Y1P6 XMQs and MS

(Total: 84 marks)

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 1. P1_Sample    Q3 .  4 marks - Y1P6 Circles

 2. P2_Specimen  Q9 .  9 marks - Y1P6 Circles

 3. P1_2018      Q6 . 10 marks - Y1P6 Circles

 4. P2_2020      Q14.  7 marks - Y1P6 Circles

 5. P1_2021      Q7 .  9 marks - Y1P6 Circles

 6. P1_2022      Q3 .  5 marks - Y1P6 Circles

 7. P1(AS)_2018  Q14.  9 marks - Y1P6 Circles

 8. P1(AS)_2019  Q10.  5 marks - Y1P6 Circles

 9. P1(AS)_2020  Q11.  9 marks - Y1P6 Circles

10. P1(AS)_2021  Q15.  9 marks - Y1P6 Circles

11. P1(AS)_2022  Q11.  8 marks - Y1P6 Circles
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3. A circle C has equation

$$x^2 + y^2 - 4x + 10y = k$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

(Total for Question 3 is 4 marks)

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Question	Scheme	Marks	AOs
3(a)	Attempts $(x-2)^2+(y+5)^2=....$	M1	1.1b
	Centre (2, −5)	A1	1.1b
		(2)	
(b)	Sets $k+2^2+5^2>0$	M1	2.2a
	$\Rightarrow k>-29$	A1ft	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts to complete the square so allow $(x-2)^2+(y+5)^2=....$

A1: States the centre is at (2, −5). Also allow written separately $x=2, y=-5$

(2, −5) implies both marks

(b)

M1: Deduces that the right hand side of their $(x\pm...)^2+(y\pm...)^2=...$ is >0 or $\geqslant 0$

A1ft: $k>-29$ Also allow $k\geqslant -29$ Follow through on their rhs of $(x\pm...)^2+(y\pm...)^2=...$

Question	Scheme	Marks	AOs
4	Writes $\int\frac{t+1}{t}\,\mathrm{d}t=\int 1+\frac{1}{t}\,\mathrm{d}t$ and attempts to integrate	M1	2.1
	$=t+\ln t\ (+c)$	M1	1.1b
	$(2a+\ln 2a)-(a+\ln a)=\ln 7$	M1	1.1b
	$a=\ln\frac{7}{2}$ with $k=\frac{7}{2}$	A1	1.1b
			(4 marks)

Notes:

M1: Attempts to divide each term by t or alternatively multiply each term by t^{-1}

M1: Integrates each term and knows $\int\frac{1}{t}\,\mathrm{d}t=\ln t$. The + c is not required for this mark

M1: Substitutes in both limits, subtracts and sets equal to ln7

A1: Proceeds to $a=\ln\frac{7}{2}$ and states $k=\frac{7}{2}$ or exact equivalent such as 3.5

9. A circle with centre $A(3, -1)$ passes through the point $P(-9, 8)$ and the point $Q(15, -10)$

(a) Show that PQ is a diameter of the circle.
(2)

(b) Find an equation for the circle.
(3)

A point R also lies on the circle.
Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.
(2)

(d) Find the size of angle ARQ, giving your answer to the nearest 0.1 of a degree.
(2)

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Question	Scheme	Marks	AOs
9(a)	E.g. midpoint $PQ = \left(\frac{-9+15}{2}, \frac{8-10}{2}\right)$	M1	1.1b
	$= (3, -1)$, which is the centre point A, so PQ is the diameter of the circle.	A1	2.1
		(2)	
(a) Alt 1	$m_{PQ} = \frac{-10-8}{15--9} = -\frac{3}{4} \Rightarrow PQ: y - 8 = -\frac{3}{4}(x - -9)$	M1	1.1b
	$PQ: y = -\frac{3}{4}x + \frac{5}{4}$. So $x = 3 \Rightarrow y = -\frac{3}{4}(3) + \frac{5}{4} = -1$ so PQ is the diameter of the circle.	A1	2.1
		(2)	
(a) Alt 2	$PQ = \sqrt{(-9-15)^2 + (8--10)^2}$ $\left\{= \sqrt{900} = 30\right\}$ **and either** • $AP = \sqrt{(3--9)^2 + (-1-8)^2}$ $\left\{= \sqrt{225} = 15\right\}$ • $AQ = \sqrt{(3-15)^2 + (-1--10)^2}$ $\left\{= \sqrt{225} = 15\right\}$	M1	1.1b
	e.g. as $PQ = 2AP$, then PQ is the diameter of the circle.	A1	2.1
		(2)	
(b)	Uses Pythagoras in a correct method to find either the radius or diameter of the circle.	M1	1.1b
	$(x-3)^2 + (y+1)^2 = 225$ $\left(\text{or } (15)^2\right)$	M1	1.1b
		A1	1.1b
		(3)	
(c)	Distance $= \sqrt{(\text{"15"})^2 - (10)^2}$ or $= \frac{1}{2}\sqrt{(2(\text{"15"}))^2 - (2(10))^2}$	M1	3.1a
	$\left\{= \sqrt{125}\right\} = 5\sqrt{5}$	A1	1.1b
		(2)	
(d)	$\sin(A\hat{R}Q) = \frac{20}{2(\text{"15"})}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\frac{10}{\text{"15"}}\right)$	M1	3.1a
	$A\hat{R}Q = 41.8103... = 41.8°$ (to 0.1 of a degree)	A1	1.1b
		(2)	
			(9 marks)

