Y1P2 XMQs and MS

(Total: 67 marks)

1. P1_Sample Q11. 9 marks - Y1P2 Quadratics
2. P1_Specimen Q6 . 6 marks - Y1P2 Quadratics
3. P2_Specimen Q5 . 5 marks - Y1P2 Quadratics
4. P2_2018 Q8 . 7 marks - Y1P2 Quadratics
5. P1_2021 Q2 . 4 marks - Y1P2 Quadratics
6. P1_2021 Q12. 9 marks - Y1P2 Quadratics
7. P1_2022 Q5 . 6 marks - Y1P2 Quadratics
8. P1(AS)_2018 Q6 . 7 marks - Y1P2 Quadratics
9. P1(AS)_2019 Q2 . 8 marks - Y1P2 Quadratics
10. P1(AS)_2019 Q9 . 6 marks - Y1P2 Quadratics

11. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.
(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.
(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.
(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.
(2)

Question	Scheme	Marks	AOs
11(a)	Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$	M1	3.4
	Solves using an appropriate method, for example $d=\dfrac{-0.4\pm\sqrt{(0.4)^2-4(-0.002)(1.8)}}{2\times -0.002}$	dM1	1.1b
	Distance = awrt $204(\text{m})$ only	A1	2.2a
		(3)	
(b)	States the initial height of the arrow above the ground.	B1	3.4
		(1)	
(c)	$1.8+0.4d-0.002d^2=-0.002(d^2-200d)+1.8$	M1	1.1b
	$=-0.002((d-100)^2-10000)+1.8$	M1	1.1b
	$=21.8-0.002(d-100)^2$	A1	1.1b
		(3)	
(d)	(i) 22.1 metres	B1ft	3.4
	(ii) 100 metres	B1ft	3.4
		(2)	
		(9 marks)	

Notes:

(a)

M1: Sets $H=0 \Rightarrow 1.8+0.4d-0.002d^2=0$

M1: Solves using formula, which if stated must be correct, by completing square (look for $(d-100)^2=10900 \Rightarrow d=..$) or even allow answers coming from a graphical calculator

A1: Awrt 204 m only

(b)

B1: States it is the initial height of the arrow above the ground. Do not allow " it is the height of the archer"

(c)

M1: Score for taking out a common factor of -0.002 from at least the d^2 and d terms

M1: For completing the square for their (d^2-200d) term

A1: $=21.8-0.002(d-100)^2$ or exact equivalent

(d)

B1ft: For their '21.8+0.3' =22.1m

B1ft: For their 100m

6.

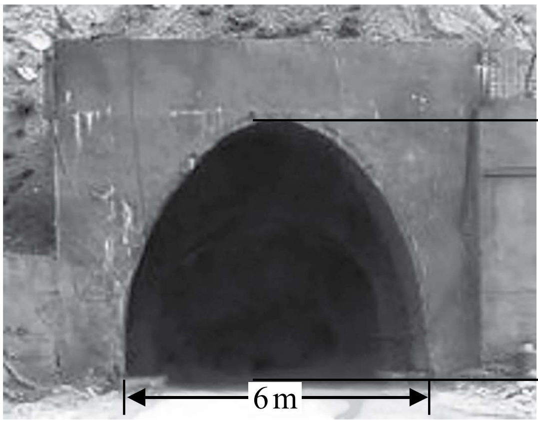

Figure 2

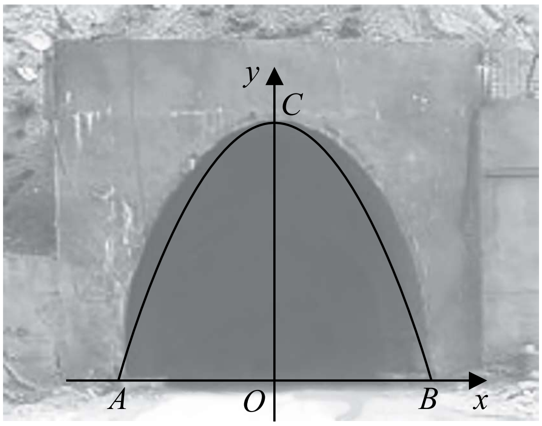

Figure 3

Figure 2 shows the entrance to a road tunnel. The maximum height of the tunnel is measured as 5 metres and the width of the base of the tunnel is measured as 6 metres.

Figure 3 shows a quadratic curve *BCA* used to model this entrance.

