Y1P7 XMQs and MS

(Total: 71 marks)

1. P2_Sample Q1 . 3 marks - Y1P7 Algebraic methods
2. P2_Sample Q6 . 6 marks - Y1P7 Algebraic methods
3. P1_2019 Q1 . 3 marks - Y1P7 Algebraic methods
4. P2_2020 Q16. 4 marks - Y1P7 Algebraic methods
5. P1_2021 Q1 . 3 marks - Y1P7 Algebraic methods
6. P1_2022 Q2 . 3 marks - Y1P7 Algebraic methods
7. P2_2022 Q11. 4 marks - Y1P7 Algebraic methods
8. P1(AS)_2018 Q2 . 5 marks - Y1P2 Quadratics
9. P1(AS)_2018 Q9 . 9 marks - Y1P7 Algebraic methods
10. P1(AS)_2019 Q15. 4 marks - Y1P7 Algebraic methods
11. P1(AS)_2020 Q13. 5 marks - Y1P7 Algebraic methods
12. P1(AS)_2021 Q6 . 6 marks - Y1P7 Algebraic methods
13. P1(AS)_2021 Q10. 5 marks - Y1P7 Algebraic methods
14. P1(AS)_2022 Q2 . 7 marks - Y1P7 Algebraic methods
15. P1(AS)_2022 Q14. 4 marks - Y1P7 Algebraic methods

Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

(Total for Question 1 is 3 marks)

Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $f(-2)=0 \Rightarrow 2\times(-2)^3-5\times(-2)^2+a\times-2+a=0$	M1	3.1a
	Solves linear equation $2a-a=-36 \Rightarrow a=$	dM1	1.1b
	$\Rightarrow a=-36$	A1	1.1b
			(3 marks)

Notes:

M1: Selects a suitable method given that $(x+2)$ is a factor of $f(x)$
Accept either setting $f(-2)=0$ or attempted division of $f(x)$ by $(x+2)$

dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a=..\Rightarrow a=$

A1: $a=-36$

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$ It should be $\frac{\sin\theta}{\cos\theta}=\tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°)\neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
			(3 marks)

Notes:

(a)

B1: Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta}=\tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta=\frac{1}{2}$ not $\tan\theta=2$'

Accept also statements such as 'it should be $\cot\theta=2$'

(b)

B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta=2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°)=+ve$ and $2\sin(-26.6°)=-ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x=5$ squared gives $x^2=25$ which has answers ± 5

6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 6 is 6 marks)

Question	Scheme	Marks	AOs
6(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2\geqslant 0 \Rightarrow (x-3)^2+1\geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b\Rightarrow x>\frac{b}{a}$ if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
			(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b\Rightarrow x<\frac{b}{a}$ or simply $-3x>6\Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

Question	Scheme	Marks	AOs
1	Attempts $\mathrm{f}(-3)=3\times(-3)^3+2a\times(-3)^2-4\times-3+5a=0$	M1	3.1a
	Solves linear equation $23a=69\Rightarrow a=...$	M1	1.1b
	$a=3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $\mathrm{f}(-3)=0$ leading to a correct equation in *a*.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $\mathrm{f}(-3)=0$ method

$$\begin{array}{r}3x^2+(2a-9)x+23-6a\\ x+3\overline{)3x^3+2ax^2-4x+5a}\\ \underline{3x^3+9x^2}\\ (2a-9)x^2-4x\\ \underline{(2a-9)x^2+(6a-27)x}\\ (23-6a)x+5a\\ (23-6a)x+69-18a\end{array}$$

So accept $5a=69-18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in *x* are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a-27+\frac{5a}{3}=-4$$

M1: This is scored for an attempt at solving a linear equation in *a*.
For the main scheme it is dependent upon having attempted $\mathrm{f}(\pm3)=0$. Allow for a linear equation in *a* leading to $a=...$. Don't be too concerned with the mechanics of this.

Via division accept $\begin{array}{r}3x^2...\\ x+3\overline{)3x^3+2ax^2-4x+5a}\end{array}$ followed by a remainder in *a* set $=0\Rightarrow a=...$

or two terms in *a* are equated so that the remainder = 0
FYI the correct remainder via division is $23a+12-81$ oe

A1: $a=3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k$, $3k+1$, $3k+2$ or equivalent e.g. $3k-1$, $3k$, $3k+1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3\times 3k^2\right)$ is a multiple of 3 $(3k+1)^2 = 9k^2+6k+1 = 3\times\left(3k^2+2k\right)+1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2+12k+4 = 3\times\left(3k^2+4k+1\right)+1$ $\left(\text{or } (3k-1)^2 = 9k^2-6k+1 = 3\times\left(3k^2-2k\right)+1\right)$ is one more than a multiple of 3	A1 **M1 on EPEN**	1.1b
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k+1$, $3k+2$ or e.g. $3k-1$, $3k$, $3k+1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square these expressions.

