6 . 6 marks - Y1P8 The binomial expansion
5. P1(AS)_2021 Q8 . 7 marks - Y1P8 The binomial expansion
6. P1(AS)_2022 Q6 . 6 marks - Y1P8 The binomial expansion

4. In the binomial expansion of

$$(a + 2x)^7 \qquad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a.

(3)

Question	Scheme	Marks	AOs
4	${}^7\mathrm{C}_4 a^3(2x)^4$	M1	1.1b
	$\frac{7!}{4!3!}a^3\times 2^4 = 15120 \Rightarrow a = ...$	dM1	2.1
	$a=3$	A1	1.1b
		(3)	
			(3 marks)

Notes:

M1: For an attempt at the correct coefficient of x^4.
The coefficient must have

- the correct binomial coefficient
- the correct power of a
- 2 or 2^4 (may be implied)

May be seen within a full or partial expansion.

Accept ${}^7\mathrm{C}_4 a^3(2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{4}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{4}a^3 16x^4$ etc.

or ${}^7\mathrm{C}_4 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{4}a^3 2^4$, $35a^3 2^4$, $560a^3$ etc.

or ${}^7\mathrm{C}_3 a^3(2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{3}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{3}a^3 16x^4$ etc.

or ${}^7\mathrm{C}_3 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{3}a^3 2^4$, $35a^3 2^4$, $560a^3$

You can condone missing brackets around the "2x" so allow e.g. $\frac{7!}{4!3!}a^3 2x^4$

An alternative is to attempt to expand $a^7\left(1+\frac{2x}{a}\right)^7$ to give $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$

Allow M1 for e.g. $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...\binom{7}{4}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...35\left(\frac{2x}{a}\right)^4...\right)$ etc.

but condone missing brackets around the $\frac{2x}{a}$

Note that ${}^7\mathrm{C}_3$, $\binom{7}{3}$ etc. are equivalent to ${}^7\mathrm{C}_4$, $\binom{7}{4}$ etc. and are equally acceptable.

If the candidate attempts $(a + 2x)(a + 2x)(a + 2x)$… etc. then it must be a complete method to reach the required term. Send to review if necessary.

dM1: For $"560"a^3 = 15120 \Rightarrow a = ...$ Condone slips on copying the 15120 but their "560" must be an attempt at ${}^7\mathrm{C}_4\times 2$ or ${}^7\mathrm{C}_4\times 2^4$ and must be attempting the <u>cube root</u> of $\frac{15120}{"560"}$. **Depends on the first mark**.

A1: $a = 3$ and no other values i.e. ±3 scores A0

Note that this is fairly common:

$${}^7\mathrm{C}_4 a^3 2x^4 = 70a^3x^4 \Rightarrow 70a^3 = 15120 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

and scores M1 dM1 A0

11. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9, \text{ where } a \text{ and } b \text{ are constants}$$

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 128 and $36x$,

(b) find the value of a,

(2)

(c) find the value of b.

(2)

Question	Scheme	Marks	AOs
11(a)	$\left(2-\frac{x}{16}\right)^9 = 2^9 + \binom{9}{1}2^8.\left(-\frac{x}{16}\right)+\binom{9}{2}2^7.\left(-\frac{x}{16}\right)^2 + ...$	M1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = 512 + ...$	B1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ... -144x + ...$	A1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ... + ... + 18x^2 \ (+ ...)$	A1	1.1b
		(4)	
(b)	Sets $'512'a = 128 \Rightarrow a = ...$	M1	1.1b
	$(a =)\frac{1}{4}$ oe	A1 ft	1.1b
		(2)	
(c)	Sets $'512'b + '-144'a = 36 \Rightarrow b = ...$	M1	2.2a
	$(b =)\frac{9}{64}$ oe	A1	1.1b
		(2)	
			(8 marks)