Question 9 Notes:	
(a)	
M1:	Uses a correct method to find the midpoint of the line segment *PQ*
A1:	Completes proof by obtaining $(3, -1)$ and gives a correct conclusion.
(a) Alt 1	
M1:	Full attempt to find the equation of the line *PQ*
A1:	Completes proof by showing that $(3, -1)$ lies on *PQ* and gives a correct conclusion.
(a) Alt 2	
M1:	Attempts to find distance *PQ* and either one of distance *AP* or distance *AQ*
A1:	Correctly shows either • $PQ = 2AP$, supported by $PQ = 30,\ AP = 15$ and gives a correct conclusion • $PQ = 2AQ$, supported by $PQ = 30,\ AQ = 15$ and gives a correct conclusion
(b)	
M1:	**Either** • uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius**. E.g. $r^2 = (-9-3)^2 + (8+1)^2$ or $r = \sqrt{(-9-3)^2 + (8+1)^2}$ or $r^2 = (15-3)^2 + (-10+1)^2$ or $r = \sqrt{(15-3)^2 + (-10+1)^2}$ **or** • uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter**. E.g. $d^2 = (15+9)^2 + (-10-8)^2$ or $d = \sqrt{(15+9)^2 + (-10-8)^2}$ **Note:** This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation)
M1:	Writes down a circle equation in the form $(x \pm \text{"3"})^2 + (y \pm \text{"}-1\text{"})^2 = (\text{their } r)^2$
A1:	$(x-3)^2 + (y+1)^2 = 225$ or $(x-3)^2 + (y+1)^2 = 15^2$ or $x^2 - 6x + y^2 + 2y - 215 = 0$
(c)	
M1:	Attempts to solve the problem by using the circle property "the perpendicular from the centre to a chord bisects the chord" and so applies Pythagoras to write down an expression of the form $\sqrt{(\text{their "15"})^2 - (10)^2}$.
A1:	$5\sqrt{5}$ by correct solution only
(d)	
M1:	Attempts to solve the problem by e.g. using the circle property "the angle in a semi-circle is a right angle" and writes down either $\sin(A\hat{R}Q) = \dfrac{20}{2(\text{their "15"})}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\dfrac{10}{\text{their "15"}}\right)$ **Note:** Also allow $\cos(A\hat{R}Q) = \dfrac{15^2 + \left(2\left(5\sqrt{5}\right)\right)^2 - 15^2}{2(15)\left(2\left(5\sqrt{5}\right)\right)}$ $\left\{= \dfrac{\sqrt{5}}{3}\right\}$
A1:	41.8 by correct solution only

6.

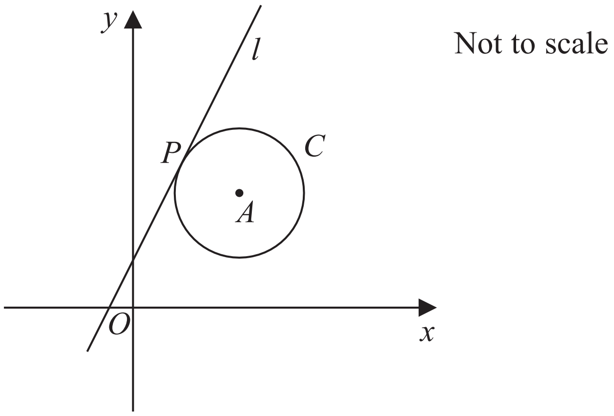

Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

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Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient $"-\frac{1}{2}"$ and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to k =...	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as c = ... to score this mark.

A1*: Completes proof with no errors or omissions $2y+x=17$

(b)
M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=\left(\sqrt{20}\right)^2$

(c)
M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to *k* . Scored for either substituting their $(11,3)$ in $y=2x+k$

or**, in the alternative**, for solving their $(4k-34)^2-4\times 5\times\left(k^2-10k+54\right)=0\Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

....

