Y1P14 XMQs and MS

(Total: 199 marks)

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 1. P1_Sample    Q6 .  7 marks - Y1P14 Exponentials and logarithms
 2. P1_Sample    Q12.  9 marks - Y1P14 Exponentials and logarithms
 3. P1_Specimen  Q5 . 10 marks - Y1P7 Algebraic methods
 4. P2_Specimen  Q3 .  4 marks - Y1P14 Exponentials and logarithms
 5. P2_Specimen  Q7 . 12 marks - Y1P14 Exponentials and logarithms
 6. P1_2018      Q12. 10 marks - Y1P14 Exponentials and logarithms
 7. P1_2019      Q7 .  7 marks - Y1P14 Exponentials and logarithms
 8. P1_2019      Q9 .  5 marks - Y1P14 Exponentials and logarithms
 9. P2_2019      Q1 .  3 marks - Y1P1 Algebraic expressions
10. P2_2019      Q9 .  9 marks - Y1P14 Exponentials and logarithms
11. P1_2020      Q2 .  3 marks - Y1P14 Exponentials and logarithms
12. P1_2020      Q8 .  2 marks - Y1P14 Exponentials and logarithms
13. P2_2020      Q3 .  5 marks - Y1P14 Exponentials and logarithms
14. P2_2020      Q5 .  4 marks - Y1P14 Exponentials and logarithms
15. P2_2020      Q9 .  6 marks - Y1P14 Exponentials and logarithms
16. P1_2021      Q8 .  9 marks - Y1P14 Exponentials and logarithms
17. P2_2021      Q3 .  3 marks - Y1P14 Exponentials and logarithms
18. P2_2021      Q10.  6 marks - Y1P14 Exponentials and logarithms
19. P1_2022      Q10.  8 marks - Y1P14 Exponentials and logarithms
20. P2_2022      Q2 .  4 marks - Y1P14 Exponentials and logarithms
21. P1(AS)_2018  Q5 .  5 marks - Y1P14 Exponentials and logarithms
22. P1(AS)_2018  Q13.  8 marks - Y1P14 Exponentials and logarithms
23. P1(AS)_2019  Q14.  9 marks - Y1P14 Exponentials and logarithms
24. P1(AS)_2020  Q8 .  9 marks - Y1P14 Exponentials and logarithms
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25. P1(AS)_2020 Q12. 7 marks - Y1P14 Exponentials and logarithms

26. P1(AS)_2021 Q11. 6 marks - Y1P14 Exponentials and logarithms

27. P1(AS)_2021 Q13. 7 marks - Y1P14 Exponentials and logarithms

28. P1(AS)_2022 Q5 . 10 marks - Y1P14 Exponentials and logarithms

29. P1(AS)_2022 Q8 . 6 marks - Y1P14 Exponentials and logarithms

30. P1(AS)_2022 Q9 . 6 marks - Y1P14 Exponentials and logarithms

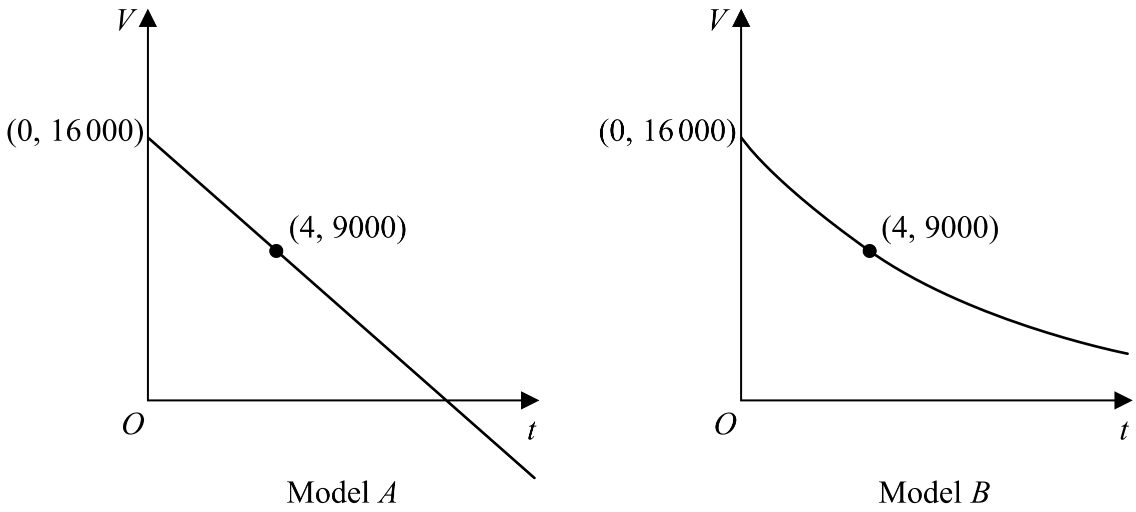
6. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

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Question	Scheme	Marks	AOs
6 (a)(i)	10750 barrels	B1	3.4
(ii)	Gives a valid limitation, for example • The model shows that the daily volume of oil extracted would become negative as t increases, which is impossible • States when $t=10, V=-1500$ which is impossible • States that the model will only work for $0 \leqslant t \leqslant \frac{64}{7}$	B1	3.5b
		(2)	
(b)(i)	Suggests a suitable exponential model, for example $V = Ae^{kt}$	M1	3.3
	Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$	dM1	3.1b
	$\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt -0.144	M1	1.1b
	$V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0.144t}$	A1	1.1b
(ii)	Uses their exponential model with $t = 3 \Rightarrow V =$ awrt 10 400 barrels	B1ft	3.4
		(5)	
		(7 marks)	

Notes:

(a)(i)

B1: 10750 barrels

(a)(ii)

B1: See scheme

(b)(i)

M1: Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^t$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for b.

dM1: Uses both $(0,16000)$ and $(4,9000)$ in their model.

With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{4k}$

With $V = Ar^t$ candidates need to proceed to $9000 = 16000r^4$

With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000-b)e^{4k} + b$ where b is given as a positive constant and $A + b = 16000$.

M1: Uses a correct method to find all constants in the model.

A1: Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0,16000)$ and $(4,9000)$. Possible equations for the model could be for example

$V = 16000e^{-0.144t}$ $\quad V = 16000\times(0.866)^t$ $\quad V = 15800e^{-0.146t} + 200$

(b)(ii)

B1ft: Follow through on their exponential model

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

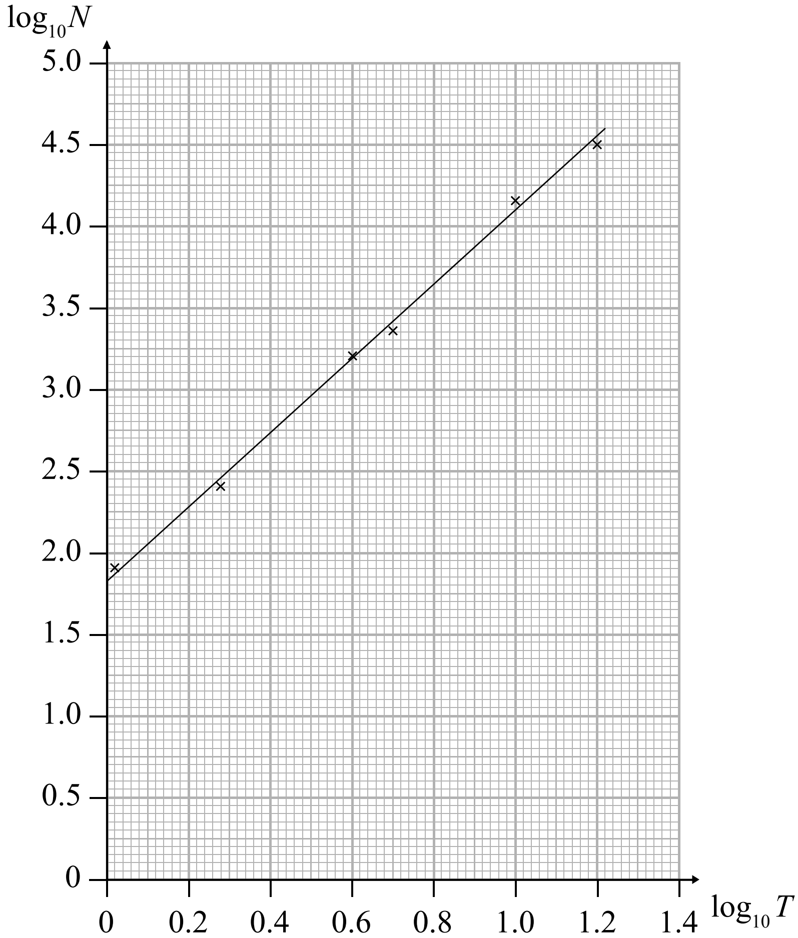

Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

Question	Scheme	Marks	AOs
12 (a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that 'a' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
			(9 marks)

Question 12 continued

Notes:

(a)

M1: Takes logs of both sides and shows the addition law

M1: Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$

(b)

M1: Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$

M1: Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$

M1: Uses $T = 3 \Rightarrow N = aT^b$ with their a and b. This is implied by an attempt at $63 \times 3^{2.3}$

A1: Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work.

There is an alternative to this using a graphical approach.

M1: Finds the value of $\log_{10} T$ from T=3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$

M1: Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48"

Accept $\log_{10} N \approx 2.9$

M1: Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$

A1: Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work

(c)

M1 For using N = 1000000 and stating that $\log_{10} N = 6$

A1: Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate"

There is an alternative approach that uses the formula.

M1: Use N = 1000000 in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\dfrac{1000000}{63}\right)}{2.3} \approx 1.83$.

A1: The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds

(d)

B1: Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at T=1

5. $$f(x) = x^3 + ax^2 - ax + 48, \quad \text{where } a \text{ is a constant}$$

Given that $f(-6) = 0$

(a) (i) show that $a = 4$

(ii) express $f(x)$ as a product of two algebraic factors.

(4)

Given that $2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$

(b) show that $x^3 + 4x^2 - 4x + 48 = 0$

(4)

(c) hence explain why

$$2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$$

has no real roots.