A1(M1 on EPEN): Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra. This must be made explicit e.g. reaches $3\times\left(3k^2+2k\right)+1$ and makes a statement that this is one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2+6k+1$ is one more than a multiple of 3 e.g. "$9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2+6k+1$ is one more than a multiple of 3"

M1(A1 on EPEN): Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra and makes a conclusion

1. $$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

(3)

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Question	Scheme	Marks	AOs
1	$\mathrm{f}(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$	M1	3.1a
	$6 - 2a = 0 \Rightarrow a = ...$	M1	1.1b
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes

Main method seen:

M1: Attempts f (1) = 0 to set up an equation in a It is implied by $a + 10 - 3a - 4 = 0$

Condone a slip but attempting f (−1) = 0 is M0

M1: Solves a linear equation in a .

Using the main method it is dependent upon having set f (±1) = 0

It is implied by a solution of $\pm a \pm 10 \pm 3a \pm 4 = 0$.

Don't be concerned about the mechanics of the solution.

A1: a = 3 (following correct work)

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Answers without working scores 0 marks. The method must be made clear. Candidates cannot guess. However if a candidate states for example, when $a = 3$, $\mathrm{f}(x) = 3x^3 + 10x^2 - 9x - 4$ and shows that $(x-1)$ is a factor of this $\mathrm{f}(x)$ by an allowable method, they should be awarded M1 M1 A1

E.g. 1: $3x^3 + 10x^2 - 9x - 4 = (x-1)(3x^2 + 13x + 4)$ Hence $a = 3$

E.g. 2: $\mathrm{f}(x) = 3x^3 + 10x^2 - 9x - 4$, $\mathrm{f}(1) = 3 + 10 - 9 - 4 = 0$ Hence $a = 3$

The solutions via this method must end with the value for a to score the A1

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Other methods are available. They are more difficult to determine what the candidate is doing.
Please send to review if you are uncertain
It is important that a correct method is attempted so look at how the two M's are scored

Amongst others are:

Alt (1) by inspection which may be seen in a table/g

	ax^2	$(10+a)x$	4
x	ax^3	$(10+a)x^2$	$4x$
-1	$-ax^2$	$-(10+a)x$	-4

$ax^3+10x^2-3ax-4=(x-1)\left(ax^2+(10+a)x+4\right)$ and sets terms in x equal

$-3a=-(10+a)+4 \Rightarrow 2a=6 \Rightarrow a=3$

M1: This method is implied by a **correct** equation, usually $-3a=-(10+a)+4$

M1: Attempts to find the quadratic factor which must be of the form $ax^2+\mathrm{g}(a)x\pm 4$ and then forms and solves a linear equation formed by linking the coefficients or terms in x

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Alt (2) By division:

$$\begin{array}{r l}
 & ax^2+(\pm 10\pm a)x+(10-2a) \\
x-1 \,\big) & ax^3 \; +10x^2 \qquad -3ax-4 \\
 & \underline{ax^3-ax^2} \\
 & \quad (10+a)x^2-3ax \\
 & \quad \underline{(10+a)x^2-(10+a)x} \\
 & \qquad\qquad (-2a+10)x
\end{array}$$

M1: This method is implied by a **correct** equation, usually $-10+2a=-4$

M1: Attempts to divide with quotient of $ax^2+(\pm 10\pm a)x+\mathrm{h}(a)$ and then forms and solves a linear equation in a formed by setting the remainder = 0.

2. $$\mathrm{f}(x) = (x-4)(x^2 - 3x + k) - 42 \quad \text{where } k \text{ is a constant}$$

Given that $(x + 2)$ is a factor of $\mathrm{f}(x)$, find the value of k.

(3)

Question	Scheme	Marks	AOs
2	Sets $f(-2)=0 \Rightarrow (-2-4)\left((-2)^2-3\times-2+k\right)-42=0$	M1	3.1a
	$-6(k+10)=42 \Rightarrow k=...$	M1	1.1b
	$k=-17$	A1	1.1b
		(3)	
(3 marks)			
Notes:			

M1: Attempts $f(-2)=0$ leading to an equation in *k*. So $(-2-4)\left((-2)^2-3\times-2+k\right)-42=0$ is fine

Condone slips but expect to see a first bracket of $(-2-4)$.