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find *k*. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2-4\times 5\times\left(k^2-10k+54\right)=0\Rightarrow k=...$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

....

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

M1: For substituting their $(11,3)$ into $y=2x+k$ and finding *k*

A1: $k=-19$ only

……………………………………………………………………………………………………

Other method are possible using trigonometry.

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

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Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=12-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(12-2x)^2-2rx-2r(12-2x)+r^2=0$ or $y=12-2x,\ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(12-2x-r)^2=r^2$	M1	1.1b
	$x^2+144-48x+4x^2-2rx-24r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-48)x+(r^2-24r+144)=0$ *	A1*	2.1
		(3)	
(b)	$b^2-4ac=0\Rightarrow(2r-48)^2-4\times5\times(r^2-24r+144)=0$	M1	3.1a
	$r^2-18r+36=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow(r-9)^2-81+36=0\Rightarrow r=...$	dM1	1.1b
	$r=9\pm3\sqrt{5}$	A1	1.1b
		(4)	
			(7 marks)

Notes:

(a)

B1: Deduces the correct equation of the circle

M1: Attempts to form an equation with terms of the form x^2, x, r^2, and xr **only** using $y=12\pm2x$ and their circle equation which must be of an appropriate form. I.e. includes or implies an x^2, y^2, r^2 such as $x^2+y^2=r^2$
If their circle equation starts off as e.g. $(x\pm a)^2+(y\pm b)^2=r^2$ then the B mark and the M mark can be awarded when the "a" and "b" are replaced by r or $-r$ as appropriate for their circle equation.

A1*: Uses correct and accurate algebra leading to the given solution.

(b)

M1: Attempts to use $b^2-4ac...0$ o.e. with $a=5, b=2r-48, c=r^2-24r+144$ and where ... is "=" or any inequality
Allow minor slips when copying the a, b and c provided it does not make the work easier and allow **their** a, b and c if they are similar expressions.
FYI $(2r-48)^2-4\times5\times(r^2-24r+144)=4r^2-192r+2304-20r^2+480r-2880=-16r^2+288r-576$

A1: Correct quadratic **equation** in r (or inequality). Terms need not be all one side but must be collected.
E.g. allow $r^2-18r=-36$ and allow any multiple of this equation (or inequality).

dM1: Correct attempt to solve their 3TQ in r. Dependent upon previous M

A1: Careful and accurate work leading to both answers in the required form (must be simplified surds)

7. The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C,

(b) find the possible values of k, giving your answers as simplified surds.

(5)

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Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2+(y+2)^2=...$	M1	1.1b
	$(5,\ -2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k \Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0 \Rightarrow (6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0 \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
		(9 marks)	

Notes

(a)(i)

M1: Attempts to complete the square on by halving both *x* and *y* terms.

Award for sight of $(x\pm5)^2,(y\pm2)^2=...$ This mark can be implied by a centre of (±5, ±2).

A1: Correct coordinates. (Allow $x = 5$, $y = -2$)

(a)(ii)

M1: Correct strategy for the radius or radius^2. For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$

or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0 \Rightarrow (x-a)^2+(y-b)^2=k \Rightarrow r^2=k$

A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5,\ -2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$

(b) Main method seen

M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0

A1: Correct 3 term quadratic equation.
The terms must be collected but this can be implied by correct *a*, *b* and *c*

M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both *b* and *c* are expressions in *k*. It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

M1: Solves 3TQ in *k*. See General Principles.
The 3TQ in *k* must have been found as a result of attempt at $b^2 - 4ac$ ... 0

A1: Correct simplified values

<u>Look carefully at the method used. It is possible to attempt this using gradients</u>

(b) Alt 1	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for *C* $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=...$ or $y=...$	M1	3.1a
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms
A1: Correct differentiation.
M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y.
M1: Uses at least one pair of coordinates and l to find at least one value for k. It is dependent upon having attempted both M's
A1: Correct simplified values

(b) Alt 2	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for l $y=3x+k,\ x+3y=1$ $\Rightarrow x=\ldots$ and $y=\ldots$ in terms of k	M1	3.1a
	$x=\frac{-3k-1}{10},\ y=\frac{k-3}{10},\ x^2+y^2-10x+4y+11=0 \Rightarrow k=\ldots$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3
M1 A1: Correct differentiation (See alt 1)
M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k
M1: Substitutes for x and y into C and solves resulting 3TQ in k
A1: Correct simplified values