(2)

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Question	Scheme	Marks	AOs
5 (a)(i)	$\mathrm{f}(x) = x^3 + ax^2 - ax + 48,\ x \in \mathbb{R}$		
	$\mathrm{f}(-6) = (-6)^3 + a(-6)^2 - a(-6) + 48$	M1	1.1b
	$= -216 + 36a + 6a + 48 = 0 \Rightarrow 42a = 168 \Rightarrow a = 4$ *	A1*	1.1b
(a)(ii)	Hence, $\mathrm{f}(x) = (x+6)(x^2 - 2x + 8)$	M1	2.2a
		A1	1.1b
		(4)	
(b)	$2\log_2(x+2) + \log_2 x - \log_2(x-6) = 3$		
	E.g. • $\log_2(x+2)^2 + \log_2 x - \log_2(x-6) = 3$ • $2\log_2(x+2) + \log_2\left(\frac{x}{x-6}\right) = 3$	M1	1.2
	$\log_2\left(\frac{x(x+2)^2}{(x-6)}\right) = 3$ $\left[\text{or } \log_2\left(x(x+2)^2\right) = \log_2\left(8(x-6)\right)\right]$	M1	1.1b
	$\left(\frac{x(x+2)^2}{(x-6)}\right) = 2^3$ $\left\{\text{i.e. } \log_2 a = 3 \Rightarrow a = 2^3 \text{ or } 8\right\}$	B1	1.1b
	$x(x+2)^2 = 8(x-6) \Rightarrow x(x^2+4x+4) = 8x - 48$		
	$\Rightarrow x^3 + 4x^3 + 4x = 8x - 48 \Rightarrow x^3 + 4x^3 - 4x + 48 = 0$ *	A1 *	2.1
		(4)	
(c)	$2\log_2(x+2) + \log_2 x - \log_2(x-6) = 3 \Rightarrow x^3 + 4x^3 - 4x + 48 = 0$		
	$\Rightarrow (x+6)(x^2 - 2x + 8) = 0$		
	Reason 1: E.g. • $\log_2 x$ is not defined when $x = -6$ • $\log_2(x-6)$ is not defined when $x = -6$ • $x = -6$, but $\log_2 x$ is only defined for $x > 0$ Reason 2: • $b^2 - 4ac = -28 < 0$, so $(x^2 - 2x + 8) = 0$ has no (real) roots		
	At least one of Reason 1 or Reason 2	B1	2.4
	Both Reason 1 and Reason 2	B1	2.1
		(2)	
			(10 marks)

Question 5 Notes:	
(a)(i)	
M1:	Applies $f(-6)$
A1*:	Applies $f(-6) = 0$ to show that $a = 4$
(a)(ii)	
M1:	Deduces $(x + 6)$ is a factor of $f(x)$ and attempts to find a quadratic factor of $f(x)$ by either equating coefficients or by algebraic long division
A1:	$(x+6)(x^2 - 2x + 8)$
(b)	
M1:	Evidence of applying a correct law of logarithms
M1:	Uses correct laws of logarithms to give either • an expression of the form $\log_2(h(x)) = k$, where *k* is a constant • an expression of the form $\log_2(g(x)) = \log_2(h(x))$
B1:	Evidence in their working of $\log_2 a = 3 \Rightarrow a = 2^3$ or 8
A1*:	Correctly proves $x^3 + 4x^3 - 4x + 48 = 0$ with no errors seen
(c)	
B1:	See scheme
B1:	See scheme

3. A cup of hot tea was placed on a table. At time t minutes after the cup was placed on the table, the temperature of the tea in the cup, θ °C, is modelled by the equation

$$\theta = 25 + Ae^{-0.03t}$$

where A is a constant.

The temperature of the tea was 75 °C when the cup was placed on the table.

(a) Find a complete equation for the model.

(1)

(b) Use the model to find the time taken for the tea to cool from 75 °C to 60 °C, giving your answer in minutes to one decimal place.

(2)

Two hours after the cup was placed on the table, the temperature of the tea was measured as 20.3 °C.

Using this information,

(c) evaluate the model, explaining your reasoning.

(1)

Question	Scheme	Marks	AOs
3 (a)	$\{t=0,\ \theta=75 \Rightarrow\ 75=25+A \Rightarrow A=50\}\ \Rightarrow\ \theta=25+50\mathrm{e}^{-0.03t}$	B1	3.3
		(1)	
(b)	$\{\theta=60 \Rightarrow\}\ \Rightarrow 60=25+\text{"50"}\mathrm{e}^{-0.03t}\ \Rightarrow\ \mathrm{e}^{-0.03t}=\dfrac{60-25}{\text{"50"}}$	M1	3.4
	$t=\dfrac{\ln(0.7)}{-0.03}=11.8891648\ =\ 11.9$ minutes (1 dp)	A1	1.1b
		(2)	
(c)	A valid evaluation of the model, which relates to the large values of *t*. E.g. • As $20.3<25$ then the model is not true for large values of t • $\mathrm{e}^{-0.03t}=\dfrac{20.3-25}{\text{"50"}}=-0.094$ does not have any solutions and so the model predicts that tea in the room will never be $20.3\ ^\circ\mathrm{C}$. So the model does not work for large values of t • $t=120 \Rightarrow \theta=25+50e^{-0.03(20)}=25.12...$ which is not approximately equal to 20.3, so the model is not true for large values of t	B1	3.5a
		(1)	
		(4 marks)	

Question 3 Notes:

(a)	
B1:	Applies $t=0,\ \theta=75$ to give the complete model $\theta=25+50\mathrm{e}^{-0.03t}$
(b)	
M1:	Applies $\theta=60$ and their value of *A* to the model and rearranges to make $\mathrm{e}^{-0.03t}$ the subject.
	Note: Later working can imply this mark.
A1	Obtains 11.9 (minutes) with no errors in manipulation seen.
(c)	
B1	See scheme

7. A bacterial culture has area p mm^2 at time t hours after the culture was placed onto a circular dish.

A scientist states that at time t hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

(a) Show that the scientist's model for p leads to the equation

$$p = ae^{kt}$$

where a and k are constants.

(4)

The scientist measures the values for p at regular intervals during the first 24 hours after the culture was placed onto the dish.

She plots a graph of $\ln p$ against t and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95

(b) Estimate, to 2 significant figures, the value of a and the value of k.

(3)

(c) Hence show that the model for p can be rewritten as

$$p = ab^t$$

stating, to 3 significant figures, the value of the constant b.

(2)

With reference to this model,

(d) (i) interpret the value of the constant a,

(ii) interpret the value of the constant b.

(2)

(e) State a long term limitation of the model for p.

(1)

Question	Scheme	Marks	AOs
7(a)	$\frac{\mathrm{d}p}{\mathrm{d}t} \propto p \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}t} = kp$	B1	3.3
	$\int \frac{1}{p}\,\mathrm{d}p = \int k\,\mathrm{d}t$	M1	1.1b
	$\ln p = kt\ \{+ c\}$	A1	1.1b
	$\ln p = kt + c \Rightarrow p = \mathrm{e}^{kt+c} = \mathrm{e}^{kt}\mathrm{e}^{c} \Rightarrow p = a\mathrm{e}^{kt}$ *	A1 *	2.1
		(4)	
(b)	$p = a\mathrm{e}^{kt} \Rightarrow \ln p = \ln a + kt$ and evidence of understanding that either • gradient $= k$ or $"M" = k$ • vertical intercept $= \ln a$ or $"C" = \ln a$	M1	2.1
	gradient $= k = 0.14$	A1	1.1b
	vertical intercept $= \ln a = 3.95 \Rightarrow a = \mathrm{e}^{3.95} = 51.935 = 52$ (2 sf)	A1	1.1b
		(3)	
(c)	e.g. • $p = a\mathrm{e}^{kt} \Rightarrow p = a(\mathrm{e}^{k})^{t} = ab^{t}$, • $p = 52\mathrm{e}^{0.14t} \Rightarrow p = 52(\mathrm{e}^{0.14})^{t}$	B1	2.2a
	$b = 1.15$ which can be implied by $p = 52(1.15)^{t}$	B1	1.1b
		(2)	
(d)(i)	Initial area (i.e. "52" mm^2) of bacterial culture that was first placed onto the circular dish.	B1	3.4
(d)(ii)	E.g. • Rate of increase per hour of the area of bacterial culture • The area of bacterial culture increases by "15%" each hour	B1	3.4
		(2)	
(e)	The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area.	B1	3.5b
		(1)	
			(12 marks)

Question 7 Notes:	
(a)	
B1:	Translates the scientist's statement regarding proportionality into a differential equation, which involves a constant of proportionality. e.g. $\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp$
M1:	Correct method of separating the variables p and t in their differential equation
A1:	$\ln p = kt$, with or without a constant of integration
A1*:	Correct proof with no errors seen in working.
(b)	
M1:	See scheme
A1:	Correctly finds $k = 0.14$
A1:	Correctly finds $a = 52$
(c)	
B1:	Uses algebra to correctly deduce either • $p = ab^t$ from $p = ae^{kt}$ • $p = "52"(e^{"0.14"})^t$ from $p = "52"e^{"0.14"t}$
B1:	See scheme
(d)(i)	
B1:	See scheme
(d)(ii)	
B1:	See scheme
(e)	
B1:	Gives a correct long-term limitation of the model for p. (See scheme).

12. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

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Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $$p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $$A = \frac{32000}{'1.0658'^4} \text{ or } A = \frac{50000}{'1.0658'^{11}}$$	M1	1.1b
	$A = 24795 \to 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p /1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8 \text{ or } 21.9$	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p = \text{awrt } 1.0658$

Both marks can be awarded from incorrect but consistent interpretations of t. Eg.

$32000 = Ap^5, 50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ' =24 800' or ' ≈ 24800 '. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

A1: 24800×1.0658^4, achieves a value between 31095 and 32005 followed by $\approx 32\,000$ hence A must be $\approx 24\,800$

(b)

B1: States that A is the value of the car on 1st January 2001.

The statement must reference **the car**, its **cost/value**, and **"0" time**

Allow 'it is the initial value of the car" "it is the cost of the car at t = 0" "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.

The statement must reference **a yearly rate** and **an increase in value or multiplier**.

They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."

Allow "p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow ' the value of the car multiplies by p each year'

Do not allow "by how much the value of the car rises each year " or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = '24800' \times '1.0658'^t$ and proceeds to their $'1.0658'^t = k$

Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model **and** (ii) translating the situation into a correct method to find 't'

A1: $(t) = \text{awrt } 21.8 \text{ or } 21.9$ or $\log_{1.0658}\left(\frac{100000}{24800}\right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks

Allow different methods in part (c).

Eg Via GP a formula

M1: $24800 \times '1.0658'^{n-1} = 100000 \Rightarrow '1.0658'^{n-1} = K$

dM1: Uses a correct method to find n.