"-42" must not be omitted but could appear as +42 with a sign slip.

There may have been attempts to expand $f(x)=(x-4)(x^2-3x+k)-42$ before attempting to set $f(-2)=0$. This is acceptable and condone slips/errors in the expansion, but the 42 must be present. FYI the expanded (and simplified) $f(x)=x^3-7x^2+(12+k)x-4k-42$

M1: Solves a **linear** equation in *k* as a result of setting $f(\pm2)=0$.

The ±42 must be there at some point when the substitution is made.

Allow minimal evidence here. A linear equation leading to a solution is fine.

If f (*x*) is expanded then it is dependent upon being a cubic which contains a *kx* term and a '42'

A1: $k=-17$ correct answer following correct work but allow recovery from invisible brackets

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Answers of $k=-17$ may appear with very little or no working, perhaps via trial and improvement. If so, then marks can only be allocated if evidence is shown.

E.g. $k=-17 \Rightarrow f(x)=(x-4)(x^2-3x-17)-42$

$f(-2)=(-6)\times(-7)-42=0$. Hence $(x+2)$ is a factor.

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More difficult alternative methods may be seen

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Alt I : You may see attempts via division / inspection

$$x+2\overline{)x^3-7x^2+(12+k)x-4k-42}\quad\text{quotient } x^2-9x+(k+30)$$

$$\overline{-6k-102}$$

Then sets remainder $-6k-102=0 \Rightarrow k=-17$

M1: For dividing their cubic by (x+2) which has both an x and a constant coefficient in k, leading to a quadratic quotient and a linear remainder in k which is then set = 0

M1: Solves a equation resulting from setting a linear remainder in k equal to 0 . It is dependent on the first M via this route

A1: Completely correct with $k=-17$

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Alt II: You may also see a grid or an attempt at factorisation via inspection

	x^2	$-9x$	$-2k-21$
x	x^3	$-9x^2$	$(-2k-21)x$
$+2$	$2x^2$	$-18x$	$-4k-42$

OR $x^3-7x^2+(12+k)x-4k-42 \equiv (x+2)(x^2-9x-2k-21)$

which should be followed by equating the x terms to form an equation in k

$$12+k=-18-2k-21 \Rightarrow 3k=-51 \Rightarrow k=-17$$

OR $x^3-7x^2+(12+k)x-4k-42 \equiv (x+2)(x^2-9x+k+30)$

which should be followed by equating the constant terms to form an equation in k

$$-4k-42=2(k+30) \Rightarrow 6k=-102 \Rightarrow k=-17$$

The above are examples. There may be other correct attempts so look at what is done.

M1: For an attempt at factorising E.g. $x^3-7x^2+(12+k)x-4k-42 \equiv (x+2)(x^2+bx+c)$ and attempting to set up three equations in b, c and k. E.g $2+b=-7,\ 2b+c=12+k,\ 2c=-4k-42$

The expanded f(x) must be a cubic which contains both a kx term and a '42'

M1: Solves the equations set up from an allowable equation to find k. It is dependent via this route.

A1: Completely correct with $k=-17$

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11. Prove, using algebra, that

$$n(n^2 + 5)$$

is even for all $n \in \mathbb{N}$.

(4)

Question	Scheme	Marks	AOs
11	$n\left(n^2+5\right)$		
	Attempts even **or** odd numbers Sets $n=2k$ **or** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	M1	3.1a
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **Or** achieves $(2k+1)\left(4k^2+4k+6\right)=2(2k+1)\left(2k^2+2k+3\right)$ (for $n=2k+1$) and states "even" Or e.g. achieves $(2k-1)\left(4k^2-4k+6\right)=2(2k-1)\left(2k^2-2k+3\right)$ (for $n=2k-1$) and states "even"	A1	2.2a
	Attempts even **and** odd numbers Sets $n=2k$ **and** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$	dM1	2.1
	Achieves $2k\left(4k^2+5\right)$ (for $n=2k$) and states "even" **and** achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ (for $n=2k\pm1$) and states "even" Correct work and states even for both WITH a final conclusion showing that true for all $n(\in\mathbb{N})$ or e.g. true for all even and odd numbers.	A1	2.4
		(4)	
			(4 marks)

Notes:

M1: For the key step attempting to find $n\left(n^2+5\right)$ when $n=2k$ **or** $n=2k\pm1$ or equivalent representation of odd or even e.g. $n=2k+2$ **or** $n=2k\pm7$

Condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1: Achieves $2k\left(4k^2+5\right)$ or e.g. $2\left(4k^3+5k\right)$ and deduces that this is even at the appropriate time.

