Y1P13 XMQs and MS

(Total: 123 marks)

1. P2_Sample   Q9 . 5 marks - Y1P13 Integration
2. P2_Sample   Q15. 10 marks - Y1P13 Integration
3. P1_2019   Q8 . 10 marks - Y1P13 Integration
4. P2_2020   Q8 . 6 marks - Y1P13 Integration
5. P2_2021   Q7 . 9 marks - Y1P13 Integration
6. P2_2022   Q8 . 6 marks - Y1P13 Integration
7. P1(AS)_2018 Q1 . 4 marks - Y1P13 Integration
8. P1(AS)_2018 Q15. 10 marks - Y1P13 Integration
9. P1(AS)_2019 Q3 . 6 marks - Y1P13 Integration
10. P1(AS)_2019 Q13. 7 marks - Y1P12 Differentiation
11. P1(AS)_2020 Q7 . 8 marks - Y1P13 Integration
12. P1(AS)_2020 Q10. 10 marks - Y1P7 Algebraic methods
13. P1(AS)_2021 Q3 . 4 marks - Y1P13 Integration
14. P1(AS)_2021 Q9 . 4 marks - Y1P13 Integration
15. P1(AS)_2021 Q14. 10 marks - Y1P2 Quadratics
16. P1(AS)_2022 Q1 . 4 marks - Y1P13 Integration
17. P1(AS)_2022 Q10. 10 marks - Y1P13 Integration

9. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

(Total for Question 9 is 5 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">9</td><td colspan="2">$\int(3x^{0.5}+A)\,dx = 2x^{1.5}+Ax(+c)$</td><td>M1
A1</td><td>3.1a
1.1b</td></tr>
<tr><td colspan="2">Uses limits and sets $= 2A^2 \Rightarrow (2\times 8+4A)-(2\times 1+A)=2A^2$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Sets up quadratic and attempts to solve</td><td>Sets up quadratic and attempts b^2-4ac</td><td>M1</td><td>1.1b</td></tr>
<tr><td>$\Rightarrow A=-2, \frac{7}{2}$ and states that there are two roots</td><td>States $b^2-4ac=121>0$ and hence there are two roots</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="5" align="right">(5 marks)</td></tr>
<tr><td colspan="5">Notes:</td></tr>
<tr><td colspan="5">
M1: Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax(+c)$ where k is a non- zero constant
A1: Correct answer but may not be simplified
M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,dx = Ax$ and not $\frac{A^2}{2}$
M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac
A1: Either $A=-2, \frac{7}{2}$ and states that there are two roots
Or states $b^2-4ac=121>0$ and hence there are two roots
</td></tr>
</table>

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">10</td><td>Attempts $S_\infty = \frac{8}{7}\times S_6 \Rightarrow \frac{a}{1-r}=\frac{8}{7}\times\frac{a(1-r^6)}{1-r}$</td><td>M1</td><td>2.1</td></tr>
<tr><td>$\Rightarrow 1=\frac{8}{7}\times(1-r^6)$</td><td>M1</td><td>2.1</td></tr>
<tr><td>$\Rightarrow r^6=\frac{1}{8}\Rightarrow r= ..$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>$\Rightarrow r=\pm\frac{1}{\sqrt{2}}$ (so $k=2$)</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="4" align="right">(4 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">
M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty=\frac{8}{7}\times S_6$
M1: Proceeds to an equation just in r
M1: Solves using a correct method
A1: Proceeds to $r=\pm\frac{1}{\sqrt{2}}$ giving $k=2$
</td></tr>
</table>

15.