A2: 2022

Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\,000$ or $t = n+1, V > 100\,000$

(So for the correct A and p this would be scored for $t = 21, V \approx £95000$ or $t = 21, V \approx £101\,000$

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest £1000 is sufficient accuracy)

A1: As before

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t=0, V=20\,000 \Rightarrow A=20\,000$	M1	1.1b
	Eg. Substitutes $t=1, V=16\,000 \Rightarrow 16\,000=20\,000e^{-1k} \Rightarrow k=..$	dM1	3.1b
	$V=20\,000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t=10$ in their $V=20\,000e^{-0.223t} \Rightarrow V=(£\ 2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V=18\,000e^{-0.223t}+2000$	B1ft	3.3
		(1)	
			(7 marks)

(a) **Option 1**

M1: For $V = Ae^{\pm kt}$ Do not allow if k is fixed, eg $k=-0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Substitutes $t=0 \Rightarrow A=20\,000$ into their exponential model

Candidates may start by simply writing $V=20\,000e^{kt}$ which would be M1 M1

dM1: Substitutes $t=1 \Rightarrow 16\,000=20\,000e^{-1k} \Rightarrow k=..$via the correct use of logs.

It is dependent upon both previous M's.

A1: $V=20\,000e^{-0.223t}$ (with accuracy to at least 3sf) or $V=20\,000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of V when t = 10.

Alternatively substitutes $V=2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares V = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of t with 10 and making a suitable comment as to the reliability of their model with a reason.

$V=20\,000e^{-0.223t} \Rightarrow 2000=20\,000e^{-0.223t} \Rightarrow t=10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. Eg states that the value of their '-0.223' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to k then refer to the model and apply the same principles.

Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

(a) Option 2

M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$

Condone different variables $V \leftrightarrow y \quad t \leftrightarrow x$ for this mark, but for A1 *V* and *t* must be used.

M1: Uses $t = 0 \Rightarrow A = 20\,000$ in their model. Alternatively uses $(0, 20000)$ and $(1, 16000)$ to give $r = \frac{4}{5}$ oe

You may award if one of the number pair $(0, 20000)$ or $(1, 16000)$ works in an allowable model

dM1: $t = 1 \Rightarrow 16\,000 = 20\,000 r^1 \Rightarrow r = ..$ Dependent upon both previous M's

In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20\,000$

You may award if both number pairs $(0, 20000)$ or $(1, 16000)$ work in an allowable model

A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct

(b)

M1: Uses a model of the form $V = Ar^t$ oe to find the value of *V* when *t* = 10. Eg. 20000×0.8^{10}

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.

Compares (£) 2147 with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

(c)

B1ft: States a value of *r* in the range $(0.8, 1)$ or states would increase the value of "0.8"

They do not need to state that "0.8" must lie in the range $(0.8, 1)$

Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

....

(a) Option 3

M1: They may suggest an exponential model with a lower bound. For example, for $V = Ae^{\pm kt} + 2000$ The bound must be stated but do not allow k to be fixed . Allow as long as the bound < 10 000

M1: $t = 0, V = 20\,000 \Rightarrow A = 18\,000$

dM1: $t = 1, V = 16\,000 \Rightarrow 16\,000 = 2\,000 + 18\,000 e^k \Rightarrow k = ..$ Dependent upon both previous M's

A1: $V = 18\,000 \times e^{-0.251t} + 2\,000$

(b)

M1: Uses their model to find the value of *V* when *t* = 10.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: For $V = 18\,000 \times e^{-0.251 \times 10} + 2\,000 = £3462.83$ Deduction: Unreliable model as £3462.83 is not close to £2 000 This can only be scored from an acceptable model with correct constants

(c)

B1: States make the value of *k* or the -0.251 greater (or less negative) so that it lies in the range $(-0.251, 0)$

Condone 'make the value of *k* or the -0.251 greater (or less negative)'

……...

It is entirely possible that they start part (a) from a differential equation.

M1: $\frac{dV}{dt} = kV \Rightarrow \int \frac{dV}{V} = \int k\,dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\frac{5}{4}\right)t + \ln 20000$

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 5 8 3 5 3 A 0 2 6 4 4

Question	Scheme	Marks	AOs
9 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \to ...... \to ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \to a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

(**a)**

B1: States or uses $\log a - \log b = \log \frac{a}{b}$. This may be awarded anywhere in the question and may be implied by a **starting line** of $\frac{a}{b} = a - b$ oe. Alternatively takes $\log b$ to the rhs and uses the addition law $\log(a-b) + \log b = \log(a-b)b$. Watch out for $\log a - \log b = \frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$ which could score 010

M1: Attempts to make '*a*' the subject. Awarded for proceeding from $\frac{a}{b} = a - b$ to a point where the two terms in *a* are on the same side of the equation and the term in *b* is on the other.

A1*: CSO. Shows clear reasoning and correct mathematics leading to $a = \frac{b^2}{b-1}$. Bracketing must be correct.

Allow a candidate to proceed from $ab - a = b^2$ to $a = \frac{b^2}{b-1}$ without the intermediate line.

(b)

B1: For deducing $b \neq 1$ as $a \to \infty$ oe such as "you cannot divide by 0" or correctly deducing that $b > 1$. They may state that *b* cannot be less than 1.

B1: For $b > 1$ and explaining that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$ (as b^2 is positive)

As a minimum accept that $b > 1$ as *a* cannot be negative.

Note that $a > b > 1$ is a correct statement but not sufficient on its own without an explanation.

....

Alt (a)

Note that it is possible to attempt part (a) by substituting $a = \frac{b^2}{b-1}$ into both sides of the given identity.

$$\log a - \log b = \log(a-b) \Rightarrow \log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b^2}{b-1} - b\right)$$

B1: Score for $\log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b}{b-1}\right)$

M1: Attempts to write $\frac{b^2}{b-1} - b$ as a single fraction $\frac{b^2}{b-1} - b = \frac{b^2 - b(b-1)}{b-1}$

Allow as two separate fractions with the same common denominator

A1*: Achieves lhs and rhs as $\log\left(\frac{b}{b-1}\right)$ **and** makes a comment such as "hence true"

Answer ALL questions. Write your answers in the spaces provided.

1. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

Question	Scheme	Marks	AOs
1	$2^x \times 4^y = \frac{1}{2\sqrt{2}} \quad \left\{= \frac{\sqrt{2}}{4}\right\}$		
Special Case	If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of • $2^x \times 4^y \to 2^{x+2y}$ • $2^x \times 4^y \to 4^{\frac{1}{2}x+y}$ • $\frac{1}{2^x 2\sqrt{2}} \to 2^{-x-\frac{3}{2}}$ • $\log 2^x + \log 4^y \to x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$ • $\ln 2^x + \ln 4^y \to x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$ • $y = \log\left(\frac{1}{2^x 2\sqrt{2}}\right)$ o.e. {base of 4 omitted}		
Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right)$ $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = ...$	M1	2.1
	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 3	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow y = ...$	M1	2.1
	$y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 4	$\log_2(2^x \times 4^y) = \log_2\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log_2 2^x + \log_2 4^y = \log_2\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
Way 5	$4^{\frac{1}{2}x} \times 4^{y} = 4^{-\frac{3}{4}}$	B1	1.1b
	$4^{\frac{1}{2}x+y} = 4^{-\frac{3}{4}} \Rightarrow \frac{1}{2}x + y = -\frac{3}{4} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	

Notes for Question 1

	Way 1
B1:	Writes a correct equation in powers of 2 only
M1:	Complete process of writing a correct equation in powers of 2 only and using correct index laws to obtain *y* written as a function of *x*.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
	Way 2, Way 3 and Way 4
B1:	Writes a correct equation involving logarithms
M1:	Complete process of writing a correct equation involving logarithms and using correct log laws to obtain *y* written as a function of *x*.
A1:	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4}$ or $y = \frac{-\ln(2\sqrt{2}) - x\ln 2}{\ln 4}$ or $y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4}$ or $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ o.e.
	Way 5
B1:	Writes a correct equation in powers of 4 only
M1:	Complete process of writing a correct equation in powers of 4 only and using correct index laws to obtain *y* written as a function of *x*.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
Note:	Allow equivalent results for A1 where *y* is written as a function of *x*
Note:	You can ignore subsequent working following on from a correct answer.
Note:	Allow B1 for $2^x \times 4^y = \frac{1}{2\sqrt{2}} \Rightarrow 4^y = \frac{1}{2^x 2\sqrt{2}} \Rightarrow \log_4(4^y) = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ followed by M1 A1 for $y = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{2^{-x}}{2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{\sqrt{2}}{4(2^x)}\right)$ or $y = -\log_4\left(2^{x+\frac{3}{2}}\right)$ or $y = -\log_4(\sqrt{2}(2^{x+1}))$

9. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V\,\text{km h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

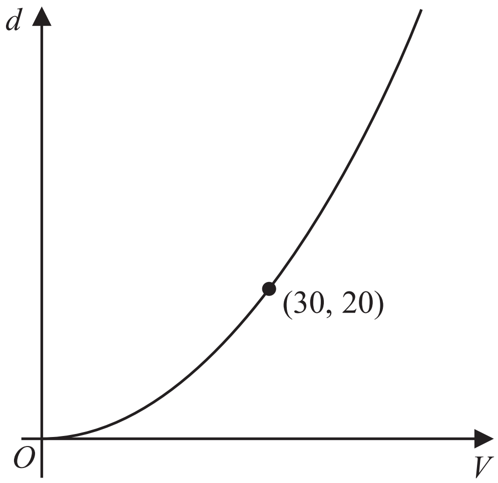


Figure 5

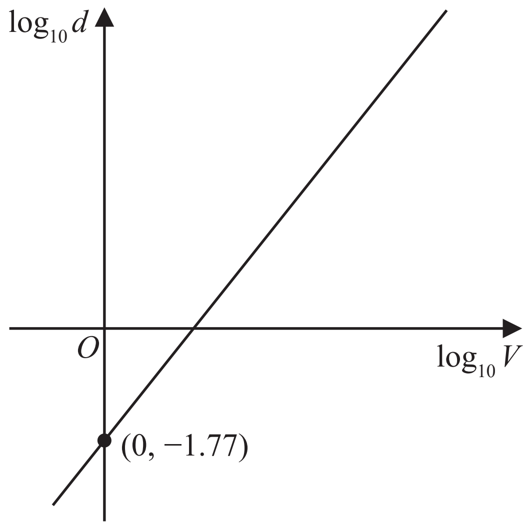


Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at $60\,\text{km h}^{-1}$ in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