The points *A*, *O*, *B* and *C* are assumed to lie in the same vertical plane and the ground *AOB* is assumed to be horizontal.

(a) Find an equation for curve *BCA*.

(3)

A coach has height 4.1 m and width 2.4 m.

(b) Determine whether or not it is possible for the coach to enter the tunnel.

(2)

(c) Suggest a reason why this model may not be suitable to determine whether or not the coach can pass through the tunnel.

(1)

Question	Scheme	Marks	AOs
6 (a)	Attempts to use an appropriate model; e.g. $y = A(3-x)(3+x)$ or $y = A(9-x^2)$	M1	3.3
	e.g. $y = A(9-x^2)$ Substitutes $x = 0, y = 5 \Rightarrow 5 = A(9-0) \Rightarrow A = \frac{5}{9}$	M1	3.1b
	$y = \frac{5}{9}(9-x^2)$ or $y = \frac{5}{9}(3-x)(3+x)$, $\{-3 \leqslant x \leqslant 3\}$	A1	1.1b
		(3)	
(b)	Substitutes $x = \frac{2.4}{2}$ into their $y = \frac{5}{9}(9-x^2)$	M1	3.4
	$y = \frac{5}{9}(9-x^2) = 4.2 > 4.1 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(b) Alt 1	$4.1 = \frac{5}{9}(9-x^2) \Rightarrow x = \frac{9\sqrt{2}}{10}$, so maximum width $= 2\left(\frac{9\sqrt{2}}{10}\right)$	M1	3.4
	$= 2.545... > 2.4 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(c)	E.g. • Coach needs to enter through the centre of the tunnel. This will only be possible if it is a one-way tunnel • In real-life the road may be cambered (and not horizontal) • The quadratic curve *BCA* is modelled for the entrance to the tunnel but we do not know if this curve is valid throughout the entire length of the tunnel • There may be overhead lights in the tunnel which may block the path of the coach	B1	3.5b
		(1)	
			(6 marks)

Question 6 Notes:

(a)	
M1:	Translates the given situation into an appropriate quadratic model – see scheme
M1:	Applies the maximum height constraint in an attempt to find the equation of the model – see scheme
A1:	Finds a suitable equation – see scheme
(b)	
M1:	See scheme
A1:	Applies a fully correct argument to infer {by assuming that curve *BCA* is quadratic and the given measurements are correct}, that is possible for the coach to enter the tunnel
(c)	
B1:	See scheme

5. The line l has equation

$$3x - 2y = k$$

where k is a real constant.

Given that the line l intersects the curve with equation

$$y = 2x^2 - 5$$

at two distinct points, find the range of possible values for k.

(5)

Question	Scheme	Marks	AOs
5	$3x - 2y = k$ intersects $y = 2x^2 - 5$ at two distinct points		
	Eliminate *y* and forms quadratic equation = 0 or quadratic expression $\{= 0\}$	M1	3.1a
	$\{3x - 2(2x^2 - 5) = k \Rightarrow\} \; -4x^2 + 3x + 10 - k = 0$	A1	1.1b
	$\{"b^2 - 4ac" > 0 \Rightarrow\} \; 3^2 - 4(-4)(10 - k) > 0$	dM1	2.1
	$9 + 16(10 - k) > 0 \;\Rightarrow\; 169 - 16k > 0$		
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
5 **Alt 1**	Eliminate *y* and forms quadratic equation = 0 or quadratic expression $\{= 0\}$	M1	3.1a
	$y = 2\left(\frac{1}{3}(k + 2y)\right)^2 - 5 \Rightarrow y = \frac{2}{9}(k^2 + 4ky + 4y^2) - 5$		
	$8y^2 + (8k - 9)y + 2k^2 - 45 = 0$	A1	1.1b
	$\{"b^2 - 4ac" > 0 \Rightarrow\} \; (8k - 9)^2 - 4(8)(2k^2 - 45) > 0$	dM1	2.1
	$64k^2 - 144k + 81 - 64k^2 + 1440 > 0 \;\Rightarrow\; -144k + 1521 > 0$		
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
5 **Alt 2**	$\frac{dy}{dx} = 4x,\; m_l = \frac{3}{2} \Rightarrow 4x = \frac{3}{2} \Rightarrow x = \frac{3}{8}$. So $y = 2\left(\frac{3}{8}\right)^2 - 5 = -\frac{151}{32}$	M1 A1	3.1a 1.1b
	$k = 3\left(\frac{3}{8}\right) - 2\left(-\frac{151}{32}\right) \Rightarrow k = ...$	dM1	2.1
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
			(5 marks)

