Y1P4 XMQs and MS

(Total: 32 marks)

1. P1_2022 Q11. 7 marks - Y1P4 Graphs and transformations
2. P1(AS)_2019 Q7 . 8 marks - Y1P4 Graphs and transformations
3. P1(AS)_2019 Q11. 10 marks - Y1P4 Graphs and transformations
4. P1(AS)_2022 Q7 . 7 marks - Y1P4 Graphs and transformations

11.

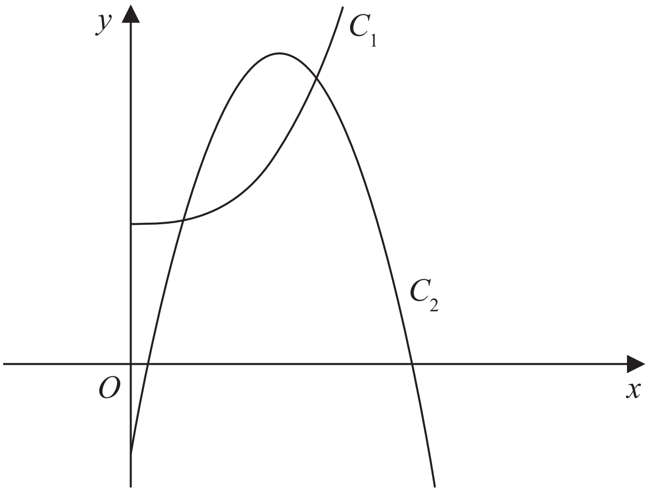


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \dfrac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

Question	Scheme	Marks	AOs
11 (a)	Substitutes $x=\frac{1}{2}$ into $y=2x^3+10$ **and** $y=42x-15x^2-7$ and finds the y values for both	M1	1.1b
	Achieves $\frac{41}{4}$ o.e. for both and makes a valid conclusion. *	A1*	2.4
		(2)	
(b)	Sets $42x-15x^2-7=2x^3+10\Rightarrow 2x^3+15x^2-42x+17=0$	M1	1.1b
	Deduces that $(2x-1)$ is a factor and attempts to divide	dM1	2.1
	$2x^3+15x^2-42x+17=(2x-1)\left(x^2+8x-17\right)$	A1	1.1b
	Solves their $x^2+8x-17=0$ using suitable method	M1	1.1b
	Deduces $x=-4+\sqrt{33}$ (see note)	A1	2.2a
		(5)	
			(7 marks)
Notes:			

(a)

M1: Substitutes $x=\frac{1}{2}$ into both $y=2x^3+10$ and $y=42x-15x^2-7$ and finds y values

Sight of just the y values at each is sufficient for this mark only.

Alternative: Sets $42x-15x^2-7=2x^3+10\Rightarrow$ cubic and substitutes $x=\frac{1}{2}$ into the expression, attempts $\mathrm{f}\left(\frac{1}{2}\right)$ or else attempts to divide the cubic = 0 by $(2x-1)$ or $\left(x-\frac{1}{2}\right)$. Condone $\mathrm{f}\left(\frac{1}{2}\right)=0$ without calculations for this mark only.

A1*: Correct calculations must be seen with a minimal conclusion that curves intersect (at $x=\frac{1}{2}$).

E.g. $2\left(\frac{1}{2}\right)^3+10=10.25$ and $42\left(\frac{1}{2}\right)-15\left(\frac{1}{2}\right)^2-7=10.25$ so curves intersect.

Acceptable alternatives are:

$\mathrm{f}(x)=42x-15x^2-7-2x^3-10, \mathrm{f}\left(\frac{1}{2}\right)=42\left(\frac{1}{2}\right)-15\left(\frac{1}{2}\right)^2-7-2\left(\frac{1}{2}\right)^3-10=0\Rightarrow$ so curves intersect

$\mathrm{f}(x)=2x^3+15x^2-42x+17\Rightarrow\left(x-\frac{1}{2}\right)\left(2x^2+16x-34\right)$ so $x=\frac{1}{2}$ is a root so curves intersect

$\mathrm{f}(x)=2x^3+15x^2-42x+17\Rightarrow(2x-1)\left(x^2+8x-17\right)$ so $(2x-1)$ is a factor hence curves intersect

Only accept verified, QED etc if there is a preamble mentioning intersection about how it will be shown.