Question	Scheme	Marks	AOs
9 (a) Way 1	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
9 (a) Way 2	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}\ \log_{10} d = \log_{10}(kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) Way 3	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ **or** $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$	A1	2.4
	$\{k =\}\ 10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}\ \ 20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = ...$	M1	1.1b
	$\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = ...$		
	$\{n = \text{awrt } 2.08 \Rightarrow\}\ d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	**(3)**	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251... \Rightarrow$ Sean stops in time • $100 - 13.333... = 86.666...$ & $d = 84.918 \Rightarrow$ Sean stops in time	M1	3.1b
		A1ft	3.2a
		(3)	
			(9 marks)

ADVICE: Ignore labelling (a), (b), (c) when marking this question

Note: Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part

Notes for Question 9	
Note:	In their solution to (a) and/or (b) condone writing $\log$ in place of $\log_{10}$
(a)	**Way 1**
M1:	See scheme
A1:	See scheme
B1*:	See scheme
(a)	**Way 2**
M1:	See scheme
A1:	Starts from $d = kV^n$ (which they do not have to state) and progresses to $\log_{10} d = \log_{10} k + n\log_{10} V$ with an intermediate step in their working.
B1*:	See scheme
(a)	**Way 3**
M1:	Starts their argument from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$
A1:	Mathematical explanation is seen by showing any of either • $\log_{10} d = m\log_{10} V + c \rightarrow d = 10^c V^m$ or $d = kV^n$ • $\log_{10} d = m\log_{10} V - 1.77 \rightarrow d = 10^{-1.77} V^m$ or $d = kV^n$ with no errors seen in their working
B1*:	See scheme
Note:	Allow B1 for $\log_{10} 0.017 = -1.77$ or $\log 0.017 = -1.77$
(b)	
M1:	Applies $V = 30$ and $d = 20$ to their model **(correct way round)**
M1:	Applies $(V, d) = (30, 20)$ or $(20, 30)$ and applies logarithms correctly leading to $n = ...$
A1:	$d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ or $\log_{10} d = \log_{10}(0.017) + 2.08\log_{10} V$
Note:	Allow $k =$ awrt 0.017 and/or $n =$ awrt 2.08 in their final model equation
Note:	M0 M1 A0 is a possible score for (b)
(c)	
M1:	Applies $V = 60$ to their exponential model or their logarithmic model
M1:	Uses their model in a correct problem-solving process of either • adding a "thinking distance" to their value of their *d* to find an overall stopping distance • applying 100 – "thinking distance" and finds their value of *d*
Note:	$\frac{1}{75}$ or 48 are examples of acceptable thinking distances
A1ft:	**Either** adds 13.3… to their *d* to find a total stopping distance and gives a correct ft conclusion **or** finds their *d* and a comparative 86.666…(m) or awrt 87 (m) and gives a correct ft conclusion
Note:	The thinking distance must be dimensionally correct for the M1 mark. i.e. $0.8\times$ their velocity
Note:	A thinking distance of awrt 13 and a value of *d* in the range [81.5, 88.5] are required for A1ft
Note:	Allow "Sean stops in time" or "Yes he stops in time" or "he misses the puddle" as relevant conclusions.
Note:	A mark of M0 M1 A0 is possible in (c)

2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

Question	Scheme	Marks	AOs
2	$4^{3p-1}=5^{210}\Rightarrow(3p-1)\log 4=210\log 5$	M1	1.1b
	$\Rightarrow 3p=\dfrac{210\log 5}{\log 4}+1\Rightarrow p=...$	dM1	2.1
	$p=$ awrt 81.6	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Takes logs of both sides and uses the power law on **each** side.
Condone a missing bracket on lhs and slips.
Award for any base including ln but the logs must be the same base.

dM1: A full method leading to a value for *p*.
It is dependent upon the previous M mark and there must be an attempt to change the subject of the equation in the correct order.

Look for $(3p-1)\log 4=210\log 5\Rightarrow 3p=\dfrac{210\log 5}{\log 4}\pm 1\Rightarrow p=...$ condoning slips.

You may see numerical versions E.g. $(3p-1)\times 0.60=210\times 0.7\Rightarrow 1.8p-0.6=147\Rightarrow p=82$

Use of incorrect log laws would be dM0. E.g $(3p-1)\log 4=210\log 5\Rightarrow 3p=210\log\dfrac{5}{4}\pm 1$

A1: awrt 81.6 following a correct method. Bracketing errors can be recovered for full marks
A correct answer with no working scores 0 marks. The demand in the question is clear.

....

There are alternatives:

E.g. A starting point could be $4^{3p-1}=5^{210}\Rightarrow\dfrac{4^{3p}}{4}=5^{210}$

but the first M mark is still for using the power law correctly on each side

In such a method the dM1 mark is for using **all** log rules correctly and proceeding to a value for *p*.

....

Using base 4 or 5

E.g. $4^{3p-1}=5^{210}\Rightarrow(3p-1)=\log_4 5^{210}$

The M mark is not scored until $(3p-1)=210\log_4 5$

....

8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation*.)

(2)

Question	Scheme	Marks	AOs
8	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = Ae^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
			(2 marks)
Notes:			

M1: Any equation of the correct form, involving n and an exponential in t.

So allow for example $n = e^{\pm t}$, $n = Ae^{\pm t}$, $n = Ae^{\pm kt}$ condoning $n = A + Be^{\pm t}$

Condone an intermediate form where n has not been made the subject.

E.g Allow $\ln n = kt + c$ but also condone $\ln n = kt$

A1: E.g. $n = Ae^{kt}$, $n = e^{kt+c}$, $n = e^{kt}e^{c}$ There is no requirement to state that A and k are positive constants

Note that the two constants need to be different.

Mark the final answer so $n = e^{kt+c}$ followed by $n = e^{kt} + e^{c}$ o.e. $n = e^{kt} + A$ such as is M1 A0

....

You may see solutions that don't include "e".

These are fine so you can include versions of $n = Ak^{t}$ using the same marking criteria as above

FYI $\frac{dn}{dt} = Ak^{t} \times \ln k = \ln k \times n$ so $\frac{dn}{dt} \propto n$

....

3. (a) Given that

$$2\log(4 - x) = \log(x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4 - x) = \log(x + 8)$$

giving a reason for your answer.

(2)

Question	Scheme	Marks	AOs
3(a)	$2\log(4-x)=\log(4-x)^2$	B1	1.2
	$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)$ $(4-x)^2=(x+8)$ **or** $2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0$ $\dfrac{(4-x)^2}{(x+8)}=1$	M1	1.1b
	$16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$ *	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-9x+8=0\Rightarrow(4-x)^2-x-8=0$	B1	1.2
	$\Rightarrow(4-x)^2=x+8$ $\Rightarrow\log(4-x)^2=\log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x)=\log(x+8)$ * Hence proved.	A1	2.1
(b)	(i) (x =) 1, 8	B1	1.1b
	(ii) 8 is **not** a solution as log(4 – 8) cannot be found	B1	2.3
		(2)	
			(5 marks)

Notes:

(a)

B1: States or uses $2\log(4-x)=\log(4-x)^2$

M1: Correct attempt at eliminating the logs to form a quadratic equation in x.

Note that this may be implied by e.g. $\log\dfrac{(4-x)^2}{(x+8)}=0\Rightarrow(4-x)^2=x+8$

A1*: Proceeds to the given answer with at least one line where the $(4-x)^2$ has been multiplied out.
There must be no errors or omissions but condone invisible brackets around the arguments of the logs e.g. allow $\log 16-8x+x^2$ for $\log(16-8x+x^2)$ and $\log x+8$ for $\log(x+8)$

Note we will allow a start of $(4-x)^2=x+8$ with no previous work for full marks.

<u>Some examples of how to mark (a) in particular cases:</u>

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)\Rightarrow\frac{\log(4-x)^2}{\log(x+8)}=1$$

$$\Rightarrow\frac{(4-x)^2}{(x+8)}=1\Rightarrow16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$$

Scores B1M1A1

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow(4-x)^2-x-8=0$$

$$\Rightarrow16-8x+x^2-x-8\Rightarrow x^2-9x+8=0$$

Scores B1M1A1

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow\frac{\log(4-x)^2}{\log(x+8)}=0$$

$$\Rightarrow\frac{(4-x)^2}{(x+8)}=1\Rightarrow16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$$

Scores B1M0A0

(a) Alternative:

B1: Writes $x^2 - 9x + 8 = 0$ as $(4 - x)^2 - x - 8 = 0$ or equivalent

M1: Proceeds correctly to reach $\log(4-x)^2=\log(x+8)$

A1: Obtains $2\log(4-x)=\log(x+8)$ and makes a (minimal) conclusion e.g. hence proved, QED, #, square etc.

(b)

B1: Writes down (x =) 1, 8

B1: Chooses 8 (no follow through here) and gives a reason why it should be rejected by referring to logs and which log it is.

They must refer to the 8 as the required value but allow e.g. $x \neq 8$ and there must be a reference to log(4 – x) or log of lhs or log(–4) or the 4 – 8. Some acceptable reasons are: log(–4) can't be found/worked out/is undefined, log(–4) gives math error, log(–4) = n/a, lhs is log(negative) so reject, you can't do the log of a negative number which would happen with 4 – 8

Do **not** allow "you can't have a negative log" unless this is clarified further and do **not** allow "you get a math error" in isolation

There must be no contradictory statements.

Note that this is an independent mark but must have $x = 8$ (i.e. may have solved to get $x = -1, 8$ for first B mark)

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

Question	Scheme	Marks	AOs
5	$15-2^{x+1}=3\times 2^{x}$	B1	1.1b
	$\Rightarrow 15-2\times 2^{x}=3\times 2^{x}\Rightarrow 2^{x}=3$ or e.g. $\Rightarrow \dfrac{15}{2^{x}}-2=3\Rightarrow 2^{x}=3$	M1	1.1b
	$2^{x}=3\Rightarrow x=\ldots$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y=3\times 2^{x}\Rightarrow 2^{x}=\dfrac{y}{3}\Rightarrow y=15-2\times\dfrac{y}{3}$	B1	1.1b
	$3y+2y=45\Rightarrow y=9\Rightarrow 3\times 2^{x}=9\Rightarrow 2^{x}=3$	M1	1.1b
	$2^{x}=3\Rightarrow x=\ldots$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
			(4 marks)

Notes:

B1: Combines the equations to reach $15-2^{x+1}=3\times 2^x$ or equivalent e.g. $15-2^{x+1}-3\times 2^x=0$

M1: Uses $2^{x+1}=2\times 2^x$ oe e.g. $\dfrac{2^{x+1}}{2^x}=2$ to obtain an equation in 2^x and attempts to make 2^x the subject.