Question 5 Notes:	
M1:	Complete strategy of eliminating *x* or *y* and manipulating the resulting equation to form a quadratic equation = 0 or a quadratic expression $\{= 0\}$
A1:	Correct algebra leading to either • $-4x^2 + 3x + 10 - k = 0$ or $4x^2 - 3x - 10 + k = 0$ or a one-sided quadratic of either $-4x^2 + 3x + 10 - k$ or $4x^2 - 3x - 10 + k$ • $8y^2 + (8k - 9)y + 2k^2 - 45 = 0$ or a one-sided quadratic of e.g. $8y^2 + (8k - 9)y + 2k^2 - 45$
dM1:	Depends on the previous M mark. Interprets $3x - 2y = k$ intersecting $y = 2x^2 - 5$ at two distinct points by applying $"b^2 - 4ac" > 0$ to their quadratic equation or one-sided quadratic.
B1:	See scheme
A1:	Correct answer, e.g. • $k < \frac{169}{16}$ • $\left\{k : k < \frac{169}{16}\right\}$
Alt 2	
M1:	Complete strategy of using differentiation to find the values of *x* and *y* where $3x - 2y = k$ is a tangent to $y = 2x^2 - 5$
A1:	Correct algebra leading to $x = \frac{3}{8},\ y = -\frac{151}{32}$
dM1:	Depends on the previous M mark. Full method of substituting their $x = \frac{3}{8},\ y = -\frac{151}{32}$ into *l* and attempting to find the value for *k*.
B1:	See scheme
A1:	Deduces correct answer, e.g. • $k < \frac{169}{16}$ • $\left\{k : k < \frac{169}{16}\right\}$

8.

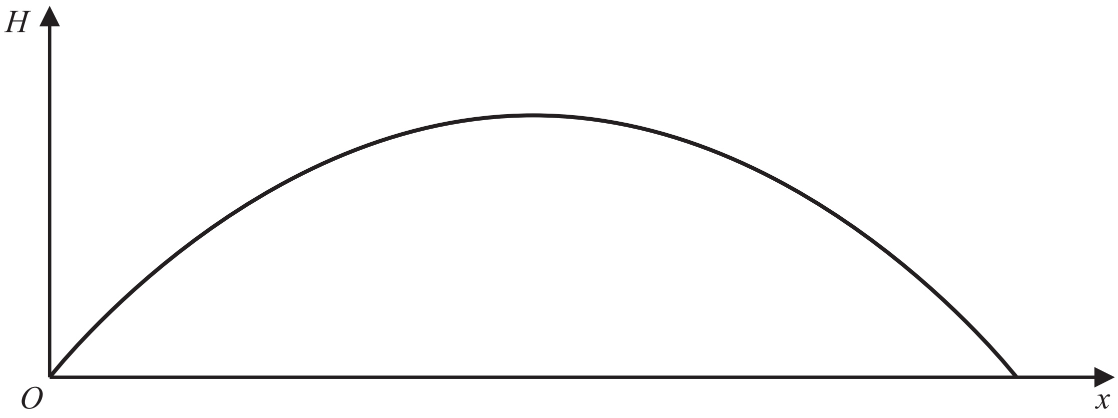

Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

Question	Scheme	Marks	AOs
8 (a) **Way 1**	$H = Ax(40-x)$ {or $H = Ax(x-40)$ }	M1	3.3
	$x = 20,\ H = 12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \dfrac{3}{100}$	dM1	3.1b
	$H = \dfrac{3}{100}x(40-x)$ or $H = -\dfrac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) **Way 2**	$H = 12 - \lambda(x-20)^2$ {or $H = 12 + \lambda(x-20)^2$ }	M1	3.3
	$x = 40,\ H = 0 \Rightarrow 0 = 12 - \lambda(40-20)^2 \Rightarrow \lambda = \dfrac{3}{100}$	dM1	3.1b
	$H = 12 - \dfrac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) **Way 3**	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x = 0,\ H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ **and either** $x = 40,\ H = 0 \Rightarrow 0 = 1600a + 40b$ **or** $x = 20,\ H = 12 \Rightarrow 12 = 400a + 20b$ **or** $\dfrac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	
(b)	$\{H = 3 \Rightarrow\}\ 3 = \dfrac{3}{100}x(40-x) \Rightarrow x^2 - 40x + 100 = 0$ **or** $\{H = 3 \Rightarrow\}\ 3 = 12 - \dfrac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \dfrac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\{$chooses $20 + \sqrt{300} \Rightarrow\}$ greatest distance $=$ awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