Special case: Scores M1 A0 with or without a conclusion

This is presumably done using a calculator and requires all three roots exact or correct to 3sf

$\mathrm{f}(x)=2x^3+15x^2-42x+17=0$

$\Rightarrow x=0.5, 1.74, -9.74$

(b) **This part requires candidates to show all stages of their working.**
Answers without working will not score any marks
A method must be seen which could be from part (a) which must then be continued in (b)

M1: Sets $42x-15x^2-7=2x^3+10$ and proceeds to 4 term cubic equation.
Condone slips, e.g. signs. Terms do not have to be on one side of the equation.

dM1: For the key step in attempting to "divide" the cubic by $(2x-1)$

If attempted via inspection look for correct first and last terms

E.g. $2x^3+15x^2-42x+17=(2x-1)\left(x^2+...\pm 17\right)$ if cubic expression is correct

If attempted via division look for correct first and second terms

$$2x-1\overset{\displaystyle x^2+8x}{\overline{\big)\,2x^3+15x^2-42x+17}}$$ if cubic expression is correct

It is acceptable for an attempt to divide by $\left(x-\frac{1}{2}\right)$. It is easily marked using the same guidelines, e.g. $2x^3+15x^2-42x+17=\left(x-\frac{1}{2}\right)\left(2x^2+16x...\right)$

A1: $2x^3+15x^2-42x+17=(2x-1)\left(x^2+8x-17\right)$ o.e. $\left(x-\frac{1}{2}\right)\left(2x^2+16x-34\right)$

This may be implied by sight of $\left(x^2+8x-17\right)$ or $\left(2x^2+16x-34\right)$ in a "division" sum.

M1: Solves their quadratic $x^2+8x-17=0$ using a suitable method including calculator. You may need to check this. It is not completely dependent upon the previous M's but an attempt at a full method must have been seen. So look for

- the two equations being set equal to each other and some attempt made to combine
- some attempt to "divide" the result by $(2x-1)$ o.e. allowing for flaws in the method

A1: Gives $x=-4+\sqrt{33}$ o.e. only. The $x=-4-\sqrt{33}$ must not be included in the final answer.

Allow exact unsimplified equivalents such as $x=\dfrac{-8+\sqrt{132}}{2}$. ISW for instance if they then put this in decimal form.

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

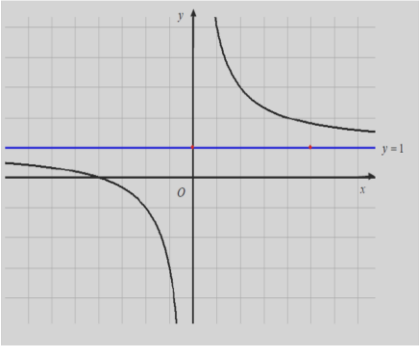
(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

Question	Scheme	Marks	AOs
7 (a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y=1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x}+1=-2x+5$	M1	1.1b
	$(\times x)\Rightarrow k^2+1x=-2x^2+5x\Rightarrow 2x^2-4x+k^2=0\,*$	A1*	2.1
		(2)	
(c)	Attempts to set $b^2-4ac=0$	M1	3.1a
	$8k^2=16$	A1	1.1b
	$k=\pm\sqrt{2}$	A1	1.1b
		(3)	
			(8 marks)

Notes

(a)

M1: For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification)

A1: Correct shape and position for both branches.
It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour

B1: Asymptote given as $y=1$. This could appear on the diagram or within the text.
Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate.