Or achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ oe e.g. $2\left(4k^3+6k^2+8k+3\right)$ and deduces that this is even.

Note that if the bracket is expanded to e.g. $8k^3+12k^2+16k+6$ then stating "even" is insufficient – they would need to say e.g. even + even + even + even = even or equivalent

Note it is also acceptable to use a divisibility argument e.g. $\dfrac{8k^3+10k}{2}=4k^3+5k$ so $8k^3+10k$ must be even.

There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered".

dM1: Attempts $n\left(n^2+5\right)$ when $n=2k$ **and** $n=2k\pm1$ or equivalent representation of odd or even e.g. $n=2k+2$ **and** $n=2k\pm7$

A1: Correct work and states even for both WITH a final conclusion e.g. so true for all $n(\in\mathbb{N})$.

There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered".

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A “solution” via just logic.

E.g.

If n is odd, then $n\left(n^2+5\right)$ is odd×(odd + odd) = odd × even = even

If n is even, then $n\left(n^2+5\right)$ is even×(even + odd) = even × odd = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

OR

E.g. $n\left(n^2+5\right)=n^3+5n$

If n is odd, then n^3 is odd and $5n$ is odd, so n^3+5n is odd + odd = even

If n is even, then n^3 is even and $5n$ is even, so n^3+5n is even + even = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

....

A solution via contradiction.

M1 A1: There exists a number n such that $n\left(n^2+5\right)$ is odd, and so deduces that both n and n^2+5 are odd. Note that M1A0 is not possible via this method.

dM1: Sets $n^2+5=2k+1$ (for some integer k) $\Rightarrow n^2=2k-4=2(k-2)$ which is even
Must use algebra here for this approach and not a “logic” argument.

A1: States that "this is a contradiction as if n^2 is even, then n is even" and then concludes so "$n\left(n^2+5\right)$ is even for all n. "

Attempts at proof by induction should be sent to review

2. (i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

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Question	Scheme	Marks	AOs
2(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ **...**

A1: For $(x-4)^2+1$ with either $(x-4)^2\geqslant 0$**,** $(x-4)^2+1\geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2>0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

....

$x^2-8x+17$

$=(x-4)^2+1\geqslant 1$ as $(x-4)^2\geqslant 0$ — scores M1 A1 A1

Hence $(x-4)^2+1>0$

....

$x^2-8x+17>0$

$(x-4)^2+1>0$ — scores M1 A1 A1

This is true because $(x-4)^2\geqslant 0$ and when you add 1 it is going to be positive

....

$x^2-8x+17>0$

$(x-4)^2+1>0$ — scores M1 A1 A0

which is true because a squared number is positive — incorrect and incomplete

....

$x^2-8x+17=(x-4)^2+1$ — scores M1 A1 A0

Minimum is (4,1) so $x^2-8x+17>0$ — correct but not explained

....

$x^2-8x+17=(x-4)^2+1$ — scores M1 A1 A1

Minimum is (4,1) so as 1 > 0 $\Rightarrow x^2-8x+17>0$ — correct and explained

....

$x^2 - 8x + 17 > 0$

$(x-4)^2 + 1 > 0$

scores M1 A0 (no explanation) A0

....

Method Two: Use of a discriminant

M1: Attempts to find the discriminant $b^2 - 4ac$ with a correct *a, b* and *c* which may be within a quadratic formula. You may condone missing brackets.

A1: Correct value of $b^2 - 4ac = -4$ **and** states or shows curve is U shaped (or intercept is (0,17)) or equivalent such as $+\text{ve } x^2$ etc

A1: Explains that as $b^2 - 4ac < 0$, there are no roots, and curve is U shaped then $x^2 - 8x + 17 > 0$

Method Three: Differentiation

M1: Attempting to differentiate and finding the turning point. This would involve attempting to find $\frac{\mathrm{d}y}{\mathrm{d}x}$, then setting it equal to 0 and solving to find the *x* value and the *y* value.