See scheme but e.g. $y=2^x \Rightarrow 3\times 2^x=15-2^{x+1} \Rightarrow 3y=15-2y \Rightarrow y=...$ is also possible

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x=k$ where $k>1$

e.g. $2^x=k \Rightarrow x=\log_2 k$

or $2^x=k \Rightarrow \log 2^x=\log k \Rightarrow x\log 2=\log k \Rightarrow x=...$

or $2^x=k \Rightarrow \ln 2^x=\ln k \Rightarrow x\ln 2=\ln k \Rightarrow x=...$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $2^x = 3$ so $x = 1.584..$ but you may need to check

A1cso: $x=\log_2 3$ or $\dfrac{\log 3}{\log 2}$ or $\dfrac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

Alternative

B1: Correct equation in y

M1: Solves their equation in y and attempts to make 2^x the subject.

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x=k$ where $k>1$

e.g. $2^x=k \Rightarrow x=\log_2 k$

or $2^x=k \Rightarrow \log 2^x=\log k \Rightarrow x\log 2=\log k \Rightarrow x=...$

or $2^x=k \Rightarrow \ln 2^x=\ln k \Rightarrow x\ln 2=\ln k \Rightarrow x=...$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $2^x = 3$ so $x = 1.584..$ but you may need to check

A1cso: $x=\log_2 3$ or $\dfrac{\log 3}{\log 2}$ or $\dfrac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ°C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

Question	Scheme	Marks	AOs
9(a)	$t=0,\ \theta=18 \Rightarrow 18 = A - B$ **or** $t=10,\ \theta=44 \Rightarrow 44 = A - B\mathrm{e}^{-0.7}$	M1	3.1b
	$t=0,\ \theta=18 \Rightarrow 18 = A - B$ **and** $t=10,\ \theta=44 \Rightarrow 44 = A - B\mathrm{e}^{-0.7}$ **and** $\Rightarrow A = \ldots, B = \ldots$	M1	3.1a
	At least one of: A = 69.6, B = 51.6 but allow awrt 70/awrt 52	A1 **M1 on EPEN**	1.1b
	$\theta = 69.6 - 51.6\mathrm{e}^{-0.07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is "69.6"(°C) (according to the model) (The model has an) upper limit of "69.6"(°C) (The model suggests that) the boiling point is "69.6"(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
			(6 marks)

Notes:

(a)

M1: Makes the first key step in the solution of the problem. Substitutes $t = 0$ and $\theta = 18$ **or** $t = 10$ and $\theta = 44$ into the equation of the model to obtain an equation connecting A and B.
Note that $18 = A - Be^0$ scores M0 unless $18 = A - B$ is seen or implied later.
If they do not obtain an equation in A and B using the first conditions e.g. they have 18 = A – 1 then they can score this mark if they substitute $A = 19$ directly into $44 = A - Be^{-0.7}$ as an equation in A and B is implied.

M1: Substitutes $t = 0$ and $\theta = 18$ **and** $t = 10$ and $\theta = 44$ to obtain 2 equations connecting A and B **and** then proceeds to solves their equations in A and B simultaneously to obtain values for both constants. Do not be too concerned with the processing as long as values for A and B are obtained.

A1(M1 on EPEN): For A = awrt 70 **or** B = awrt 52

A1: For $\theta = 69.6 - 51.6e^{-0.07t}$ **Must be a <u>fully correct equation as shown</u> but allow recovery if seen in (b).**
Note that some candidates evaluate e^0 as 0 and so obtain $A = 18$ and then write $44 = 18 - Be^{-0.7}$ and solve for B. Such attempts can score M1M0A0A0 only.

(b)

B1ft: Identifies A as the boiling point/maximum temperature in the model. Follow through their A.

B1ft: Makes a valid conclusion (valid/not valid, good/not good etc.) that refers to the 78 and includes a reference to a significant/large difference

Alternative provided their $A < 78$

B1ft: $\theta = 69.6 - 51.6e^{-0.07t} = 78 \Rightarrow 51.6e^{-0.07t} = 69.6 - 78 = -8.4$

$\Rightarrow e^{-0.07t} = -\frac{7}{43}$ and $\ln\left(-\frac{7}{43}\right)$ and makes a reference to the fact that the equation cannot be solved or e.g. cannot take log of a negative number. You can condone numerical slips in the calculation.

B1ft: Model is not appropriate as 69.6(°C) is much lower than 78(°C)

Minimum for both marks: The model is not appropriate as "69.6"(°C) is much lower than 78(°C)

Note that these marks are not available if their equation is solvable. Note also that B0B1 is not possible.

8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \qquad t \geqslant 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

Question	Scheme	Marks	AOs
8(a)	$A=1000$	B1	3.4
	$2000=1000e^{5k}$ or $e^{5k}=2$	M1	1.1b
	$e^{5k}=2 \Rightarrow 5k=\ln 2 \Rightarrow k=...$	M1	2.1
	$N=1000e^{\left(\frac{1}{5}\ln 2\right)t}$ or $N=1000e^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{dN}{dt}=1000\times\left(\frac{1}{5}\ln 2\right)e^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{dN}{dt}=1000\times 0.139e^{0.139t}$ $\left(\frac{dN}{dt}\right)_{t=8}=1000\times\left(\frac{1}{5}\ln 2\right)e^{8\times\frac{1}{5}\ln 2}$ or $\left(\frac{dN}{dt}\right)_{t=8}=1000\times 0.139e^{0.139\times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500e^{1.4\times\left(\frac{1}{5}\ln 2\right)T}=1000e^{\left(\frac{1}{5}\ln 2\right)T}$ or $500e^{1.4\times"0.139"t}=1000e^{"0.139"t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2=\ln 2$ or $1.4\times"0.339"T=\ln 2+"0.339"t$	M1	2.1
	$T=12.5$ hours	A1	1.1b
		(3)	
			(9 marks)

Notes

Mark as one complete question. Marks in (a) can be awarded from (b)

(a)

B1: Correct value of A for the model. Award if equation for model is of the form $N=1000e^{...t}$

M1: Uses the model to set up a correct equation in k. Award for substituting $N=2000, t=5$ following through on their value for A.

M1: Uses correct ln work to solve an equation of the form $ae^{5k}=b$ and obtain a value for k

A1: Correct equation of model. Condone an ambiguous $N=1000e^{\frac{1}{5}\ln 2t}$ unless followed by something incorrect. Watch for $N=1000\times 2^{\frac{1}{5}t}$ which is also correct

(b)

M1: Differentiates αe^{kt} to βe^{kt} and substitutes t = 8 (Condone $\alpha=\beta$ so long as you can see an attempt to differentiate)

A1: For awrt 420 (2sf).

(c)

M1: Uses both models to set up an equation in T using their value for k, but also allow in terms of k

M1: Uses correct processing using lns to obtain a linear equation in T (or t)

A1: Awrt 12.5

....

Answers to (b) and (c) appearing without working (i.e. from a calculator).

It is important that candidates show sufficient working to make their methods clear.

(b) If candidate has for example $N=1000e^{0.139t}$, and then writes at t = 8 $\frac{dN}{dt}$ = awrt 420 award both marks. Just the answer from a correct model equation score SC 1,0.

(c) The first M1 should be seen E.g $500e^{1.4\times"0.139"t}=1000e^{"0.139"t}$

If the answer $T=12.5$ appears without any further working score SC M1 M1 A0

....

3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
3	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3\frac{12y+5}{1-3y}=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=...$ or e.g. $\log_3(12y+5)=\log_3\left(3^2(1-3y)\right) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=...$	M1	2.1
	$y=\frac{4}{39}$	A1	1.1b
		(3)	
			(3 marks)

Notes

B1(M1 on EPEN): Applies at least one addition or subtraction law of logs correctly.
Can also be awarded for using $2=\log_3 9$. This may be implied by e.g. $\log_3 ...=2 \Rightarrow ...=9$

M1: A rigorous argument with no incorrect working to remove the log or logs correctly and obtain a correct equation in any form **and** solve for *y*.

A1: Correct **exact** value. Allow equivalent fractions.

Guidance on how to mark particular cases:

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M1A1

10. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

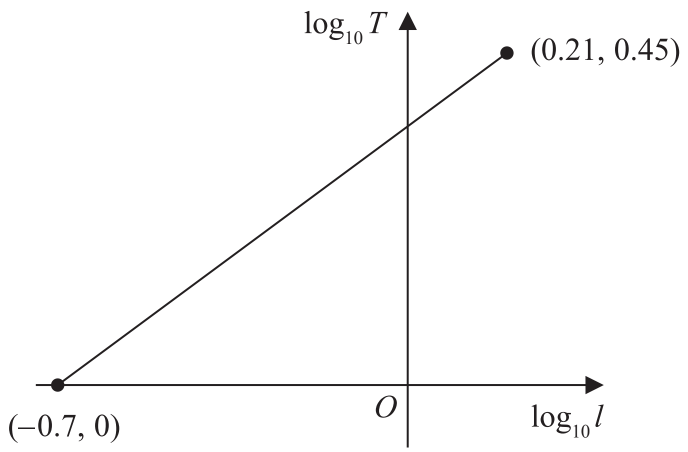


Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data.
The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \frac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

Notes

(a)

M1: Takes logs of both sides and shows the addition law.

Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$

A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer.
Also allow *t* rather than *T* and *A* rather than *a*.

Allow working backwards e.g.

$$\log_{10} T = b\log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$$

$$\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b \, *$$

M1: Uses the given answer and uses the power law and addition law correctly

A1: Reaches the given equation with no errors as above

(b)

B1: Deduces the correct value for *b* (Allow awrt 0.495 or $\frac{45}{91}$)

M1: Correct strategy to find the value of *a*.
E.g. substitutes one of the given points and their value for *b* into $\log_{10} T = \log_{10} a + b\log_{10} l$ and uses correct log work to identify the value of *a*. Allow slips in rearranging their equation but must be correct log work to find *a*.
Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of *a*.
E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0.346} = ...$

A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$

(Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$)

Must see the <u>equation</u> not just correct values as it is a requirement of the question.

(c)

B1: Correct interpretation

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study,

(1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year.

(3)

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places.