(b) Alt 3	$y=3x+k \Rightarrow m=3 \Rightarrow m_r=-\frac{1}{3}$	M1
	$y+2=-\frac{1}{3}(x-5)$	A1
	$(x-5)^2+(y+2)^2=18,\ y+2=-\frac{1}{3}(x-5)$ $\Rightarrow \frac{10}{9}(x-5)^2=18 \Rightarrow x=\ldots$ or $\Rightarrow 10(y+2)^2=18 \Rightarrow y=\ldots$	M1
	$x=\frac{25\pm9\sqrt{5}}{5},\ y=\frac{-10\pm3\sqrt{5}}{5},\ k=y-3x \Rightarrow k=\ldots$	M1
	$k=-17\pm6\sqrt{5}$	A1

M1: Applies negative reciprocal rule to obtain gradient of radius
A1: Correct equation of radial line passing through the centre of C
M1: Solves simultaneously to find x or y
Alternatively solves "$y=-\frac{1}{3}x-\frac{1}{3}$" and $y=3x+k$ to get x in terms of k which they substitute in $x^2+(3x+k)^2-10x+4(3x+k)+11=0$ to form an equation in k.
M1: Applies $k = y - 3x$ with at least one pair of values to find k
A1: Correct simplified values

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

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Question	Scheme	Marks	AOs
3 (a)	(i) $x^2+y^2-10x+16y=80 \Rightarrow (x-5)^2+(y+8)^2=...$	M1	1.1b
	Centre $(5,-8)$	A1	1.1b
	(ii) Radius 13	A1	1.1b
		(3)	
(b)	Attempts $\sqrt{"5"^2+"8"^2}+"13"$	M1	3.1a
	$13+\sqrt{89}$ but ft on their centre and radius	A1ft	1.1b
		(2)	
		(5 marks)	
Notes:			

(a)(i)

M1: Attempts to complete the square on **both** x and y terms.

Accept $(x\pm5)^2+(y\pm8)^2=...$ or imply this mark for a centre of $(\pm5,\pm8)$

Condone $(x\pm5)^2....(y\pm8)^2=...$ where the first ... could be , or even –

A1: Correct centre $(5,-8)$.

Accept without brackets. May be written $x=5, y=-8$

(a)(ii)

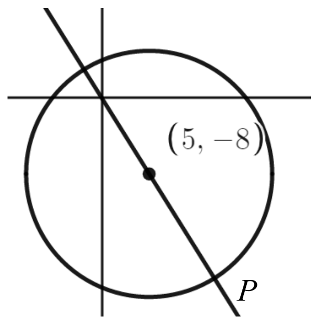
A1: 13. The M mark must have been awarded, so it can be scored following a centre of $(\pm5,\pm8)$.

Do not allow for $\sqrt{169}$ or ±13

(b)

M1: Attempts $\sqrt{"5"^2+"8"^2}+"13"$ for their centre $(5,-8)$ and their radius 13.

Award when this is given as a decimal, e.g. 22.4 for correct centre and radius. Look for $\sqrt{a^2+b^2}+r$ where centre is $(\pm a,\pm b)$ and radius is r



A1ft: $13+\sqrt{89}$ Follow through on their $(5,-8)$ and their 13 leading to an exact answer. ISW for example if they write $13+\sqrt{89}=22.4$

....

There are more complicated attempts which could involve finding P by solving $y="-\frac{8}{5}x"$ and $x^2+y^2-10x+16y=80$ simultaneously and choosing the coordinate with the greatest modulus. The method is only scored when the distance of the largest coordinate from O is attempted. Such methods are unlikely to result in an exact value but can score 1 mark for the method. Condone slips

FYI. Solving $y=-\frac{8}{5}x$ and $x^2+y^2-10x+16y=80 \Rightarrow 89x^2-890x-2000=0 \Rightarrow P=(11.89,-19.02)$

Hence $OP=\sqrt{"11.89"^2+"19.02"^2}\ (=22.43)$ scores M1 A0 but $OP=\sqrt{258+26\sqrt{89}}$ is M1 A1

14. The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k.

(6)

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Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = ...$	M1	1.1b
	(i) Centre $(3, -5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Substitute $y = kx$ in $x^2 + y^2 - 6x + 10y + 9 = 0$	M1	3.1a
	Collects terms to form correct 3TQ $(1 + k^2)x^2 + (10k - 6)x + 9 = 0$	A1	1.1b
	Attempts $b^2 - 4ac...0$ for their *a*, *b* and *c* leading to values for *k* $"(10k-6)^2 - 36(1+k^2)...0" \to k = ..., ...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2 - 4ac > 0$ and chooses the outside region (see note) for **their** critical values (Both *a* and *b* must have been expressions in *k*)	dM1	3.1a
	Deduces $k < 0, k > \frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)