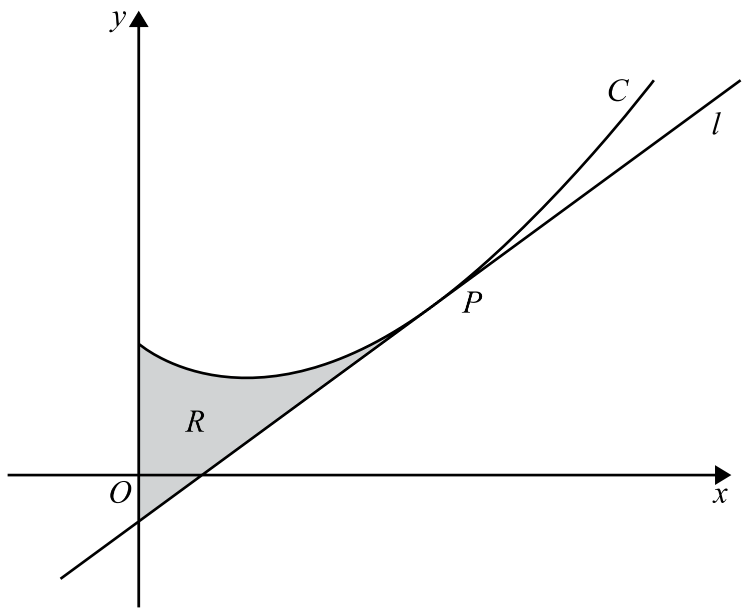


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, \; x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
15	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{15}{2}x^{\frac{1}{2}}-9$	M1 A1	3.1a 1.1b
	Substitutes $x=4\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y-15=6(x-4)$	M1	2.1
	Equation of l is $y=6x-9$	A1	1.1b
	Area $R=\int_0^4\left(5x^{\frac{3}{2}}-9x+11\right)-(6x-9)\,\mathrm{d}x$	M1	3.1a
	$=\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x\right]_0^4=2\times4^{\frac{5}{2}}-\frac{15}{2}\times4^2+20\times4-0$	M1	2.1
	Area of $R=24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
			(10 marks)

Question 15 continued

Notes:

M1: Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

A1: $\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

M1: Substitutes $x = 4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,dx$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

A1: $= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve $= \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx\right]_0^4$

A1: $= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x\right]_0^4 = 36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R = \text{their } 36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2}\right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$

8.

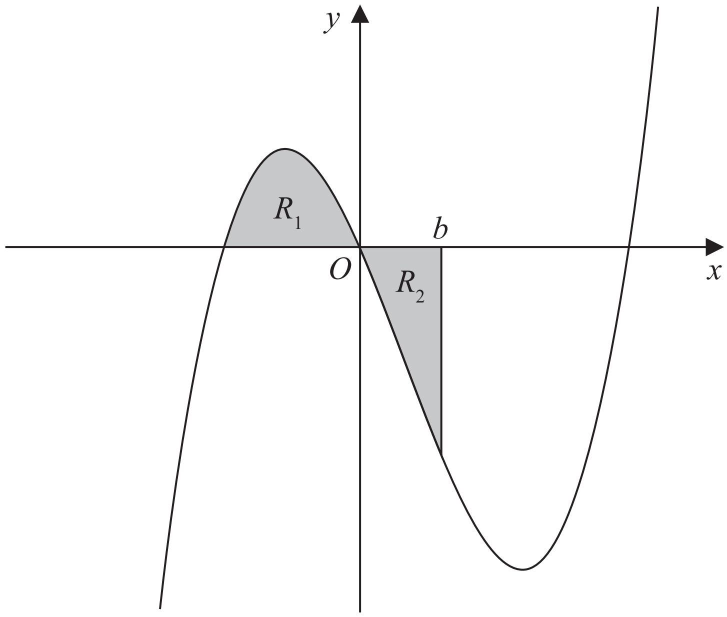


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

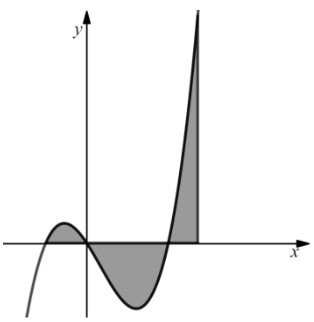
$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

Question	Scheme	Marks	AOs
8 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \, dx \to \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y \, dx$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) \quad = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2\left(3b^2 - 20b + 20\right) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x -axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to $x^3 - 2x^2 - 8x$ (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \to x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function.