(b)

M1: Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x

A1*: Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips.
Condone if the order of the terms are different $2x^2+k^2-4x=0$

(c)

M1: Deduces that $b^2-4ac=0$ or equivalent for **the given equation.**
If a, b and c are stated only accept $a=2, b=\pm4, c=k^2$ so $4^2-4\times2\times k^2=0$
Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0\Rightarrow(x-1)^2=1-\frac{1}{2}k^2\Rightarrow "1-\frac{1}{2}k^2"=0$

A1: $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$

If *a*, *b* and *c* are stated they must be correct. Note that b^2 appearing as 4^2 is correct

Note on Question 7 continue

A1: $k = \pm\sqrt{2}$ and following correct *a*, *b* and *c* if stated

A solution via differentiation would be awarded as follows

M1: Sets the gradient of the curve $= -2 \Rightarrow -\frac{k^2}{x^2} = -2 \Rightarrow x = (\pm)\frac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2 - 4x + k^2 = 0$

A1: $2k^2 = (\pm)2\sqrt{2}k$ oe

A1: $k = \pm\sqrt{2}$

11. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

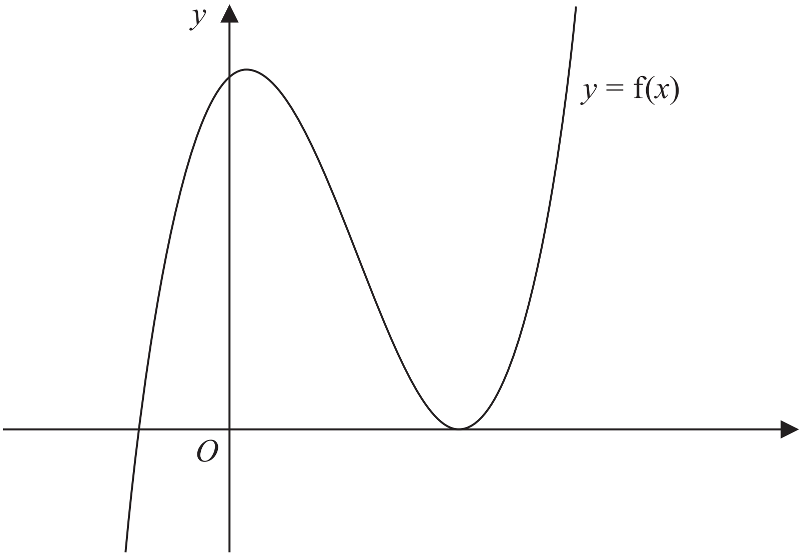


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

Question	**Scheme**	**Marks**	**AOs**
11 (a)	Attempts $\mathrm{f}(4)=2\times4^3-13\times4^2+8\times4+48$	M1	1.1b
	$\mathrm{f}(4)=0\Rightarrow(x-4)$ is a factor	A1	1.1b
		(2)	
(b)	$2x^3-13x^2+8x+48=(x-4)\left(2x^2...x-12\right)$	M1	2.1
	$=(x-4)\left(2x^2-5x-12\right)$	A1	1.1b
	Attempts to factorise quadratic factor or solve quadratic eqn	dM1	1.1b
	$\mathrm{f}(x)=(x-4)^2(2x+3)\Rightarrow\mathrm{f}(x)=0$ has only two roots, 4 and -1.5	A1	2.4
		(4)	
(c)	Deduces either three roots or deduces that $\mathrm{f}(x)$ is moved down two units	M1	2.2a
	States three roots, as when $\mathrm{f}(x)$ is moved down two units there will be three points of intersection (with the x - axis)	A1	2.4
		(2)	
(d)	For sight of $k=\pm4,\pm\frac{3}{2}$	M1	1.1b
	$k=4,-\frac{3}{2}$	A1ft	1.1b
		(2)	
			(10 marks)

Notes

(a)

M1: Attempts to calculate $\mathrm{f}(4)$.