11(a) alt	$\left(2-\frac{x}{16}\right)^9 = 2^9\left(1-\frac{x}{32}\right)^9 = 2^9\left(1+\binom{9}{1}\left(-\frac{x}{32}\right)+\binom{9}{2}\left(-\frac{x}{32}\right)^2 + ...\right)$	M1	1.1b
	$= 512 + ...$	B1	1.1b
	$= ... -144x + ...$	A1	1.1b
	$= ... + ... + 18x^2 \ (+ ...)$	A1	1.1b

Notes Mark (a)(b) and (c) as one complete question

(a)

M1: Attempts the binomial expansion. May be awarded on either term two and/or term three Scored for a correct binomial coefficient combined with a correct power of 2 and a correct power of $\left(\pm\frac{x}{16}\right)$ Condone $\binom{9}{2}2^7.\left(-\frac{x^2}{16}\right)$ for term three.

Allow any form of the binomial coefficient. Eg $\binom{9}{2} = {}^9C_2 = \frac{9!}{7!2!} = 36$

In the alternative it is for attempting to take out a factor of 2 (may allow 2^n outside bracket) and having a correct binomial coefficient combined with a correct power of $\left(\pm\frac{x}{32}\right)$

B1: For 512
A1: For $-144x$

A1: For $+\ 18x^2$ Allow even following $\left(+\frac{x}{16}\right)^2$

Listing is acceptable for all 4 marks
(b)
M1: For setting their $512a = 128$ and proceeding to find a value for *a*. Alternatively they could substitute $x = 0$ into both sides of the identity and proceed to find a value for *a*.

A1 ft: $a = \frac{1}{4}$ oe Follow through on $\frac{128}{\text{their } 512}$

(c)
M1: Condone $512b \pm 144 \times a = 36$ following through on their 512, their -144 and using their value of "a" to find a value for "b"

A1: $b = \frac{9}{64}$ oe

8. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

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Question	Scheme	Marks	AOs
8(a)	2^6 or 64 as the constant term	B1	1.1b
	$\left(2+\frac{3x}{4}\right)^6 = ...+{}^6C_1 2^5\left(\frac{3x}{4}\right)^1+{}^6C_2 2^4\left(\frac{3x}{4}\right)^2+...$ $= ...+6\times 2^5\left(\frac{3x}{4}\right)^1+\frac{6\times 5}{2}\times 2^4\left(\frac{3x}{4}\right)^2+...$	M1 A1	1.1b 1.1b
	$= 64+144x+135x^2+...$	A1	1.1b
		(4)	
(b)	$\frac{3x}{4}=-0.075 \Rightarrow x=-0.1$ So find the value of $64+144x+135x^2$ with $x=-0.1$	B1ft	2.4
		(1)	
			(5 marks)

Notes

(a)

B1: Sight of either 2^6 or 64 as the constant term

M1: An attempt at the binomial expansion. This may be awarded for a correct attempt at either the second **OR** third term. Score for the correct binomial coefficient with the correct power of 2 and the correct power of $\frac{3x}{4}$ condoning slips. Correct bracketing is not essential for this M mark.

Condone ${}^6C_2 2^4 \frac{3x}{4}^2$ for this mark

A1: Correct (unsimplified) second **AND** third terms.

The binomial coefficients must be processed to numbers /numerical expression e.g $\frac{6!}{4!2!}$ or $\frac{6\times 5}{2}$

They cannot be left in the form ${}^6C_1$ and/or $\binom{6}{2}$

A1: $64+144x+135x^2+...$ Ignore any terms after this. Allow to be written $64, 144x, 135x^2$

(b)

B1ft: $x=-0.1$ or $-\frac{1}{10}$ **with** a comment about substituting this into their $64+144x+135x^2$

If they have written (a) as $64, 144x, 135x^2$ candidate would need to say substitute $x=-0.1$ into the sum of the first three terms.