Notes for Question 8	
(a)	
M1:	Translates the situation given into a suitable equation for the model. E.g. **Way 1:** {Uses $(0, 0)$ and $(40, 0)$ to write} $H = Ax(40-x)$ o.e. {or $H = Ax(x-40)$ }
	Way 2: {Uses $(20, 12)$ to write} $H = 12 - \lambda(x-20)^2$ or $H = 12 + \lambda(x-20)^2$
	Way 3: Writes $H = ax^2 + bx + c$, **and** uses $(0, 0)$ to deduce $c = 0$ **and** an attempt at using either $(40, 0)$ or $(20, 12)$ **Special Case:** Allow SC M1dM0A0 for not deducing $c = 0$ but attempting to apply both $(40, 0)$ and $(20, 12)$
dM1:	Applies a complete strategy with appropriate constraints to find all constants in their model. **Way 1:** Uses $(20, 12)$ on their model and finds $A = ...$ **Way 2:** Uses either $(40, 0)$ or $(0, 0)$ on their model to find $\lambda = ...$ **Way 3:** Uses $(40, 0)$ and $(20, 12)$ on their model to find $a = ...$ and $b = ...$
A1:	Finds a correct equation linking *H* to *x* E.g. $H = \frac{3}{100}x(40-x)$, $H = 12 - \frac{3}{100}(x-20)^2$ or $H = -0.03x^2 + 1.2x$
Note:	Condone writing *y* in place of *H* for the M1 and dM1 marks.
Note:	Give final A0 for $y = -0.03x^2 + 1.2x$
Note:	Give special case M1dM0A0 for writing down any of $H = 12 - (x-20)^2$ or $H = x(40-x)$ or $H = x(x-40)$
Note:	Give M1 dM1 for finding $-0.03x^2 + 1.2x$ or $a = -0.03, b = 1.2, c = 0$ in an implied $ax^2 + bx$ or $ax^2 + bx + c$ (with no indication of $H = ...$)
(b)	
M1:	Substitutes $H = 3$ into their quadratic equation and proceeds to obtain a 3TQ or a quadratic in the form $(x \pm \alpha)^2 = \beta;\ \alpha,\ \beta \neq 0$
Note:	E.g. $1.2x - 0.03x^2 = 3$ or $40x - x^2 = 100$ are acceptable for the 1st M mark
Note:	Give M0 dM0 A0 for $(\text{their } A)x^2 = 3 \Rightarrow x = ...$ or their $(\text{their } A)x^2 + (\text{their } k) = 3 \Rightarrow x = ...$
dM1:	Correct method of solving their quadratic equation to give ***at least one solution***
A1:	Interprets their solution in the original context by selecting the larger correct value ***and states correct units for their value***. E.g. Accept awrt 37.3 m or $\left(20 + \sqrt{300}\right)$ m or $\left(20 + 10\sqrt{3}\right)$ m
Note:	Condone the use of inequalities for the method marks in part (b)
(c):	
B1:	See scheme
Note:	Give no credit for the following reasons • *H* (or the height of ball) is negative when $x > 40$ • Bounce of the ball should be considered after hitting the ground • Model will not be true for a different rugby ball • Ball may not be kicked in the same way each time

2. Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express f(x) in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P

(ii) the coordinates of Q

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
2(a)	$\mathrm{f}(x) = (x-2)^2 \pm ...$	M1	1.2
	$\mathrm{f}(x) = (x-2)^2 + 1$	A1	1.1b
		(2)	
(b)(i)	$P = (0, 5)$	B1	1.1b
(b)(ii)	$Q = (2, 1)$	B1ft	1.1b
		(2)	
			(4 marks)
Notes			

(a)

M1: Achieves $(x-2)^2 \pm ...$ or states $a = -2$

A1: Correct expression $(x-2)^2 + 1$ ISW after sight of this

Condone $a = -2$ and $b = 1$. Condone $(x-2)^2 + 1 = 0$

(b)

(i) B1: Correct coordinates for P. Allow to be expressed $x = 0,\ y = 5$

(ii) B1ft: Correct coordinates for Q. Allow to be expressed $x = 2,\ y = 1$ (Score for the correct answer or follow through their part (a) so allow $(-a, b)$ where a and b are numeric)

Score in any order if they state $P = (0, 5)$ and $Q = (2, 1)$

....