A1: For differentiating $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 8 \Rightarrow (4,1)$ is the **turning point**

A1: Shows that $(4,1)$ is the minimum point (second derivative or U shaped), hence $x^2 - 8x + 17 > 0$

Method 4: Sketch graph using calculator

M1: Attempting to sketch $y = x^2 - 8x + 17$, U shape with minimum in quadrant one

A1: As above with minimum at (4,1) marked

A1: Required to state that quadratics only have one turning point and as "1" is above the *x*-axis then $x^2 - 8x + 17 > 0$

(ii)

Numerical approach

Do not allow any marks if the candidate just mentions "positive" and "negative" numbers. Specific examples should be seen calculated if a numerical approach is chosen.

M1: Attempts a value (where it is not true) and shows/implies that it is not true for that value.

For example, for -4 : $(-4+3)^2 > (-4)^2$ and indicates not true (states not true, ✗)

or writing $(-4+3)^2 < (-4)^2$ is sufficient to imply that it is not true

A1: Shows/implies that it can be true for a value **AND** states sometimes true.

For example for +4 : $(4+3)^2 > 4^2$ and indicates true ✓

or writing $(4+3)^2 > 4^2$ is sufficient to imply this is true following $(-4+3)^2 < (-4)^2$

condone incorrect statements following the above such as 'it is only true for positive numbers' as long as they state "sometimes true" and show both cases.

Algebraic approach

M1: Sets the problem up algebraically Eg. $(x+3)^2 > x^2 \Rightarrow x > k$ Any inequality is fine. You may condone one error for the method mark. Accept $(x+3)^2 > x^2 \Rightarrow 6x + 9 > 0$ oe

A1: States sometimes true a**nd** states/implies true for $x > -\frac{3}{2}$ or states/implies not true for $x < -\frac{3}{2}$ In both cases you should expect to see the statement "sometimes true" to score the A1

9. $$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

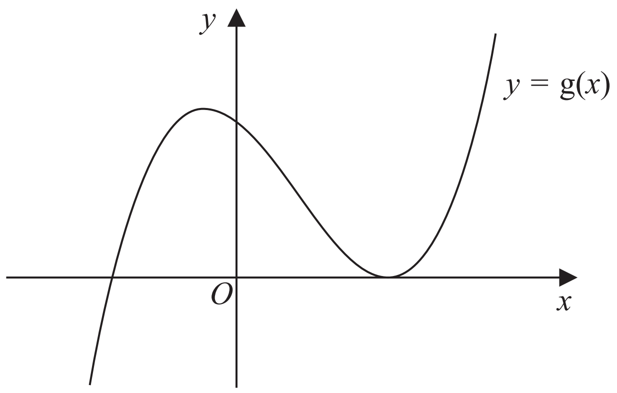

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

Question	Scheme	Marks	AOs
9(a)	$(\mathrm{g}(-2)) = 4\times -8 - 12\times 4 - 15\times -2 + 50$	M1	1.1b
	$\mathrm{g}(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$	M1 A1	1.1b 1.1b
	$= (x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x = 2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x = -1,\ x = 1.25$	B1ft	2.2a
		(3)	
			(9 marks)

Notes

(a)

M1: Attempts $\mathrm{g}(-2)$ Some sight of (-2) embedded or calculation is required.

So expect to see $4\times(-2)^3 - 12\times(-2)^2 - 15\times(-2) + 50$ embedded

Or $-32 - 48 + 30 + 50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

A1: $\mathrm{g}(-2) = 0 \Rightarrow (x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$\mathrm{g}(-2) = 0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $\mathrm{g}(-2) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to divide $\mathrm{g}(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 \text{.............} \pm 25)$

If algebraic / long division is used expect to see

$$x+2\overline{\smash{)}\,4x^3 - 12x^2 - 15x + 50} \quad \text{quotient: } 4x^2 \pm 20x$$

A1: Correct quadratic factor is $(4x^2 - 20x + 25)$ may be seen and awarded from part (a)

M1: Attempts to factorise their $(4x^2 - 20x + 25)$ usual rule $(ax+b)(cx+d),\ ac = \pm 4,\ bd = \pm 25$

A1: $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $\mathrm{g}(x) = (x+2)(x-2.5)^2 = (x+2)(2x-5)^2$

(c)(i)

M1: For identifying that the solution will be where the curve is on or below the axis. Award for either $x \leqslant -2$ or $x = 2.5$ Follow through on their $\mathrm{g}(x) = (x+2)(ax+b)^2$ only where $ab < 0$ (that is a positive root). Condone $x < -2$ See SC below for $\mathrm{g}(x) = (x+2)(2x+5)^2$

A1ft: BOTH $x \leqslant -2$, $x = 2.5$ Follow through on their $-\frac{b}{a}$ of their $g(x) = (x+2)(ax+b)^2$

May see $\{x \leqslant -2 \cup x = 2.5\}$ which is fine.