(4)

Question	Scheme	Marks	AOs
10 (a)	265 thousand	B1	3.4
		(1)	
(b)	Attempts $\frac{dN_b}{dt} = 11e^{0.05t}$	M1	1.1b
	Substitutes $t = 10$ into their $\frac{dN_b}{dt}$	M1	3.4
	$\frac{dN_b}{dt} =$ awrt 18.1 which is approximately 18 thousand per year *	A1*	2.1
		(3)	
(c)	Sets $45 + 220e^{0.05t} = 10 + 800e^{-0.05t} \Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$	M1	3.1b
	Correct quadratic equation $\Rightarrow 220\left(e^{0.05t}\right)^2 + 35e^{0.05t} - 800 = 0$	A1	1.1b
	$e^{0.05t} = 1.829, (-1.988) \Rightarrow 0.05t = \ln(1.829)$	M1	2.1
	$T = 12.08$	A1	1.1b
		(4)	
			(8 marks)
Notes:			

(a) May be seen in the question so watch out.

B1: Accept 265 thousand or 265 000 or equivalent such as 265 k but not just 265.

(b)

M1: Differentiates to a form $ke^{0.05t}, k > 0, k \neq 220$. Do not be too concerned about the lhs.

M1: Substitutes $t = 10$ into a changed function that was formed from an attempt at differentiation.

The left hand side must have implied differentiation. E.g. Rate = , $N', \frac{dN_b}{dt}, \frac{dN}{dt}$ or even $\frac{dy}{dx}$

A1*: Full and complete proof that requires

- some correct lhs seen at some point. E.g. "Rate = , " $\frac{dN_b}{dt}, \frac{dN}{dt}$ but condone N'.
- an intermediate line/answer of either $11e^{0.05\times10}$ or awrt 18.1 before a minimal conclusion which must be referencing the 18 000 or 18 thousand

(c)

M1: Attempts to set both equations equal to each other and simplify the constant terms.

Look for $220\,e^{0.05t}+35=800\,e^{-0.05t}$ o.e but condone slips

It is also possible to set $\frac{N-45}{220}=\left(e^{0.05t}=\right)\frac{800}{N-10}$ and form an equation in N

A1: Correct quadratic form.

Look for $220\left(e^{0.05t}\right)^2+35e^{0.05t}-800=0$ or $220\,e^{0.1t}+35e^{0.05t}-800=0$ but allow with terms in different order such as $220\,e^{0.1t}+35e^{0.05t}=800$

FYI the equation in N is $N^2-55N-175550=0$

M1: Full attempt to find the value of t (or a constant multiple of t)

This involves the key step of recognising and solving a 3TQ in $e^{0.05t}$ followed by the use of lns.
If the answers to the quadratic just appear (from a calculator) you will need to check.
Accuracy should be to 3sf.
You may see different variables used such as x

$x=e^{0.05t},\ 220\,e^{0.1t}+35e^{0.05t}=800\Rightarrow 220x^2+35x=800\Rightarrow x=1.82...\Rightarrow t=20\ln 1.82...$

Allow use of calculator for solving the quadratic and for $e^{0.05t}=1.82..\Rightarrow t=12.08$

Via the N route it will involve substituting the positive solution to their quadratic into either equation to find a value for t/T using same rules as above.

A1: AWRT 12.08

....

Answers with limited or no working in (b) and (c)

(b) A derivative in the correct form must be seen

(c) Candidates who state $45+220\,e^{0.05t}=10+800\,e^{-0.05t}$ followed by awrt 12.08 (presumably from using num-solv on their calculators) can score SC 1100. Rubric on the front of the paper states that "Answers without working may not gain full credit" so we demand a method in this part.

....

2. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

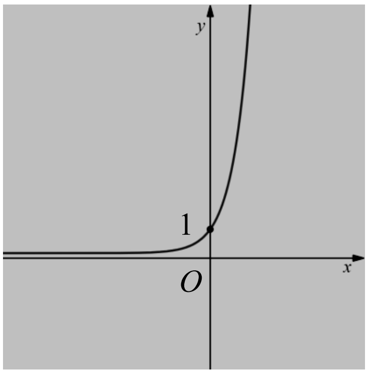
(2)

(b) Solve

$$4^x = 100$$

giving your answer to 2 decimal places.

(2)

Question	Scheme		Marks	AOs
2(a)		Correct shape or correct intercept – see notes	B1	1.2
		Fully correct – see notes	B1	1.1b
			(2)	
(b)	$4^x = 100 \Rightarrow x = \log_4 100$ or e.g. $x \log 4 = \log 100 \Rightarrow x = \frac{\log 100}{\log 4}$		M1	1.1b
	$\Rightarrow (x =)$ awrt 3.32		A1	1.1b
			(2)	
				(4 marks)

Notes:

Note that B0B1 is not possible in part (a)

(a) **Axes do not need to be labelled. No sketch is no marks.**

B1: Correct shape or correct intercept.

Shape: A positive exponential curve in quadrants 1 and 2 only, passing through a point on the positive y-axis. Must "level out" in quadrant 2 but not necessarily asymptotic to the x-axis and allow if the curve bends up slightly for $x < 0$ but do not allow a clear "U" shape. It must not clearly "stop" on the x-axis to the left of the y-axis.

OR

Intercept: The intercept can be marked as 1 or (0, 1) or $y = 1$ or (1, 0) as long as it is in the correct place. May also be seen away from the sketch but must be seen as (0, 1) or possibly these coordinates in a table but it must correspond to the sketch. If there is any ambiguity, the sketch takes precedence.

B1: Fully correct.

Shape: A positive exponential curve in quadrants 1 and 2 only, passing through a point on the positive y-axis. The curve must appear to be asymptotic to the x-axis **and it must level out at least half way below the intercept**. Allow if the curve bends up slightly for $x < 0$ but do not allow a clear "U" shape. The curve must not bend back on itself on the rhs of the y-axis. There must be no suggestion that the curve approaches another horizontal asymptote other than the x-axis e.g. a horizontal dotted line that the curve approaches.

AND

Intercept: As above

See practice items and below for some examples:

(b)

M1: Uses logs in an attempt to solve the equation. E.g. takes log base 4 and obtains $x = \log_4 100$

Alternatively takes logs (any base) to obtain $x\log 4 = \log 100$ and proceeds to $x = \dfrac{\log 100}{\log 4}$

Allow if this subsequently becomes e.g. log 25 as long as $\dfrac{\log 100}{\log 4}$ is seen **but**

$x\log 4 = \log 100 \Rightarrow x = \log 25$ or $x\log 4 = \log 100 \Rightarrow x = \log 100 - \log 4$ scores M0

A1: awrt 3.32 . A correct answer only of awrt 3.32 scores M1A1

Note that a common incorrect answer is $x = 3.218875\ldots$ and comes from ln 25 or ln 100 – ln4 and unless $x = \dfrac{\ln 100}{\ln 4}$ is seen previously, this scores M0A0

5. A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$	
$2\log_2 \left(\dfrac{x}{\sqrt{x}}\right) = 3$	using the subtraction law for logs
$2\log_2 \left(\sqrt{x}\right) = 3$	simplifying
$\log_2 x = 3$	using the power law for logs
$x = 3^2 = 9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme		Marks	AOs
5 (a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x = 2^3 = 8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$	$\frac{3}{2}\log_2(x) = 3$	M1	1.1b
	$x^{\frac{3}{2}} = 2^3$ or $\frac{x^2}{\sqrt{x}} = 2^3$	$x = 2^2$	M1	1.1b
	$x = \left(2^3\right)^{\frac{2}{3}} = 4$	$x = 4$	A1	1.1b
			(3)	
				(5 marks)

(a)

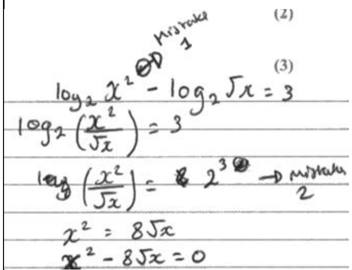
B1: States one of the two errors.

Error One: Either in words states 'They cannot use the subtraction law without dealing with the 2 first' or writes ' that line 2 should be $\log_2\left(\frac{x^2}{\sqrt{x}}\right) \quad (=3)$' If they rewrite line two it must be correct. Allow 'the coefficient of each log term is different so we cannot use the subtraction law' Allow responses such as 'it must be $\log x^2$ before subtracting the logs'

Do not accept an incomplete response such as "the student ignored the 2". **There must be some reference to the subtraction law as well.**

Error Two: Either in words states 'They undo the log incorrectly' or writes that 'if $\log_2 x = 3$ then $x = 2^3 = 8$' If it is rewritten it must be correct. Eg $x = \log_2 9$ is B0

B1: States both of the two errors. (See above)



Cases like these please send to review.

(b)

M1: Uses a correct method of combining the two log terms. Either uses both the power law and the subtraction law to reach a form $\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$ oe. Or uses both the power law and subtraction to reach $\frac{3}{2}\log_2(x) = 3$

M1: Uses correct work to "undo" the log. Eg moves from $\log_2\left(Ax^n\right) = b \Rightarrow Ax^n = 2^b$

This is independent of the previous mark so allow following earlier error.

A1: cso $x = 4$ achieved with at least one intermediate step shown. Extra solutions would be A0

SC: If the "answer" rather than the "solution" is given score 100.

13.

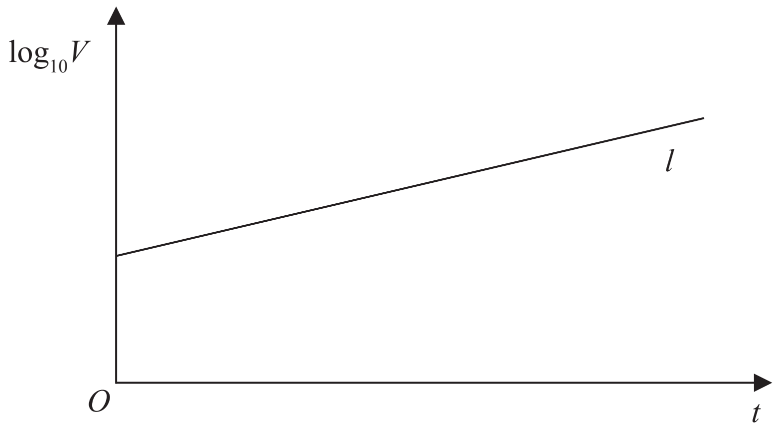


Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

Question	Scheme	Marks	AOs
13(a)	For a correct equation in p or q $p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a) This is now being marked M1 A1 M1 A1 and in this order on e pen

M1: For a correct equation in p or q This is usually $p=10^{4.8}$ or $q=10^{0.05}$ but may be $\log q=0.05$ or $\log p=4.8$

A1: For $p=$ awrt 63100 or $q=$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p=10^{4.8}$ and $q=10^{0.05}$ but may be $\log q=0.05$ and $\log p=4.8$

A1: For $p=$ awrt 63100 and $q=$ awrt 1.122 Both these values implies M1 M1

....