As they do not have to perform the calculation allow

Set $2+\frac{3x}{4}=1.925$, solve for *x* and then substitute this value into the expression from (a)

If a value of *x* is found then it must be correct

Alternative to part (a)

$$\left(2+\frac{3x}{4}\right)^6 = 2^6\left(1+\frac{3x}{8}\right)^6 = 2^6\left(1+{}^6C_1\left(\frac{3x}{8}\right)^1+{}^6C_2\left(\frac{3x}{8}\right)^2+...\right)$$

B1: Sight of either 2^6 or 64

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term. Score for the correct binomial coefficient with the correct power of $\frac{3x}{8}$ Correct bracketing is not essential for this mark.
A1: A correct attempt at the binomial expansion on the second and third terms.
A1: $64+144x+135x^2+...$ Ignore any terms after this.

6. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient x^3 is 3 times the coefficient of x,

(b) find the possible values of k.

(3)

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Question	Scheme	Marks	AOs
6 (a)	$(1+kx)^{10}=1+\binom{10}{1}(kx)^1+\binom{10}{2}(kx)^2+\binom{10}{3}(kx)^3...$	M1 A1	1.1b 1.1b
	$=1+10kx+45k^2x^2+120k^3x^3...$	A1	1.1b
		(3)	
(b)	Sets $120k^3=3\times 10k$	B1	1.2
	$4k^2=1\Rightarrow k=...$	M1	1.1b
	$k=\pm\frac{1}{2}$	A1	1.1b
		(3)	
			(6 marks)

(a)

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term or fourth term. The coefficients may be of the form ${}^{10}C_1$, $\binom{10}{2}$ etc or eg $\frac{10\times 9\times 8}{3!}$

A1: A correct unsimplified binomial expansion. The coefficients must be numerical so cannot be of the form ${}^{10}C_1$, $\binom{10}{2}$. Coefficients of the form $\frac{10\times 9\times 8}{3!}$ are acceptable for this mark.

The bracketing must be correct on $(kx)^2$ but allow recovery

A1: $1+10kx+45k^2x^2+120k^3x^3...$ or $1+10(kx)+45(kx)^2+120(kx)^3...$
Allow if written as a list.

(b)

B1: Sets their $120k^3=3\times$ their $10k$ (Seen or implied)
For candidates who haven't cubed allow $120k=3\times$ their $10k$

If they write $120k^3x^3=3\times$ their $10kx$ only allow recovery of this mark if x disappears afterwards.

M1: Solves a cubic of the form $Ak^3=Bk$ by factorising out/cancelling the k and proceeding correctly to at least one value for k. Usually $k=\sqrt{\frac{B}{A}}$

A1: $k=\pm\frac{1}{2}$ o.e ignoring any reference to 0

8. $$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

8 (a)	$(2+ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5x^5$	M1 A1	1.1a 1.1b
	Sets $448a^5 = 3402 \Rightarrow a^5 = \dfrac{243}{32}$	M1	1.1b
	$\Rightarrow a = \dfrac{3}{2}$	A1	1.1b
		(4)	
(b)	Attempts either term. So allow for 2^8 or ${}^8C_4 2^4 a^4$	M1	1.1b
	Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$	dM1	2.1
	= 256 + 5670 = 5926	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: An attempt at selecting the correct term of the binomial expansion. If all terms are given then the correct term must be used. Allow with a missing bracket ${}^8C_5 2^3 ax^5$ and left without the binomial coefficient expanded

A1: $448a^5x^5$ Allow unsimplified but ${}^8C_5$ must be "numerical"

M1: Sets their $448a^5 = 3402$ and proceeds to $\Rightarrow a^k = ...$ where $k \in \mathbb{N}$ $k \neq 1$

A1: Correct work leading to $a = \dfrac{3}{2}$

(b)