M1: Attempts $(x \pm 3)^2 + (y \pm 5)^2 = ..$

This mark may be implied by candidates writing down a centre of $(\pm 3, \pm 5)$ or $r^2 = 25$

(i) A1: Centre $(3, -5)$

(ii) A1: Radius 5. Do not accept $\sqrt{25}$

Answers only scores all three marks

(b)

B1: Uses a sketch or their subsequent quadratic to deduce that $k = 0$ is a critical value.
You may award for the correct $k < 0$ but award if $k \leqslant 0$ or even with greater than symbols

M1: Substitutes $y = kx$ in $x^2 + y^2 - 6x + 10y + 9 = 0$ or their $(x \pm 3)^2 + (y \pm 5)^2 = ...$ to form an equation in just *x* and *k* . It is possible to substitute $x = \frac{y}{k}$ into their circle equation to form an equation in just *y* and *k*.

A1: Correct 3TQ $(1 + k^2)x^2 + (10k - 6)x + 9 = 0$ with the terms in *x* collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as $x^2 + k^2x^2 + (10k - 6)x + 9 = 0$ so long as the correct values of *a*, *b* and *c* are used in $b^2 - 4ac...0$.

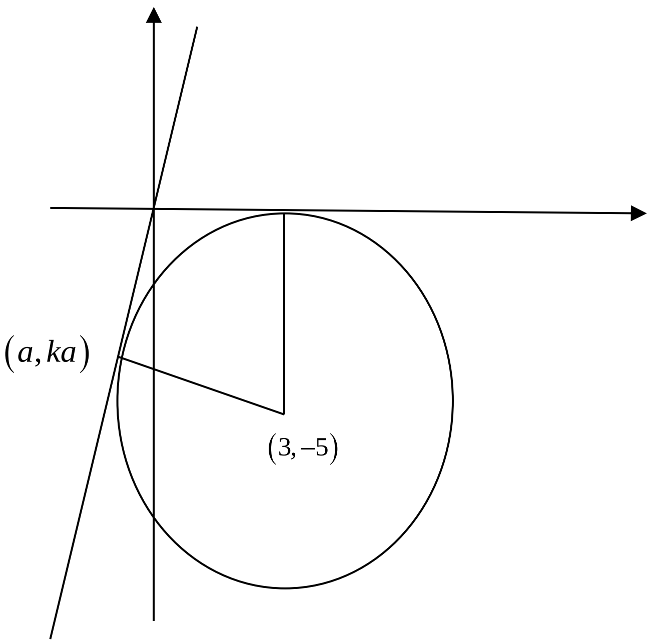
FYI The equation in *y* and *k* is $(1 + k^2)y^2 + (10k^2 - 6k)y + 9k^2 = 0$ oe

M1: Attempts to find two critical values for *k* using $b^2 - 4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both a and b must have been expressions in k.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.
$b^2 - 4ac = (k-6)^2 - 4\times(1+k^2)\times 9 > 0 \Rightarrow -35k^2 - 12k > 0 \Rightarrow k(35k+12) < 0$

A1: Deduces $k < 0, k > \frac{15}{8}$. This must be in terms of k.
Allow exact equivalents such as $k < 0 \cup k > 1.875$
but not allow $0 > k > \frac{15}{8}$ or the above with AND, & or $\cap$ between the two inequalities

Alternative using a geometric approach with a triangle with vertices at $(0,0)$, and $(3,-5)$



Alt (b)	Uses a sketch or otherwise to deduce $k = 0$ is a critical value	B1	2.2a
	Distance from (a, ka) to $(0,0)$ is $3 \Rightarrow a^2(1+k^2) = 9$	M1	3.1a
	Tangent and radius are perpendicular $\Rightarrow k \times \frac{ka+5}{a-3} = -1 \Rightarrow a(1+k^2) = 3-5k$	M1	3.1a
	Solve simultaneously, (dependent upon both M's)	dM1	1.1b
	$k = \frac{15}{8}$	A1	1.1b
	Deduces $k < 0, k > \frac{15}{8}$	A1	2.2a
		(6)	

10. A circle C has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The straight line with equation $x = k$, where k is a constant, is a tangent to C.

(b) Find the possible values for k.