Do not accept $\mathrm{f}(4)=0$ without sight of embedded values or calculations.

If values are not embedded look for two correct terms from $\mathrm{f}(4)=128-208+32+48$

Alternatively attempts to divide by $(x-4)$. Accept via long division or inspection.

See below for awarding these marks.

A1: Correct reason with conclusion. Accept $\mathrm{f}(4)=0,$ hence factor as long as M1 has been scored.

This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $\mathrm{f}(4)=0,$ then $(x-4)$ is a factor before doing the calculation and then writing hence proven or ✓ oe.

If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x-4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor

(b)

M1: Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)

Notes on Question 11 continue

So for inspection award for $2x^3-13x^2+8x+48=(x-4)(2x^2...x\pm12)$

$$\begin{array}{r|l} & 2x^2-5x \\ \hline x-4 & 2x^3-13x^2+8x+48 \end{array}$$

For division look for $\begin{array}{l} 2x^3-8x^2 \\ \hline \quad -5x^2 \end{array}$

A1: Correct quadratic factor $(2x^2-5x-12)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x-4)$ for this mark.

If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $(2x^2-5x-12)$.

dM1: Correct attempt to solve or factorise their $(2x^2-5x-12)$ including use of formula

Apply the usual rules $(2x^2-5x-12)=(ax+b)(cx+d)$ where $ac=\pm2$ and $bd=\pm12$

Allow the candidate to move from $(x-4)(2x^2-5x-12)$ to $(x-4)^2(2x+3)$ for this mark.

A1: Via factorisation

Factorises twice to $f(x)=(x-4)(2x+3)(x-4)$ or $f(x)=(x-4)^2(2x+3)$ or

$f(x)=2(x-4)^2\left(x+\frac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x=4$ and $-\frac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $f(x)=\underset{\nwarrow}{(x-4)^2}\underset{\uparrow}{(2x+3)}$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorsises to $(x-4)(2x^2-5x-12)$ and solves $2x^2-5x-12=0\Rightarrow x=4,-\frac{3}{2}$ followed by an explanation that the roots are $4,4,-\frac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

(c)

M1: For a valid **deduction.**

Accept **either** there are 3 roots **or** states that it is a solution of $f(x)=2$ or $f(x)-2=0$

A1: Fully explains:

Eg. States three roots, as $f(x)$ is moved down by **two** units (giving three points of intersection with the x - axis)

Eg. States three roots, as it is where $f(x)=2$ (You may see $y=2$ drawn on the diagram)

Notes on Question 11 continue
(d) **M1:** For sight of ± 4 **and** $\pm\frac{3}{2}$ Follow through on ± their roots. **A1ft:** $k = 4, -\frac{3}{2}$ Follow through on their roots. Accept $4, -\frac{3}{2}$ but not $x = 4, -\frac{3}{2}$

7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

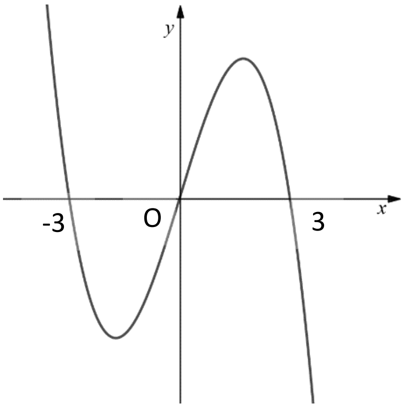
The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

Question	Scheme	Marks	AOs
7(a)	$9x-x^3=x\left(9-x^2\right)$	M1	1.1b
	$9x-x^3=x\left(3-x\right)\left(3+x\right)$ oe	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	B1	1.1b
	Passes though origin, (3, 0) and (–3, 0)	B1	1.1b
		(2)	
(c)	$y=9x-x^3 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=9-3x^2=0 \Rightarrow x=\left(\pm\right)\sqrt{3} \Rightarrow y=...$	M1	3.1a
	$y=\left(\pm\right)6\sqrt{3}$	A1	1.1b
	$\left\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\right\}$ oe	A1ft	2.5
		(3)	
		(7 marks)	

Notes

(a)

M1: Takes out a factor of x or $-x$. Scored for $\pm x(\pm 9 \pm x^2)$ May be implied by the correct answer or $\pm x\left(\pm x \pm 3\right)\left(\pm x \pm 3\right)$.