Allow part (b) to be awarded from a sketch. So award

First B1 from a sketch crossing the y-axis at 5

Second B1 from a sketch with minimum at $(2, 1)$

....

12.

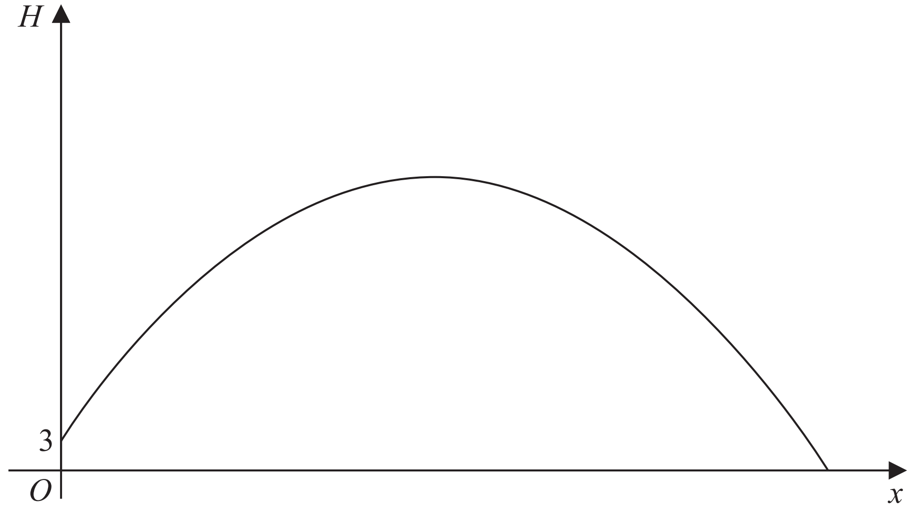

Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x = 0,\ H = 3 \Rightarrow H = ax^2 + bx + 3$	M1	3.3
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **or** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 3$ and $x = 120, H = 27 \Rightarrow 27 = 14400a + 120b + 3$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 90 \Rightarrow 180a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 90 \Rightarrow H\left(= -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3\right) = 30\,\text{m}$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{300}x^2 + \frac{3}{5}x + 3 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-4.868...,)\ 184.868...$ $\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • *H* is not likely to be a quadratic function in *x*	B1	3.5b
		(1)	
		(9 marks)	

Notes

(a)

M1: Translates the problem into a suitable model and uses $H = 3$ when $x = 0$ to establish $c = 3$
Condone with $a = \pm 1$ so $H = x^2 + bx + 3$ will score M1 but little else

M1: For a correct attempt at **using one of the two other pieces** of information within a quadratic model **Either** uses $H = 27$ when $x = 120$ (with $c = 3$) to produce a linear equation connecting a and b for the model **Or** differentiates and uses $\frac{dH}{dx} = 0$ when $x = 90$. Alternatives exist here, using the symmetrical nature of the curve, so they could use $x = -\frac{b}{2a}$ at vertex or use point (60, 27) or (180,3).

A1: At least one correct equation connecting a and b. Remember "a" could have been set as negative so an equation such as $27 = -14400a + 120b + 3$ would be correct in these circumstances.

dM1: Fully correct strategy that uses $H = ax^2 + bx + 3$ with the two other pieces of information in order to establish the values of **both** a **and** b for the model

A1: Correct equation, not just the correct values of a, b and c. Award if seen in part (b)

(b)(i)

B1: Correct height including the units. CAO

(b)(ii)

M1: Uses $H = 0$ and attempts to solve for x. Usual rules for quadratics.