(2)

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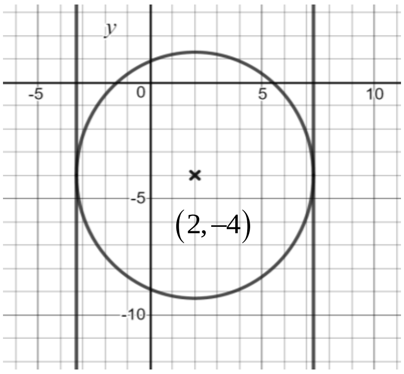
Question	Scheme	Marks	AOs
10(a)	$x^2+y^2-4x+8y-8=0$		
	Attempts $(x-2)^2+(y+4)^2-4-16-8=0$	M1	1.1b
	(i) Centre $(2,-4)$	A1	1.1b
	(ii) Radius $\sqrt{28}$ oe Eg $2\sqrt{7}$	A1	1.1b
		(3)	
(b)	 Attempts to add/subtract '*r*' from '2'	M1	3.1a
	$k=2\pm\sqrt{28}$	A1ft	1.1b
		(2)	
			(5 marks)

Notes

(a)

M1: Attempts to complete the square. Look for $(x\pm2)^2+(y\pm4)^2$ ...

If a candidate attempts to use $x^2+y^2+2gx+2fy+c=0$ then it may be awarded for a centre of $(\pm2,\pm4)$ Condone $a=\pm2, b=\pm4$

A1: Centre $(2,-4)$ This may be written separately as $x=2, y=-4$ BUT $a=2, b=-4$ is A0

A1: Radius $\sqrt{28}$ or $2\sqrt{7}$ isw after a correct answer

(b)

M1: Attempts to add **or** subtract their radius from their 2.

Alternatively substitutes $y=-4$ into circle equation and finds *x*/*k* by solving the quadratic equation by a suitable method.

A third (and more difficult) method would be to substitute $x=k$ into the equation to form a quadratic eqn in $y \Rightarrow y^2+8y+k^2-4k-8=0$ and use the fact that this would have one root. E.g. $b^2-4ac=0 \Rightarrow 64-4(k^2-4k-8)=0 \Rightarrow k=..$ Condone slips but the method must be sound.

A1ft: $k=2+\sqrt{28}$ and $k=2-\sqrt{28}$ Follow through on their 2 and their $\sqrt{28}$

If decimals are used the values must be calculated. Eg $k=2\pm5.29 \to k=7.29, k=-3.29$

Accept just $2\pm\sqrt{28}$ or equivalent such as $2\pm2\sqrt{7}$

Condone $x=2+\sqrt{28}$ and $x=2-\sqrt{28}$ but not $y=2+\sqrt{28}$ and $y=2-\sqrt{28}$

11. (i) A circle C_1 has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line l is the tangent to C_1 at the point $P(-5, 7)$.

Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k.

(4)

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Question	Scheme	Marks	AOs
11. (i)	$x^2+y^2+18x-2y+30=0 \Rightarrow (x+9)^2+(y-1)^2=...$	M1	1.1b
	Centre $(-9,1)$	A1	1.1b
	Gradient of line from $P(-5,7)$ to "$(-9,1)$" = $\frac{7-1}{-5+9}=\left(\frac{3}{2}\right)$	M1	1.1b
	Equation of tangent is $y-7=-\frac{2}{3}(x+5)$	dM1	3.1a
	$3y-21=-2x-10 \Rightarrow 2x+3y-11=0$	A1	1.1b
		(5)	
(ii)	$x^2+y^2-8x+12y+k=0 \Rightarrow (x-4)^2+(y+6)^2=52-k$	M1	1.1b
	Lies in Quadrant 4 if radius $<4 \Rightarrow$ "$52-k$" $<4^2$	M1	3.1a
	$\Rightarrow k>36$	A1	1.1b
	Deduces $52-k>0 \Rightarrow$ Full solution $36<k<52$	A1	3.2a
		(4)	
			(9 marks)

Notes

(i)

M1: Attempts $(x\pm9)^2....(y\pm1)^2=...$ It is implied by a centre of $(\pm9,\pm1)$

A1: States or uses the centre of C is $(-9,1)$

M1: A correct attempt to find the gradient of the radius using their $(-9,1)$ and P. E.g. $\frac{7-"1"}{-5-"-9"}$

dM1: For the complete strategy of using perpendicular gradients and finding the equation of the tangent to the circle. It is dependent upon both previous M's. $y-7=-\frac{1}{\text{gradient } CP}(x+5)$

Condone a sign slip on one of the -7 or the 5.

A1: $2x+3y-11=0$ oe such as $k(2x+3y-11)=0, k\in \mathbb{Z}$

....

Attempt via implicit differentiation. The first three marks are awarded

M1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow ...x+...y\frac{dy}{dx}+18-2\frac{dy}{dx}....=0$

A1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow 2x+2y\frac{dy}{dx}+18-2\frac{dy}{dx}=0$

M1: Substitutes $P(-5,7)$ into their equation involving $\frac{dy}{dx}$

....