(c) (ii)

B1ft: For deducing that the solutions of $g(2x) = 0$ will be where $x = -1$ and $x = 1.25$

Condone the coordinates appearing $(-1, 0)$ and $(1.25, 0)$

Follow through on their 1.25 of their $g(x) = (x+2)(ax+b)^2$

……..

SC: If a candidate reaches $g(x) = (x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

In (i) M1 A0 for $x \leqslant -2$ or $x < -2$

In (ii) B1 for $x = -1$ and $x = -1.25$

………

Alt (b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(ax+b)^2$ $= a^2x^3 + (2ba + 2a^2)x^2 + (b^2 + 4ab)x + 2b^2$		
	Compares terms to get either *a* or *b*	M1	1.1b
	Either $a = 2$ or $b = -5$	A1	1.1b
	Multiplies out expression $(x+2)(\pm 2x \pm 5)^2$ and compares to $4x^3 - 12x^2 - 15x + 50$	M1	
	All terms must be compared or else expression must be multiplied out and establishes that $4x^3 - 12x^2 - 15x + 50 = (x+2)(2x-5)^2$	A1	1.1b
		(4)	

15. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

DO NOT WRITE IN THIS AREA

Question 15

Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for *n* being even **OR** odd
A1: Acceptable proof for *n* being even **OR** odd
M1: Suitable approach to answer the question for *n* being even **AND** odd
A1: Acceptable proof for *n* being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of *n* and then drawing conclusions. So $n = 5 \Rightarrow n^3 + 2 = 127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8} = \frac{1}{8}n^3 + \frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8} = \frac{1}{8}n^3 + \frac{3}{8}n^2 + \frac{3}{8}n + \frac{3}{8}$ which is not a whole number

There must be an attempt to generalise either logic or algebra.

Example of a logical approach

Logical approach	States that if *n* is odd, n^3 is odd	M1	2.1
	so $n^3 + 2$ is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if *n* is even, n^3 is a multiple of 8	M1	2.1
	so $n^3 + 2$ cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example
"odd number + 2 is still odd and odd numbers are not divisible by 8"
"$n^3 + 2$ is odd and cannot be divided by 8 **exactly**''
and for even numbers accept
"a multiple of 8 add 2 is not a multiple of 8, so $n^3 + 2$ is not divisible by 8"
"if n^3 is a multiple of 8 then $n^3 + 2$ cannot be divisible by 8
Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

Example of algebraic approaches

Question	Scheme	Marks	AOs
15 **Algebraic approach**	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=8k^3+12k^2+6k+3$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$	M1	2.1
	$=k^3+\frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$	M1	2.1
	$=\frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $8k^3+12k^2+6k+3$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Notes

Correct expressions are required for the M's. There is no need to state "**If *n* is even**," $n=2k$ and "**If *n* is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathbb{N}$

Some students will use $2k-1$ for odd numbers

There is no requirement to change the variable. They may use $2n$ and $2n\pm1$

Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!)

Also **"$=\frac{8k^3+12k^2+6k+3}{8}=k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

:

13. (a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b.

(1)

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Question	Scheme	Marks	AOs
13 (a)	States $(2a-b)^2 \geqslant 0$	M1	2.1
	$4a^2 + b^2 \geqslant 4ab$	A1	1.1b
	(As $a>0, b>0$) $\dfrac{4a^2}{ab}+\dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$	M1	2.2a
	Hence $\dfrac{4a}{b}+\dfrac{b}{a} \geqslant 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \dfrac{4a}{b}+\dfrac{b}{a}=-20-\dfrac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \geqslant 0$

A1: Reaches $4a^2 + b^2 \geqslant 4ab$

M1: Divides each term by $ab \Rightarrow \dfrac{4a^2}{ab}+\dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

....

Proof by contradiction: Scores all marks

M1: Assume that there exists an $a, b > 0$ such that $\frac{4a}{b} + \frac{b}{a} < 4$

A1: $4a^2 + b^2 < 4ab \Rightarrow 4a^2 + b^2 - 4ab < 0$

M1: $(2a - b)^2 < 0$

A1*: States that this is not true, hence we have a contradiction so $\frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

....

Attempt starting with the left-hand side

M1: $(\text{lhs} =) \frac{4a}{b} + \frac{b}{a} - 4 = \frac{4a^2 + b^2 - 4ab}{ab}$

A1: $= \frac{(2a - b)^2}{ab}$

M1: $= \frac{(2a - b)^2}{ab} \ldots 0$

A1*: Hence $\frac{4a}{b} + \frac{b}{a} - 4 \ldots 0 \Rightarrow \frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- ab is positive as $a > 0$ and $b > 0$

....