A1: Discards the negative solution (may not be seen) and identifies awrt 185 m. Condone lack of units

(c)
B1: Candidate should either refer to an issue with one of the four aspects of how the situation has been modelled or give a situation where the model breaks down

- the ball has been modelled as a particle
- there may be trees (or other hazards) in the way that would affect the motion

Condone answers (where the link to the model is not completely made) such as

- the ball will spin
- ground is not flat

Do not accept answers which refer to the situation after it hits the ground (this isn't what was modelled)

- the ball will bounce after hitting the ground
- it gives a negative height for some values for x

Do not accept answers that do not refer to the model in question, or else give single word vague answers

- the height of tee may have been measured incorrectly
- "friction", "spin", "force" etc
- it does not take into account the weight of the ball
- it depends on how good the golfer is
- the shape of the ball will affect the motion
- you cannot hit a ball the same distance each time you hit it

The method using an alternative form of the equation can be scored in a very similar way.
The first M is for the completed square form of the quadratic showing a maximum at $x = 90$
So award M1 for $H = \pm a(x-90)^2 + c$ or $H = \pm a(90-x)^2 + c$. Condone for this mark an equation with
$a = 1 \Rightarrow H = (x-90)^2 + c$ or $c = 3 \Rightarrow H = a(x-90)^2 + 3$ but will score little else

Alt (a)	$H = a(x+b)^2 + c$ and $x = 90$ at $H_{max} \Rightarrow H = a(x-90)^2 + c$	M1	3.3
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **or** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$	M1 A1	3.1b 1.1b
	$H = 3$ when $x = 0 \Rightarrow 3 = 8100a + c$ **and** $H = 27$ when $x = 120 \Rightarrow 27 = 900a + c$ $\Rightarrow a = \ldots, c = \ldots$	dM1	3.1b
	$H = -\frac{1}{300}(x-90)^2 + 30$ o.e	A1	1.1b
		(5)	
(b)	$x = 90 \Rightarrow H = 0^2 + 30 = 30\,\text{m}$	B1	3.4
		(1)	
	$H = 0 \Rightarrow 0 = -\frac{1}{300}(x-90)^2 + 30 \Rightarrow x = \ldots$	M1	3.4
	$\Rightarrow x = 185\,(\text{m})$	A1	3.2a
		(2)	

Note that $H = -\frac{1}{300}(x-90)^2 + 30$ is equivalent to $H = -\frac{1}{300}(90-x)^2 + 30$

Other versions using symmetry are also correct so please look carefully at all responses

E.g. Using a starting equation of $H = a(x-60)(x-120) + b$ leads to $H = -\frac{1}{300}(x-60)(x-120) + 27$

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures.
(4)

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.
(2)

Question	Scheme	Marks	AOs
5 (a)	Attempts to use $h^2 = at + b$ with either $t = 2, h = 2.6$ or $t = 10, h = 5.1$	M1	3.1b
	Correct equations $2a + b = 6.76$, $10a + b = 26.01$	A1	1.1b
	Solves simultaneously to find values for a and b	dM1	1.1b
	$h^2 = 2.41t + 1.95$ cao	A1	3.3
		(4)	
(b)	Substitutes $t = 20$ into their $h^2 = 2.41t + 1.95$ and finds h or h^2 Or substitutes $h = 7$ into their $h^2 = 2.41t + 1.95$ and finds t	M1	3.4
	Compares the model with the true values and concludes "good model" with a minimal reason E.g. I Finds $h = 7.08\,(\text{m})$ and states that it is a good model as 7.08 (m) is close to 7 (m) E.g II Finds t =19.5 years and states that the model is accurate as 19.5 (years) $\approx$ 20 (years)	A1	3.5a
		(2)	
			(6 marks)
Notes:			

(a)

M1: For translating the problem into mathematics. Attempts to use the given equation o.e. with either of the pieces of information to form one correct equation.

Award for unsimplified equations as well, such as $2.6^2 = 2a + b$ or $2.6 = \sqrt{2a+b}$

A1: Two correct (and different) equations which may be unsimplified

dM1: Solves simultaneously to find values for *a* and *b*. It is dependent upon the previous M

Don't be too concerned with the process here as calculators may be used.

Score if values of *a* and *b* are reached from a pair of simultaneous equations

A1: Establishes **the full equation of the model** with values of *a* and *b* given to **exactly** 3sf. Award if seen in either (a) or (b). It is not scored for the values of *a* and *b*.

Allow either $h^2 = 2.41t + 1.95$ or $h = \sqrt{2.41t + 1.95}$

If they go on to square root each term from $h^2 = 2.41t + 1.95$ then it is A0. E.g. $h = 1.55t + 1.40$

....

Special case for candidates who mistakenly use $h = at + b$

For $2.6 = 2a + b, \quad 5.1 = 10a + b \quad \Rightarrow h = 0.3125t + 1.975$ or $h = 0.313t + 1.98$

can score M1 correct equations with attempt to solve and A1 for either correct answer shown above. These are the only marks available to them for a maximum mark of 1100 00

....