The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2} = \frac{20}{3}$ oe for this mark

A1*: For a rigorous argument leading to area of $R_1 = \frac{20}{3}$ For this to be awarded the integration must be correct and the limits must be the correct way around and embedded or calculated values must be seen.

Eg. Look for $-\left(4+\frac{16}{3}-16\right)$ or $-\left(\frac{1}{4}(-2)^4-\frac{2}{3}(-2)^3-4(-2)^2\right)$ oe before you see the $\frac{20}{3}$

Note: It is possible to do this integration by parts.

(b)

M1: For setting their $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=\pm\frac{20}{3}$ or $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{b}=0$

A1: Deduces that $3b^4-8b^3-48b^2+80=0$. Terms may be in a different order but expect integer coefficients.

It must have followed $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=-\frac{20}{3}$ oe.

Do not award this mark for $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2+\frac{20}{3}=0$ unless they attempt the second part of this question by expansion and then divide the resulting expanded expression by 12

M1: Attempts to factorise $3b^4-8b^3-48b^2\pm80=(b+2)(b+2)(3b^2...b...20)$ via repeated division or inspection. FYI $3b^4-8b^3-48b^2+80=(b+2)(3b^3-14b^2-20b+40)$ Allow an attempt via inspection

$3b^4-8b^3-48b^2\pm80=(b^2+4b+4)(3b^2...b...20)$ but do not allow candidates to just write out

$3b^4-8b^3-48b^2\pm80=(b+2)^2(3b^2-20b+20)$ which is really just copying out the given answer.

Alternatively attempts to expand $(b+2)^2(3b^2-20b+20)$ achieving terms of a quartic expression

A1*: Correctly reaches $(b+2)^2(3b^2-20b+20)=0$ with no errors and must have = 0

In the alternative obtains both equations in the same form **and states that they are same**. Allow ✓ QED etc here.

(c) Please watch for candidates who answer this on Figure 2 which is fine

B1: Sketches the curve and a vertical line to the right of 4 (x = 5.442 may not be labelled.)

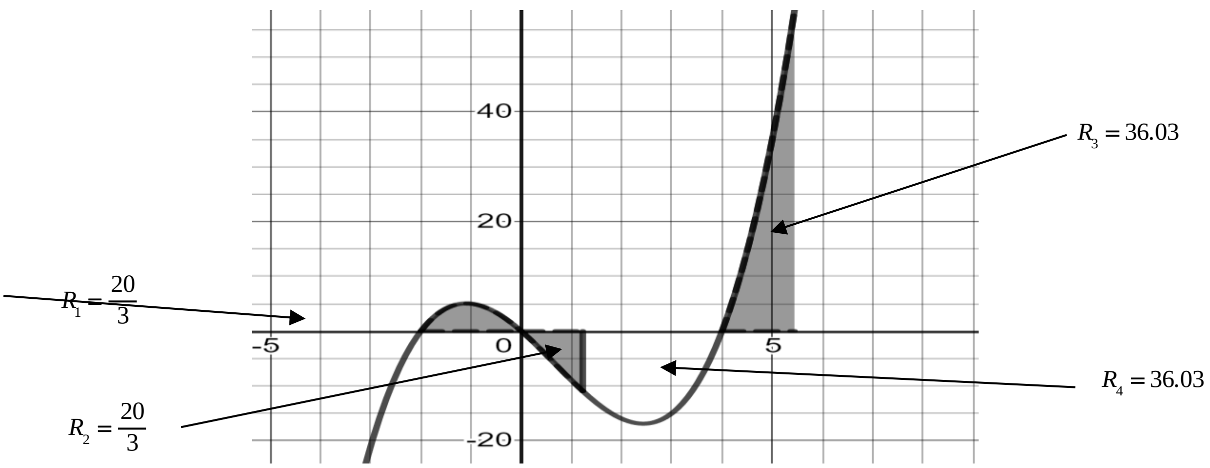
B1: Explains that (between $x=-2$ and $x=5.442$) the area above the x-axis = area below the x -axis with appropriate areas shaded or labelled**.**