Attempt using given result: For 3 out of 4

$\frac{4a}{b} + \frac{b}{a} \ldots 4$ M1 $\Rightarrow 4a^2 + b^2 \ldots 4ab \Rightarrow 4a^2 + b^2 - 4ab \ldots 0$

A1 $\Rightarrow (2a - b)^2 \ldots 0$ oe

M1 gives both reasons why this is true

- "square numbers are greater than or equal to 0"
- "multiplying by *ab* does not change the sign of the inequality because *a* and *b* are positive"

6. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

(3)

Question	Scheme	Marks	AOs
6 (a)	$3x^3-17x^2-6x=0 \Rightarrow x\left(3x^2-17x-6\right)=0$	M1	1.1a
	$\Rightarrow x(3x+1)(x-6)=0$	dM1	1.1b
	$\Rightarrow x=0,-\frac{1}{3},6$	A1	1.1b
		(3)	
(b)	Attempts to solve $(y-2)^2=n$ where n is any solution $\geqslant 0$ to (a)	M1	2.2a
	Two of $2, 2\pm\sqrt{6}$	A1ft	1.1b
	All three of $2, 2\pm\sqrt{6}$	A1	2.1
		(3)	
		(6 marks)	

Notes

(a)

M1: Factorises out or cancels by x to form a quadratic equation.

dM1: Scored for an attempt to find x. May be awarded for factorisation of the quadratic or use of the quadratic formula.

A1: $x=0,-\frac{1}{3},6$ and no extras

(b)

M1: Attempts to solve $(y-2)^2=n$ where n is any solution $\geqslant 0$ to (a). At least one stage of working must be seen to award this mark. Eg $(y-2)^2=0 \Rightarrow y=2$

A1ft: Two of $2, 2\pm\sqrt{6}$ but follow through on $(y-2)^2=n \Rightarrow y=2\pm\sqrt{n}$ where n is a positive solution to part (a). (Provided M1 has been scored)

A1: All three of $2, 2\pm\sqrt{6}$ and no extra solutions. (Provided M1A1 has been scored)

10. A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers.

(4)

(b) Use a counterexample to show that the statement is not always true.

(1)

Question	Scheme	Marks	AOs
10(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = ...$	M1	2.1
	……….and factorise $8k^3 \pm 12k^2 \pm 4k = 4k(2k^2 \pm 3k \pm 1) =$	dM1	1.1b
	Correct work with statement $4\times..$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
			(5 marks)
Alt (a)	Selects a correct strategy. Factorises $k^3 - k = k(k-1)(k+1)$	B1	3.1a
	States that if k is odd then both $k-1$ and $k+1$ are even	M1	2.1
	States that $k-1$ multiplied by $k+1$ is therefore a multiple of 4	dM1	1.1b
	Concludes that $k^3 - k$ is a multiple of 4 as it is odd $\times$ multiple of 4	A1	2.4
		(4)	

Notes:

(a)

Note: May be in any variable (condone use of *n*)

B1: Selects a correct strategy. E.g uses an odd number is $2k \pm 1$

M1: Attempts $(2k \pm 1)^3 - (2k \pm 1) = ...$ Condone errors in multiplying out the brackets and invisible brackets for this mark. Either the coefficient of the *k* term or the constant of $(2k \pm 1)^3$ must have changed from attempting to simplify.

dM1: Attempts to take a factor of 4 or 4*k* from their cubic

A1: Correct work with statement $4\times..$ is a multiple of 4

(b)

B1: Any counter example with correct statement.

Question	Scheme	Marks	AOs

2.

$$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

Question	Scheme	Marks	AOs
2(a)	$f(-3)=2(-3)^3+5(-3)^2+2(-3)+15$ $=-54+45-6+15$	M1	1.1b
	$f(-3)=0 \Rightarrow (x+3)$ is a factor	A1	2.4
		(2)	
(b)	At least 2 of: $a=2,\ b=-1,\ c=5$	M1	1.1b
	All of: $a=2,\ b=-1,\ c=5$	A1	1.1b
		(2)	
(c)	$b^2-4ac=(-1)^2-4(2)(5)$	M1	2.1
	$b^2-4ac=-39$ which is <0 so the quadratic has no real roots so f(x) = 0 has only 1 real root	A1	2.4
		(2)	
(d)	(x =) 2	B1	2.2a
		(1)	
			(7 marks)