M1: Finds either term required. So allow for 2^8 or ${}^8C_4 2^4 a^4$ (even allowing with a)

dM1: Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$

A1: cso 5926

6. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

(b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

Question	Scheme	Marks	AOs
6(a)	3^8 or 6561 as the constant term	B1	1.1b
	$\left(3-\frac{2x}{9}\right)^8 = ...+{}^8C_1(3)^7\left(-\frac{2x}{9}\right)+{}^8C_2(3)^6\left(-\frac{2x}{9}\right)^2+{}^8C_3(3)^5\left(-\frac{2x}{9}\right)^3+...$ $= ...+8\times(3)^7\left(-\frac{2x}{9}\right)+28\times(3)^6\left(-\frac{2x}{9}\right)^2+56(3)^5\left(-\frac{2x}{9}\right)^3$	M1 A1	1.1b 1.1b
	$= 6561-3888x+1008x^2-\frac{448}{3}x^3+...$	A1	1.1b
		(4)	
(b)	Coefficient of x^2 is $\frac{1}{2}\times\text{"}1008\text{"}-\frac{1}{2}\times\text{"}-\frac{448}{3}\text{"}$	M1	3.1a
	$=\frac{1736}{3}$ (or $578\frac{2}{3}$)	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1: Sight of 3^8 or 6561 as the constant term.

M1: An attempt at the binomial expansion. This can be awarded for the correct structure of the 2nd, 3rd or 4th term. The correct binomial coefficient must be associated with the correct power of 3 and the correct power of $(\pm)\frac{2x}{9}$. Condone invisible brackets eg ${}^8C_2(3)^6-\frac{2x}{9}^2$ for this mark.

A1: For a correct simplified or unsimplified **second** or **fourth term** (with binomial coefficients evaluated).

$+8\times(3)^7\left(-\frac{2x}{9}\right)$ or $+56(3)^5\left(-\frac{2x}{9}\right)^3$

A1: $6561-3888x+1008x^2-\frac{448}{3}x^3$ Ignore any extra terms and allow the terms to be listed.

Allow the exact equivalent to $-\frac{448}{3}$ eg $-149.\dot{3}$ but not -149.3.

Condone x^1 and eg $+-3888x$. Do not isw if they multiply all the terms by eg 3

Alt(a)

B1: Sight of $3^8(1+.....)$ or 6561 as the constant term

M1: An attempt at the binomial expansion $\left(1-\frac{2}{27}x\right)^8$. This can be awarded for the correct structure of the 2nd, 3rd or 4th term. The correct binomial coefficient must be associated with the correct power of $(\pm)\frac{2x}{27}$. Condone invisible brackets for this mark.

Score for any of:

$8\times-\frac{2}{27}x, \quad \frac{8\times7}{2}\times\left(-\frac{2}{27}x\right)^2, \quad \frac{8\times7\times6}{6}\times\left(-\frac{2}{27}x\right)^3$ which may be implied by any of

$-\frac{16}{27}x, \ +\frac{112}{729}x^2, \ -\frac{448}{19683}x^3$

A1: For a correct simplified or unsimplified **second** or **fourth** term including being multiplied by 3^8

A1: $6561-3888x+1008x^2-\frac{448}{3}x^3$ Ignore any extra terms and allow the terms to be listed.

Allow the exact equivalent to $-\frac{448}{3}$ eg $-149.\dot{3}$ but not -149.3.

Condone x^1 and eg $+-3888x$

(b)

M1: Adopts a correct strategy for the required coefficient. This requires an attempt to calculate $\pm$ ½ their coefficient of x^2 from part (a) $\pm$ ½ their coefficient of x^3 from part (a).

There must be an attempt to bring these terms together to a single value. ie they cannot just circle the relevant terms in the expansion for this mark. The strategy may be implied by their answer.
Condone any appearance of x^2 or x^3 appearing in their intermediate working.

A1: $\frac{1736}{3}$ or $578\frac{2}{3}$ Do not accept $578.\dot{6}$ or $\frac{1736}{3}x^2$