(b)

M1: A full and valid attempt to

either substitute $t = 20$ into their $h^2 = 2.41t + 1.95$ o.e. and find a value for *h* or h^2

or substitute $h = 7$ into their $h^2 = 2.41t + 1.95$ o.e. and find a value for *t*

(to enable the candidate to compare real life data with that of the model.)

The equation of the model must be of the correct form, either $h^2 = at + b$ or $h = \sqrt{at+b}$

Do not be too concerned with the mechanics of the solution but the square or $\sqrt{\ }$ must have been used appropriately to enable the comparison to be made.

In cases with no working you will need to check the calculation

A1: Compares their *h*=7.08m to 7m o.e using h^2 or their *t* =19.5 years to 20 years and makes valid conclusion with reason.

For this mark you require

- a statement that it is a "good" or "accurate' ' model or similar wording
- a reason such as "the values are close", "the values are similar" or "the predicted values are within 5% of the true values."
- a model with equation $h^2 = a\,t + b$ o.e. where $a =$ awrt 2.4 and $b \in [1.9, 2.0]$
- correct calculations

Condone a statement like ' the model is pretty accurate as it predicted 7.08m and the actual value is 7m'

Do not allow incorrect statements such as the model is incorrect as it does not give 7 metres.

Do not allow just "the model gives an underestimate of the true value."

Do not allow 'bad' or 'poor' model

6.

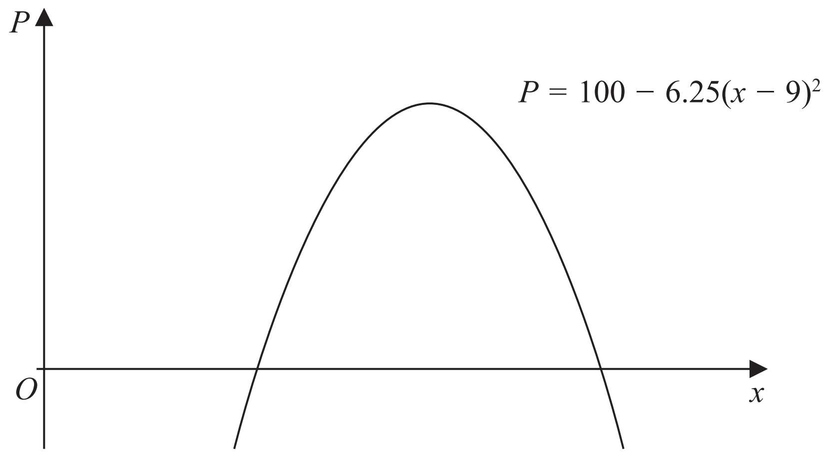

Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 5 8 3 4 6 A 0 1 2 4 8

Question	Scheme	Marks	AOs
6 (a)	Attempts $P=100-6.25(15-9)^2$	M1	3.4
	$=-125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P>80 \Rightarrow (x-9)^2<3.2$ or $P=80 \Rightarrow (x-9)^2=3.2$	M1	3.1b
	$\Rightarrow 9-\sqrt{3.2}<x<9+\sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

(a)

M1: Substitutes $x=15$ into $P=100-6.25(x-9)^2$ and attempts to calculate. This is implied by an answer of -125. Some candidates may have attempted to multiply out the brackets before they substitute in the *x* = 15. This is acceptable as long as the function obtained is quadratic. There must be a calculation seen or implied by the value of -125.

A1: Finds $P=-125$ or states that $P<0$ **and** explains that (this is not sensible as) the company would make a loss.

Condone $P=-125$ followed by an explanation that it is not sensible as the company would make a loss of £125 rather than £125 000. (They will lose marks later in the question). An explanation that it is not sensible as "the profit cannot be negative" , "the profit is negative" or "the company will not make any money" , "they might make a loss" is incomplete/incorrect. You may ignore any misconceptions or reference to the price of the toy being too cheap for this mark.