(ii)

M1: For reaching $(x \pm 4)^2 + (y \pm 6)^2 = P - k$ where P is a positive constant. Seen or implied by centre coordinates $(\mp 4, \mp 6)$ and a radius of $\sqrt{P-k}$

M1: Applying the strategy that it lies entirely within quadrant if "their radius" < 4 and proceeding to obtain an inequality in k only (See scheme). Condone $\ldots \leqslant 4$ for this mark.

A1: Deduces that $k > 36$

A1: A rigorous argument leading to a full solution. In the context of the question the circle exists so that as well as $k > 36$ $52 - k > 0 \Rightarrow 36 < k < 52$ Allow $36 < k \leqslant 52$

15.

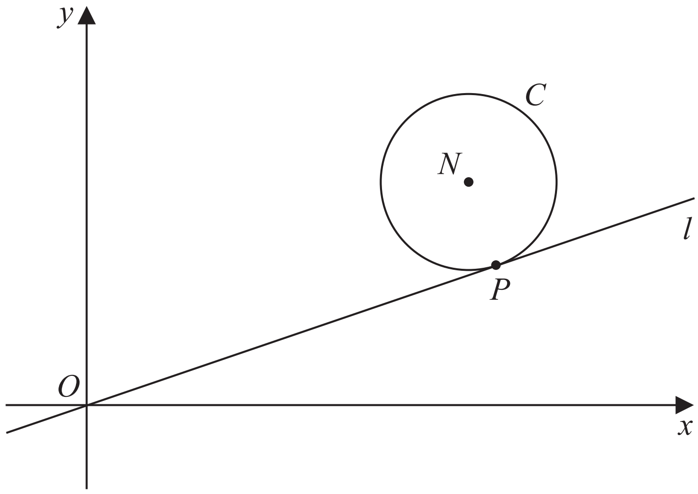

Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) an equation for C.

(4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C.

(c) Find the value of k.

(3)

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Question	Scheme	Marks	AOs
15 (a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	

Notes:
(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PN is -3 with point $(7,4)$ to find the equation of line PN

So sight of $y-4=-3(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1: Achieves $y=-3x+25$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving their $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates.

A1: $P=\left(\frac{15}{2},\frac{5}{2}\right)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=\left(\frac{15}{2},\frac{5}{2}\right)$ and $(7,4)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: Full and careful work leading to a correct equation. Eg $(x-7)^2+(y-4)^2=\frac{5}{2}$ or its expanded form. Do not accept $(x-7)^2+(y-4)^2=\left(\sqrt{\frac{5}{2}}\right)^2$

(c)

M1: Attempts to find where $y=\frac{1}{3}x+k$ meets *C* using a vector approach

M1: For a full method leading to k . Scored for substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$

A1: $k=\frac{10}{3}$ only

Alternative I

M1: For solving $y=\frac{1}{3}x+k$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$ oe

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=\frac{10}{3}$ only

Alternative II

M1: For solving $y=-3x+25$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$, creating a 3TQ and solving.

M1: For substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ into $y=\frac{1}{3}x+k$ and finding *k*

A1: $k=\frac{10}{3}$ only

11.

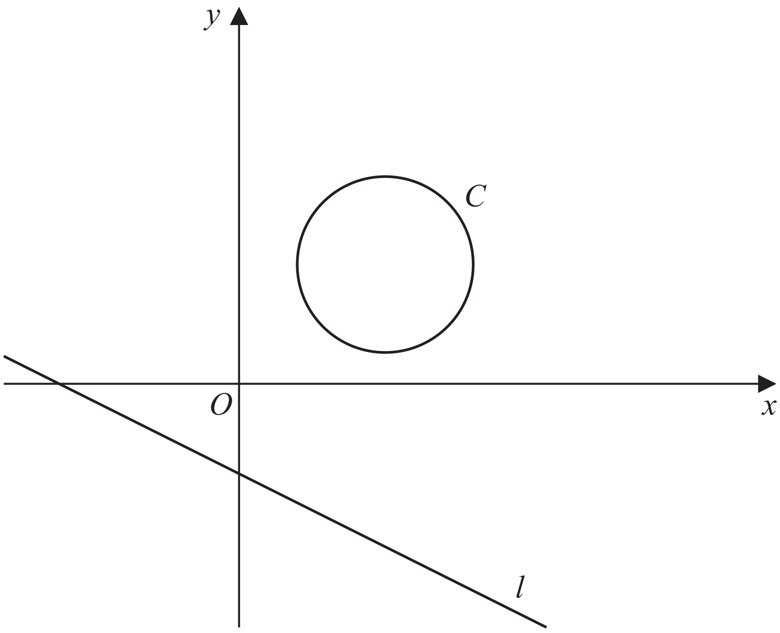

Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the radius of C.