ALT I(a)

M1: Substitutes $t=0$ and states that $\log p=4.8$

A1: $p=$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p=$ awrt 63100 and $q=$ awrt 1.122

....

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled "*p* is........... " and "*q* is ...."

(c)

M1: For substituting $t=30$ into $V=pq^t$ using their values for p and q or substituting t =30 into $\log_{10}V=0.05t+4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign.

Remember to isw after a correct answer

14. The value of a car, £V, can be modelled by the equation

$$V = 15\,700\mathrm{e}^{-0.25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is t years.

Using the model,

(a) find the initial value of the car.

(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925\mathrm{e}^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

The model predicts that the value of the car approaches, but does not fall below, £A.

(c) State the value of A.

(1)

(d) State a limitation of this model.

(1)

Question	Scheme	Marks	AOs
14 (a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
		(9 marks)	

Notes

(a)

B1: £18 000 There is no requirement to have the units

(b)(i)

M1: Award for making the link between gradient and rate of change.

Score for attempting to differentiate V to $\frac{dV}{dt} = ke^{-0.25t}$ An attempt at both sides are required.

For the left hand side you may condone attempts such as $\frac{dy}{dx}$

A1: Achieves $\frac{dV}{dt} = -3925e^{-0.25t}$ or $\frac{dV}{dt} = 15700 \times -0.25e^{-0.25t}$ with both sides correct

A1*: Sets $-3925e^{-0.25T} = -500$ oe and proceeds to $3925e^{-0.25T} = 500$

This is a given answer and to achieve this mark, all aspects must be seen and be correct.

t must be changed to T at some point even if just at the end of their solution/proof

SC: Award SC 110 candidates who simply write

$-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

Or $15700 \times -0.25e^{-0.25t} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$

(b)(ii)

M1: Proceeds from $e^{-0.25T} = A, A > 0$ using ln's to $\pm 0.25T = ..$

Alternatively takes lns first $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 - 0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$

but $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 \times -0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ is M0

A1: $T =$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t =$ awrt 8.24

Notes on Question 14 continue
A1: 8 years 3 months. Correct answer and solution only Answers obtained numerically score 0 marks. The M mark must be scored. **(c)** **B1:** 2 300 but condone £ 2 300 **(d)** **B1:** Any suitable reason. See scheme Accept "Scrappage" schemes may pay more (or less) than £ 2 300. Do not accept "does not take into account inflation" It asks for a limitation of the model so candidates cannot score marks by suggesting other suitable models " the value may fall by the same amount each year"

8. The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(1)

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35 °C.

(3)

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15 °C.

(1)

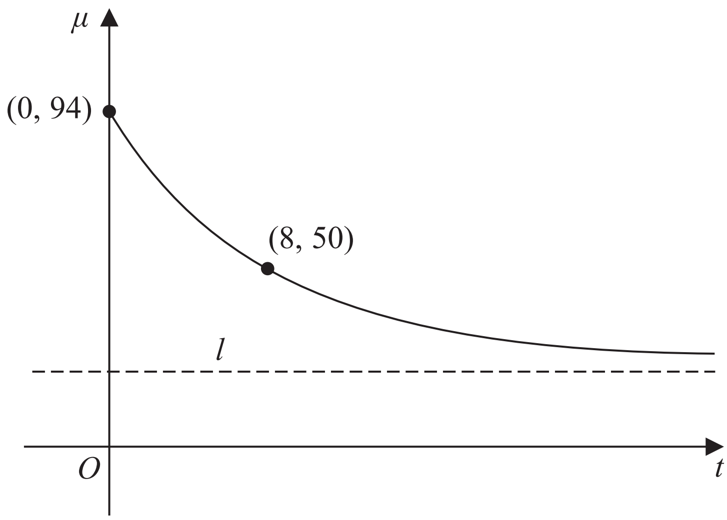


Figure 2

The temperature, μ°C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.

(4)

Question	Scheme		Marks	AOs
8 (a)	Temperature = 83°C		B1	3.4
			(1)	
(b)	$18+65e^{-\frac{t}{8}}=35 \Rightarrow 65e^{-\frac{t}{8}}=17$		M1	1.1b
	$t=-8\ln\left(\frac{17}{65}\right)$	$\ln 65-\frac{t}{8}=\ln 17 \Rightarrow t=...$	dM1	1.1b
	$t=10.7$		A1	1.1b
			(3)	
(c)	States a suitable reason • As $t\to\infty, \theta\to 18$ from above. • The minimum temperature is 18°C		B1	2.4
			(1)	
(d)	$A+B=94$ or $A+Be^{-1}=50$		M1	3.4
	$A+B=94$ and $A+Be^{-1}=50$		A1	1.1b
	Full method to find at least a value for A		dM1	2.1
	Deduces $\mu=\frac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4		A1	2.2a
			(4)	
			(9 marks)	

Notes

(a)

B1: Uses the model to state that the temperature = 83°C Requires units as well

(b)

M1: Uses the information and proceeds to $Pe^{\pm\frac{t}{8}}=Q$ condoning slips

dM1: A full method using correct log laws and a knowledge that e^x and ln$x$ are inverse functions. This cannot be scored from unsolvable equations, e.g $P>0, Q<0$. Condone one error in their solution.

A1: $t=$ awrt 10.7

(c)

B1: States a suitable reason with minimal conclusion

- As $t\to\infty, \theta\to 18$ from above.
- The minimum temperature is 18°C (so it cannot drop to 15°C)
- Substitutes $\theta=15$ (or a value between 15 and 18) into $18+65e^{-\frac{t}{8}}=15$ (may be impied by $15-18=-3$ or similar) and makes a statement that $e^{-\frac{t}{8}}$ cannot be less than zero or else that $\ln(-ve)$ is undefined and hence $\theta\neq 15$. All calculations must be correct
- (According to the model) the room temperature is 18°C (so cannot fall below this)

(d)

M1: Attempts to use $(0, 94)$ or $(8, 50)$ in order to form at least one equation in *A* and *B*
Allow this to be scored with an equation containing e^0

A1: Correct equations $A + B = 94$ and $A + Be^{-1} = 50$ or equivalent. $e^0 = 1$ must have been processed. Condone $A + B = 94$ and $A + 0.37B = 50$ where $e^{-1} =$ awrt 0.37

dM1: Dependent upon having two equations in *A* and *B* formed from the information given. It is a full and correct method leading to a value of *A*. Allow this to be solved from a calculator.
Note $B = 69.6..$ or $\dfrac{44}{1 - e^{-1}} \Rightarrow A = 94 - "B"$

A1: Deduces that $\mu = \dfrac{50e - 94}{e - 1}$ or accept $\mu =$ awrt 24.4. Condone $y = \ldots$

12. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \qquad 1 \leqslant t \leqslant 30, t \in \mathbb{N}$$

is used to model the total number of views of the advert, V, in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab.

(1)

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live. Give your answer to 2 significant figures.

(2)

Question	Scheme		Marks	AOs
12 (a)	$\log_{10} V = 0.072t + 2.379$ $\Rightarrow V = 10^{0.072t+2.379}$ $\Rightarrow V = 10^{0.072t} \times 10^{2.379}$	$V = ab^t$ $\Rightarrow \log_{10} V = \log_{10} a + \log_{10} b^t$ $\Rightarrow \log_{10} V = \log_{10} a + t\log_{10} b$	B1	2.1
	States either $a = 10^{2.379}$ or $b = 10^{0.072}$	States either $\log_{10} a = 2.379$ or $\log_{10} b = 0.072$	M1	1.1b
	$a = 239$ or $b = 1.18$	$a = 239$ or $b = 1.18$	A1	1.1b
	Either $V = 239 \times 1.18^t$ or imply by $a = 239, b = 1.18$		A1	1.1b
			(4)	
(b)	The value of ab is the (total) number of views of the advert **1 day after** it went live.		B1	3.4
			(1)	
(c)	Substitutes $t = 20$ in either equation and finds V Eg $V = 239 \times 1.18^{20}$		M1	3.4
	Awrt 6500 or 6600		A1	1.1b
			(2)	
				(7 marks)

(a) **Condone** $\log_{10}$ **written** log **or** lg **written throughout the question**

B1: Scored for showing that $\log_{10} V = 0.072t + 2.379$ can be written in the form $V = ab^t$ or vice versa

Either starts with $\log_{10} V = 0.072t + 2.379$ (may be implied) and **shows lines**

$$V = 10^{0.072t+2.379} \text{ and } V = 10^{0.072t} \times 10^{2.379}$$

Or starts with $V = ab^t$ (implied) and **shows the lines**

$$\log_{10} V = \log_{10} a + \log_{10} b^t \text{ and } \log_{10} V = \log_{10} a + t\log_{10} b$$

M1: For a correct equation in a or a correct equation in b

A1: Finds either constant. Allow $a = \text{awrt } 240$ or $b = \text{awrt } 1.2$ following a correct method

A1: Correct solution: Look for $V = 239 \times 1.18^t$ or $a = 239, b = 1.18$

Note that this is NOT awrt

(b)

B1: See scheme. Condone not seeing total. Do not allow number of views at the start or similar.

(c)

M1: Substitutes $t = 20$ in either their $V = 239 \times 1.18^t$ or $\log_{10} V = 0.072t + 2.379$ and uses a correct method to find V

A1: Awrt 6500 or 6600

11. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

(1)

11 (a)	35 (km^2)	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45e^{14c} \Rightarrow 45e^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \frac{1}{14}\ln\left(\frac{20}{45}\right) = \cdots - 0.0579$	dM1	3.1b
	$A = 80 - 45e^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
			(6 marks)

Notes

(a)

B1: Uses the equation of the model to find that 35 (km^2) of the reserve was covered on 1st January 2005. Do not accept eg. 35 m^2

(b)

M1: Sets their $60 = 80 - 45e^{14c} \Rightarrow Ae^{14c} = B$

A1: $45e^{14c} = 20$ or equivalent.

dM1: A full and careful method using precise algebra, correct log laws and a knowledge that e^x and ln$x$ are inverse functions and proceeds to a value for c.

A1: Gives a complete equation for the model $A = 80 - 45e^{-0.0579t}$

(c)

B1: Gives a suitable interpretation (See scheme)

13.