Notes

(a)

M1: Attempts f(–3). Attempted division by $(x+3)$ or $f(3)$ is M0
Look for evidence of embedded values or two correct terms of
$f(-3)=-54+45-6+15=...$

A1: Achieves and states f(–3) = 0, and makes a suitable conclusion. Sight of f(x)=0 when $x=-3$ is also acceptable.
It must follow M1. Accept, for example, $f(-3)=0 \Rightarrow (x+3)$ is a factor

This may be seen in a preamble before finding $f(-3)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Correct method implied by values for at least 2 correct constants. Allow embedded in their f(x) or within their working if they use algebraic division/other methods which may be seen in part (a) and used in part (b).

A1: All values correct. Allow embedded in their f(x) or seen as the quotient from algebraic division. Isw incorrectly stated values of a b and c following a correct quadratic expression seen.

$$
\begin{array}{r}
2x^2 - x + 5 \\
x+3 \overline{\smash{\big)}\, 2x^3 + 5x^2 + 2x + 15} \\
2x^3 + 6x^2 \phantom{{}+2x+15} \\
-x^2 + 2x \phantom{{}+15} \\
-x^2 - 3x \phantom{{}+15} \\
5x + 15 \\
5x + 15 \\
0
\end{array}
$$

scores M1A1

(c)

M1: Either:

- considers the discriminant using their *a*, *b* and *c* (does not need to be evaluated) $\left(b^2 - 4ac =\right)\ (-1)^2 - 4(2)(5)$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2). Discriminant = −39 is sufficient for M1
- attempts to complete the square so score for $2\left(x \pm \frac{1}{4}\right)^2 + ...$
- attempts to find the roots of the quadratic using the formula. The values embedded in the formula score this mark. $\frac{1 \pm \sqrt{(-1)^2 - 4\times 2\times 5}}{2\times 2}$ (the $(-1)^2$ may appear as 1^2 and condone missing brackets for this mark for -1^2)
- Sketches a graph of the quadratic. It must be a U shaped quadratic which does not cross the *x*-axis.

A1: Provides a correct explanation from correct working. They must

- Have a correct calculation
- Explanation that the quadratic has no (real) roots
- Minimal conclusion stating that $f(x) = 0$ has only one root

eg $b^2 - 4ac = -39 < 0$ so only one root is M1A0 (needs to explain the quadratic has no real roots)

eg $2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8} > 0$ **so no real roots** (for the quadratic) **so** (f(*x*) has) **only one (**real**) root** is M1A1

The value of the discriminant, completed square form $2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8}$ or roots of the quadratic $\left(= \frac{1 \pm \sqrt{39}i}{4}\right)$ must be correct.

If they sketch the quadratic graph it must be a U shaped quadratic which crosses the *y*-axis at 5 and has a minimum in the 1st quadrant. They must explain that the graph does not cross the *x*-axis so no real roots for the quadratic so only one root for $f(x) = 0$.

(d)

B1: 2 condone (2, 0)

14. (i) A student states

"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

Question	Scheme	Marks	AOs
14(i)	The statement is **not true** because e.g. when $x = -4$, $x^2 = 16$ (which is > 9 but $x < 3$)	B1	2.3
		(1)	
(ii)	$n^3 + 3n^2 + 2n = n\left(n^2 + 3n + 2\right) = n\left(n+1\right)\left(n+2\right)$	M1	2.1
	$n(n + 1)(n + 2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n + 1)(n + 2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n	A1	2.4
		(3)	
			(4 marks)

Notes

(i)

B1: Identifies the error in the statement by giving

- a counter example and a reason eg $x = -4$ with $x^2 = 16$ eg $x = -4$ with $(-4)^2 > 9$
- concludes **not true**

There should be no errors seen including the use of brackets. The conclusion could be a preamble. Do not accept "sometimes true" or equivalent.

Alternatively, explains why the statement is **not true**

Eg. It is not true as when $x < -3$ then $x^2 > 9$ so x does not have to be greater than 3.

Eg. $x^2 > 9 \Rightarrow x < -3$ or $x > 3$ so not true

(ii)

M1: Takes out a factor of n and attempts to factorise the resulting quadratic.

A1: Deduces that the expression is the product of 3 consecutive integers

A1: Explains that as the expression is a multiple of 3 **and** 2, it must be a multiple of 6 and so is divisible by 6

If you see any method which appears to be credit worthy but is not covered by the scheme then send to review