(3)

(b) Find the shortest distance between C and l.

(5)

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P 6 9 2 0 1 A 0 3 4 4 8

Question	Scheme	Marks	AOs
11(a)	$(x \pm 5)^2 + (y \pm 4)^2$	M1	1.1b
	(i) Centre is (5, 4)	A1	1.1b
	(ii) Radius is 3	A1	1.1b
		(3)	
(b)	$2y + x + 6 = 0 \Rightarrow y = -\frac{1}{2}x + ... \Rightarrow -\frac{1}{2} \to 2$	B1	2.2a
	$m_N = 2 \Rightarrow y - 4 = 2(x-5)$ $y - 4 = 2(x-5),\ 2y + x + 6 = 0 \Rightarrow x = ...,\ y = ...$	M1	3.1a
	Intersection is at $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(5-\frac{6}{5}\right)^2 + \left(4+\frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left("5"-"\frac{6}{5}"\right)^2 + \left("4"+"\frac{18}{5}"\right)^2} - "3"$	dM1	3.1a
	$= \frac{19\sqrt{5}}{5} - 3$ (or awrt 5.50)	A1	1.1b
		(5)	
			(8 marks)

Notes

(a)

M1: Attempts to complete the square for both *x* and *y* terms $(x \pm 5)^2 .... (y \pm 4)^2$ which may be implied by a centre of (±5, ±4)

A1: Centre (5, 4)

A1: Radius 3

(b)

B1: Deduces the gradient of the perpendicular to *l* is 2. May be seen in the equation for the perpendicular line to *l*

M1: A fully correct strategy for finding the intersection. This requires use of their gradient of the perpendicular which cannot be the gradient of *l*
Look for $y - "4" = "2"(x - "5")$ where $(5, 4)$ is their centre being solved simultaneously with the equation of *l*
Do not be concerned with the mechanics of their rearrangement when solving simultaneously.
Many are finding the *y*-intercept of *l* $(0, -3)$ which is M0

A1: $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ or equivalent eg $(1.2,\ -3.6)$

They do not have to be written as coordinates and may be seen within their working rather than explicitly stated. They may also be written on the diagram.

dM1: Fully correct strategy for finding the required distance e.g. correct use of Pythagoras to find the distance between their centre and their intersection and then completes the problem by subtracting their radius. Condone a sign slip subtracting their $-\frac{18}{5}$.

It is dependent on the previous method mark.

Alternatively, they solve simultaneously their $y = 2x - 6$ with the equation of the circle and then find the distance between this intersection point and the point of intersection between *l* and the normal. They must choose the smaller positive root of the solution to their quadratic.

Eg

$(x-5)^2 + (2x-6-4)^2 = 9 \Rightarrow 5x^2 - 50x + 125 = 9$

$x = \frac{25-3\sqrt{5}}{5},\ y = \frac{20-6\sqrt{5}}{5}$

Distance between two points:

$$\sqrt{\left(\text{"}\frac{25-3\sqrt{5}}{5}\text{"}-\text{"}\frac{6}{5}\text{"}\right)^2 + \left(\text{"}\frac{20-6\sqrt{5}}{5}\text{"}+\text{"}\frac{18}{5}\text{"}\right)^2}$$

A1: Correct value e.g. $\sqrt{\frac{361}{5}} - 3$ or $\frac{19\sqrt{5}-15}{5}$). Also allow awrt 5.50

Isw after a correct answer is seen.

Alt (b) Be aware they may use vector methods:

B1M1: Attempts to find the perpendicular distance between their $(5,4)$ and $x + 2y + 6 = 0$ by substituting the values into the formula to find the distance between a point (x, y) and a line $ax + by + c = 0$

$$\Rightarrow \frac{|ax+by+c|}{\sqrt{a^2+b^2}} = \frac{|\text{"}5\text{"}\times\text{"}1\text{"}+\text{"}4\text{"}\times\text{"}2\text{"}+\text{"}6\text{"}|}{\sqrt{\text{"}1\text{"}^2+\text{"}2\text{"}^2}}$$

A1: $\frac{|5\times1+4\times2+6|}{\sqrt{1^2+2^2}}\ \left(=\frac{19}{\sqrt{5}}\right)$

dM1: Distance $= \text{"}\frac{19\sqrt{5}}{5}\text{"} - 3$

A1: $\frac{19\sqrt{5}-15}{5}$