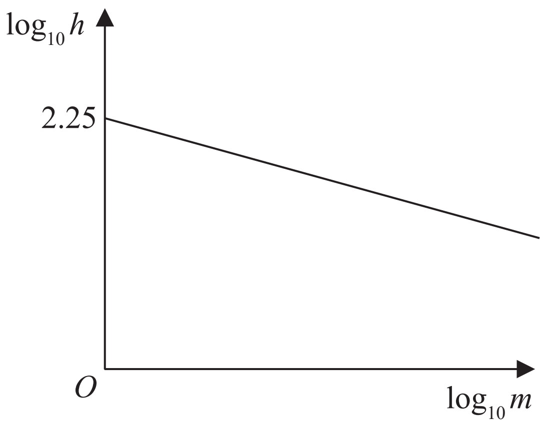


Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

P 6 6 5 8 5 A 0 3 0 4 4

Question	Scheme		Marks	AOs
13 (a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25-0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235" \log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
				(7 marks)

Notes

(a)

M1: Establishes a link between $h = pm^q$ and $\log_{10} h = 2.25 - 0.235\log_{10} m$.
May be implied by a correct equation in p or q

A1: For a correct equation in p or q

A1: $p = 178$ and $q = -0.235$

(b)

M1: Uses either model to set up an equation in h (or m)

A1: h = awrt 122. Condone h = awrt 122 bpm

A1ft: Comments on the suitability of the model. Follow through on their answer.

Requires a comment consistent with their answer from using the model.

E.g. It is a suitable model as it is only “3” bpm away from the real value ✓
Do not allow an argument stating that it should be the same.
It is an unsuitable model as “122” bpm is not equal to 119 bpm ×

(c)

B1: "p" would be the (resting) heart rate of a mammal with a mass of 1 kg

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

Question	Scheme	Marks	AOs
5(a)	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) or $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	M1	1.1b
	$p =$ awrt 3.162 or $q =$ awrt 1.072	A1	1.1b
	$p = 10^{0.5}$ (or $\log_{10} p = 0.5$) and $q = 10^{0.03}$ (or $\log_{10} q = 0.03$)	dM1	3.1a
	$A = 3.162 \times 1.072^t$	A1	3.3
		(4)	
(b)(i)	The initial mass (in kg) of algae (in the pond).	B1	3.4
(b)(ii)	The ratio of algae from one week to the next.	B1	3.4
		(2)	
(c)(i)	5.5 kg	B1	2.2a
(c)(ii)	$4 = "3.162" \times "1.072"^t$ or $\log_{10} 4 = 0.03t + 0.5$	M1	3.4
	awrt 3.4 (weeks)	A1	1.1b
		(3)	
(d)	• The model predicts unlimited growth. • The weather may affect the rate of growth	B1	3.5b
		(1)	
			(10 marks)

Notes

(a)

M1: A correct equation in *p* **or** *q*. May be implied by a correct value for *p* or *q*.
Also score for rearranging the equation to the form $A = 10^{0.5} ... 10^{0.03t}$

A1: For *p* = awrt 3.162 or *q* = awrt 1.072. May be embedded within the equation.

dM1: Correct equations in *p* **and** *q*. Also score for rearranging the equation to the form $A = 10^{0.5} \times 10^{0.03t}$

A1: Complete equation with *p* = awrt 3.162 and *q* = awrt 1.072. **Must be seen in (a)**
If *p* and *q* are just stated but the equation is not written with the values embedded then withhold this mark.
Withhold the final mark if the correct values for *p* and *q* result from incorrect working such as $A = 10^{0.5} + 10^{0.03t} \Rightarrow A = 3.162 \times 1.072^t$.
If *p* and *q* are stated the wrong way round, take the stated equation as their final answer and isw.

(b)
(i)
B1: Must reference mass of algae and relating to initially/at the start/beginning
Examples of acceptable answers:
The mass of algae originally (in the pond)
p is the mass of algae when *t* = 0
Examples of answers we would not accept
p is how much algae there is at the beginning
The relationship between algae and number of weeks

(ii)

B1: Must reference the rate of change/multiplier and the time frame eg per week/every week/each week.

Examples of acceptable answers:

q is the rate at which the mass of algae increases for every week

The amount of algae increases by 7.2% each week (condone amount for mass in ii)

The proportional increase in mass of the algae each week

Examples of answers we would not accept:

q is how much algae will increase when *t* increases by 1

The amount that grows per unit of time

The rate at which the mass of algae in the small pond increases after *t* number of weeks

The rate in which the algae mass increases

(c)

B1: cao (including units)

M1: Setting up a correct equation to find *t* using the given equation or their part (a)
Substitution of $A = 4$ into their equation for *A* or the given equation is sufficient for this mark.

A1: awrt 3.4 (weeks). Accept any acceptable method (including trial and improvement)
Condone lack of units. isw if they subsequently convert to weeks and days. Allow awrt 3.5 (weeks) following p = awrt 3.16 and q = awrt 1.07.
An answer of only awrt 3.4 is M1A1, but an answer of 4 (weeks) with no working is M0A0

(d)

B1: Any reason why the rate of change, growth or the mass of algae might change or why the model in not realistic.
Be generous with the awarding of this mark as long as the answer has engaged with the context of the problem or the model

Examples of acceptable answers:

Seasonal changes (which would affect the growth rate)

Overcrowding (as it is a small pond)

Algae may stop growing (the model predicts unlimited growth)

Algae may die / be removed / eaten (so the rate of growth may not continue at the same rate)

Examples of answers we would not accept:

There could be other factors that affect the amount of algae (too vague)

The mass of algae might change

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \qquad t \in \mathbb{R} \qquad t \geqslant 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2

(a) state the value of k.

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

Question	Scheme	Marks	AOs
8(a)	$(k =)\ 0.8$	B1	1.1b
		(1)	
(b)	$1 = 0.8 + 1.4e^{-0.5t} \Rightarrow 1.4e^{-0.5t} = 0.2$	M1	3.1b
	$-0.5t = \ln\left(\frac{0.2}{1.4}\right) \Rightarrow t = ...$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\frac{dP}{dt} =\right)\ -0.7e^{-0.5t}$ $\left(\frac{dP}{dt}\right)_{t=2} = -0.7e^{-0.5\times 2}$	M1	3.1b
	$=$ awrt 0.258 (kg/cm^2 per minute)	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1: Completes the equation for the model by obtaining (k =) 0.8 or equivalent.

(b) ***Be aware this could be solved entirely using a calculator which is not acceptable***

M1: For using the model with P = 1 and their value for k from (a) and proceeding to $Ae^{\pm 0.5t} = B$. Condone if A or B are negative for this mark.

M1: Uses correct log work to solve an equation of the form $Ae^{\pm 0.5t} = B$ leading to a value for t. They cannot proceed directly to awrt 3.9 without some intermediate working seen.
Eg $t = 2\ln 7$ or $-2\ln\left(\frac{1}{7}\right)$ is acceptable.
Also allow $1.4e^{-0.5t} = 0.2 \Rightarrow -0.5t = -1.9459... \Rightarrow t = ...$
This cannot be scored from an unsolvable equation (eg when their $k \geq 1$ so that $e^{\pm 0.5t} \leq 0$).

A1: Accept awrt 3.9 minutes or t = awrt 3.9 with correct working seen.
eg $1.4e^{-0.5t} = 0.2 \Rightarrow t = 3.9$ would be M1M0A0

(c) ***Be aware this can be solved entirely using a calculator which is not acceptable***

M1: Links rate of change to gradient and differentiates to obtain an expression of the form $Ae^{-0.5t}$ and substitutes t = 2. Do not accept $Ate^{-0.5t}$ as the derivative.
Beware that substituting t = 2 and proceeding from e^{-1} to e^{-2} is M0A0

A1: Obtains awrt 0.258 with differentiation seen. (Units not required) Condone awrt -0.258
Awrt ± 0.258 with no working is M0A0. Isw after a correct answer is seen.

(Ignore in (c) any spurious notation on the LHS when differentiating such as $P = ...$ or $\frac{dy}{dx} = ...$)

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\dfrac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

Question	Scheme	Marks	AOs
9(a)(i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=p-2$	B1	1.2
(ii)	$\log_3\left(\sqrt{x}\right)=\frac{1}{2}p$	B1	1.1b
		(2)	
(b)	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow 2p-4+\frac{3}{2}p=-11\Rightarrow p=...$	M1	1.1b
	$p=-2$	A1	1.1b
	$\log_3 x=-2\Rightarrow x=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
		(4)	
	Alternative for (b) not using (a):		
	$2\log_3\left(\frac{x}{9}\right)+3\log_3\left(\sqrt{x}\right)=-11\Rightarrow\log_3\left(\frac{x}{9}\right)^2+\log_3\left(\sqrt{x}\right)^3=-11$ $\Rightarrow\log_3\frac{x^{\frac{7}{2}}}{81}=-11$	M1	1.1b
	$\Rightarrow\frac{x^{\frac{7}{2}}}{81}=3^{-11}$ or equivalent eg $x^{\frac{7}{2}}=3^{-7}$	A1	1.1b
	$x^{\frac{7}{2}}=81\times3^{-11}\Rightarrow x^{\frac{7}{2}}=3^4\times3^{-11}=3^{-7}\Rightarrow x=\left(3^{-7}\right)^{\frac{2}{7}}=3^{-2}$	M1	1.1b
	$x=\frac{1}{9}$	A1	1.1b
			(6 marks)

Notes

(a)(i)
B1: Recalls the subtraction law of logs and so obtains $p-2$

(a)(ii)
B1: $\frac{1}{2}p$ oe

(b) ***Be aware this should be solved by non-calculator methods***

M1: Uses their results from part (a) to form a linear equation in p and attempts to solve leading to a value for p. Allow slips in their rearrangement when solving. Allow a misread forming the equation equal to 11 instead of −11

A1: Correct value for p

M1: Uses $\log_3 x=p\Rightarrow x=3^p$ following through on what they consider to be their p. It must be a value rather than p

A1: $(x=)\ \frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.

Alternative:

M1: Correct use of log rules to achieve an equation of the form $\log_3 ... = \log_3 ...$ or $\log_3 ... = \text{a number}$ (typically −11). Condone arithmetical slips.

A1: Correct equation with logs removed.

M1: Uses inverse operations to find *x*. Condone slips but look for proceeding from $x^{\frac{a}{b}} = ... \Rightarrow x = ...^{\frac{b}{a}}$ where they have to deal with a fractional power.

A1: $(x=)\ \frac{1}{9}$ cao with correct working seen. Must be this fraction. Do not penalise invisible brackets as long as the intention is clear.