Alt: **M1:** Sets *P* = 0 and finds $x=5,13$ A1: States $15>13$ and states makes a loss

(b)

M1: Uses $P ... 80$ where ... is any inequality or "$=$"in $P=100-6.25(x-9)^2$ and proceeds to $(x-9)^2 ... k$ where *k* >0 and ... is any inequality or "$=$"

Eg. Condone $P<80$ in $P=100-6.25(x-9)^2 \Rightarrow (x-9)^2<k$ where *k* >0 If the candidate attempts to multiply out then allow when they achieve a form $ax^2+bx+c=0$

dM1: Award for solving to find the two positive values for *x*. Allow decimal answers

FYI correct answers are $\Rightarrow 9-\sqrt{3.2}<x<9+\sqrt{3.2}$ Accept $\Rightarrow x=9\pm\sqrt{3.2}$

Condone incorrect inequality work $100-6.25(x-9)^2>80 \Rightarrow (x-9)^2>3.2 \Rightarrow x>9\pm\sqrt{3.2}$

Alternatively award if the candidate selects the lower of their two positive values $9-\sqrt{3.2}$

A1: Deduces that the minimum Price = £7.22 (£7.21 is not acceptable)

Trial and improvement or just answers of £7.22 or £7.21(with no working) then please send to review.

(c)

(i) B1: Maximum Profit = £ 100 000 with units. Accept 100 thousand pound.

(ii) B1: Selling price = £9 with units

SC 1: Missing units in (b) and (c) only penalise once, withhold the final mark. Eg correct values in (c) would be scored B1 B0.

SC 2: If the answers to (c) are both correct, but in the wrong order score SC B1 B0

If (i) and (ii) are not written out score in the order given.

2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

P 5 8 3 5 1 A 0 4 4 4

Question	Scheme		Marks	AOs
2(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow \left(b^2 + 9\right)\left(b^2 - 2\right) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}\ ,\ -\sqrt{2}$ only		A1	1.1b
			(4)	
				(8 marks)

Notes

(i)

M1: Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $\left(C \neq 0\right)$

An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$

Eg. $\ldots a^4 = \ldots a \Rightarrow a^{\pm 3} = k$ or $\ldots a^4 = \ldots a \Rightarrow \ldots a^4 - \ldots a = 0 \Rightarrow \ldots a\left(a^3 - \ldots\right) = 0 \Rightarrow a^3 = \ldots$

Allow for slips on coefficients.

M1: Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible)
You may even see logs used.

A1: $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25

B1: Deduces that a = 0 is a solution.

(ii)

M1: Attempts to solve as a quadratic equation in b^2
Accept $\left(b^2 + m\right)\left(b^2 + n\right) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u

A1: Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given.
Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen

dM1: Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$

A1: $b=\sqrt{2}$, $-\sqrt{2}$ only. The solution asks for real values so if $3i$ is given then score A0
Notes on Question 2 continue

Answers with minimal or no working:

In part **(i)**

- no working, just answer(s) with they can score the B1
- If they square and proceed to the quartic equation $256a^4 = 4a$ oe, and then write down the answers they can have access to all marks.

In part (ii)

- Accept for 4 marks $b^2 = 2 \Rightarrow b = \pm\sqrt{2}$
- No working, no marks.

9. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)

Question	Scheme	Marks	AOs
9 (a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \leqslant 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
			(6 marks)

Notes

Note: Only withhold the mark for a lack of tonnes, once, the first time that it occurs.

(a)
B1: 117 tonnes or 117 t.
(b)
B1: 1200 tonnes or 1200 t.
(c)
M1: Attempts $T_5 - T_4 = \{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$ May be implied by $525-432$

Condone for this mark an attempt at $T_4 - T_3 = \{1200-3\times(4-20)^2\}-\{1200-3\times(3-20)^2\}$

A1: 93 tonnes or 93 t
(d)
For one mark

Shows an appreciation of the model

- States $n \leqslant 20$ or $n < 20$
- Condone for one mark $n \leqslant 40$ or $n < 40$ **with** "the mass of tin mined cannot be negative" oe
- Condone for one mark $n = 40$ **with** a statement that "the mass of tin mined becomes 0" oe
- after 20 years the (total) amount of tin mined starts to go down (n may not be mentioned and total may be missing)
- after 20 years the (total) mass reaches a maximum value. (Similar to above)
- States T_{max} is reached when $n = 20$

For two marks

States the limitation on n and explains fully. (Total mass, not mass must be used)

- States that $n \leqslant 20$ and explains that the total mass of tin cannot decrease.
- Alternatively states that n cannot be more than 20 and the total mass of tin would be decreasing
- $0 < n \leqslant 20$ as the maximum total amount of tin mined is reached at 20 years