Also allow if they attempt to take out a factor of $(\pm x \pm 3)$ so score for $(\pm x \pm 3)(\pm 3x \pm x^2)$

A1: Correct factorisation. $x\left(3-x\right)\left(3+x\right)$ on its own scores M1A1.

Allow eg $-x\left(x-3\right)\left(x+3\right)$, $x\left(x-3\right)\left(-x-3\right)$ or other equivalent expressions

Condone an = 0 appearing on the end and condone eg x written as $(x+0)$.

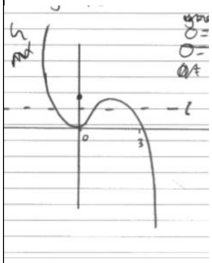
(b)

B1: Correct shape (negative cubic) appearing anywhere on a set of axes. It must have a minimum to the left and maximum to the right. Be tolerant of pen slips. Judge the intent of the shape. (see examples)

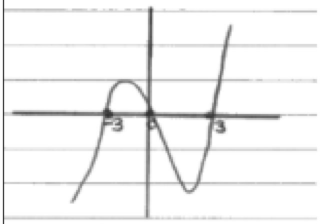
B1: Passes **through** each of the origin, (3, 0) and (–3, 0) and no other points on the x axis. (The graph should not turn on any of these points).
The points may be indicated as just 3 and -3 on the axes. Condone x and y to be the wrong way round eg $(0,-3)$ for (–3, 0) as long as it is on the correct axis but do not allow (–3, 0) to be labelled as (3, 0).

Examples

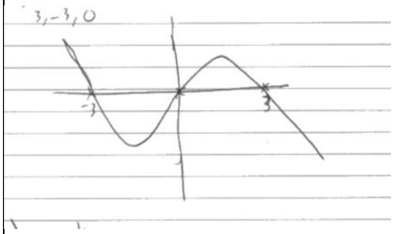
B1B0

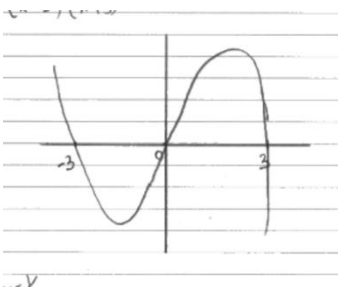


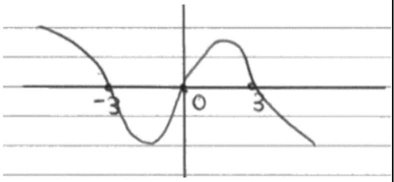
B0B1



B1B1







(c) ***Be aware the value of *y* can be solved directly using a calculator which is not acceptable***

M1: Uses a correct strategy for the *y* value of either the maximum or minimum. E.g. differentiates to achieve a quadratic, solves $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and uses their *x* to find *y*

A1: Either or both of the values $(\pm)6\sqrt{3}$.

Cannot be scored for an answer without any working seen.

A1ft: Correct answer in any acceptable set notation following through their $6\sqrt{3}$.

Condone $\left\{"-6\sqrt{3}" < k < "6\sqrt{3}"\right\}$ or $\left\{"-6\sqrt{3}" < k\right\} \cap \left\{k < "6\sqrt{3}"\right\}$ but not $\left\{"-6\sqrt{3}" < k\right\} \cup \left\{k < "6\sqrt{3}"\right\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Must be in terms of *k*