Alternatively states that the area between 1.225 and 4 is the same as the area between 4 and 5.442

Another correct statement is that the net area between 0 and 5.442 is $-\frac{20}{3}$

Look carefully at what is written. There are many correct statements/ deductions.

Eg. " (area between 0 and 4) - (area between 4 and 5.442) = 20/3". Diagram below for your information.



8. A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $\mathrm{f}(x)$

find, in simplest form, $\mathrm{f}(x)$

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
8	$f'(x) = 6x^2 + ax - 23 \Rightarrow f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$	M1 A1	1.1b 1.1b
	$"c" = -12$	B1	2.2a
	$f(-4) = 0 \Rightarrow 2\times(-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$	dM1	3.1a
	$a = \ldots (6)$	dM1	1.1b
	$(f(x) =) 2x^3 + 3x^2 - 23x - 12$ Or Equivalent e.g. $(f(x) =)(x+4)(2x^2 - 5x - 3)$ $(f(x) =)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Integrates $f'(x)$ with two correct indices. There is no requirement for the + c

A1: Fully correct integration (may be unsimplified). The + c must be seen (or implied by the −12)

B1: Deduces that the constant term is −12

dM1: Dependent upon having done some integration. It is for setting up a linear equation in a by using $f(-4) = 0$

May also see long division attempted for this mark. Need to see a complete method leading to a remainder in terms of a which is then set = 0.

For reference, the quotient is $2x^2 + \left(\frac{a}{2} - 8\right)x + 9 - 2a$ and the remainder is $8a - 48$

May also use $(x + 4)(px^2 + qx + r) = 2x^3 + \frac{1}{2}ax^2 - 23x - 12$ and compare coefficients to find p, q and r and hence a. Allow this mark if they solve for p, q and r

Note that some candidates use 2f(*x*) which is acceptable and gives the same result if executed correctly.

dM1: Solves the linear equation in a or uses p, q and r to find a.

It is dependent upon having attempted some integration and used $f(\pm 4) = 0$ or long division/comparing coefficients with $(x + 4)$ as a factor.

A1cso: For $(f(x) =) 2x^3 + 3x^2 - 23x - 12$ oe. Note that "f(x) =" does not need to be seen and ignore any "= 0"

Via firstly using factor

Question	Scheme	Marks	AOs
8 Alt	$\mathrm{f}(x)=(x+4)\left(Ax^2+Bx+C\right)$	M1 A1	1.1b 1.1b
	$\mathrm{f}(x)=Ax^3+(4A+B)x^2+(4B+C)x+4C \Rightarrow C=-3$	B1	2.2a
	$\mathrm{f}'(x)=3Ax^2+2(4A+B)x+(4B+C)$ and $\mathrm{f}'(x)=6x^2+ax-23$ $\Rightarrow A=\ldots$	dM1	3.1a
	Full method to get A, B and C	dM1	1.1b
	$\mathrm{f}(x)=(x+4)\left(2x^2-5x-3\right)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Uses the fact that f(x) is a cubic expression with a factor of $(x + 4)$
A1: For $\mathrm{f}(x) = (x + 4)(Ax^2 + Bx + C)$
B1: Deduces that $C = -3$
dM1: Attempts to differentiate either by product rule or via multiplication and compares to $\mathrm{f}'(x) = 6x^2 + ax - 23$ to find A.
dM1: Full method to get A, B and C
A1cso: $\mathrm{f}(x) = (x + 4)(2x^2 - 5x - 3)$ or $\mathrm{f}(x) = (x + 4)(2x + 1)(x - 3)$

7. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

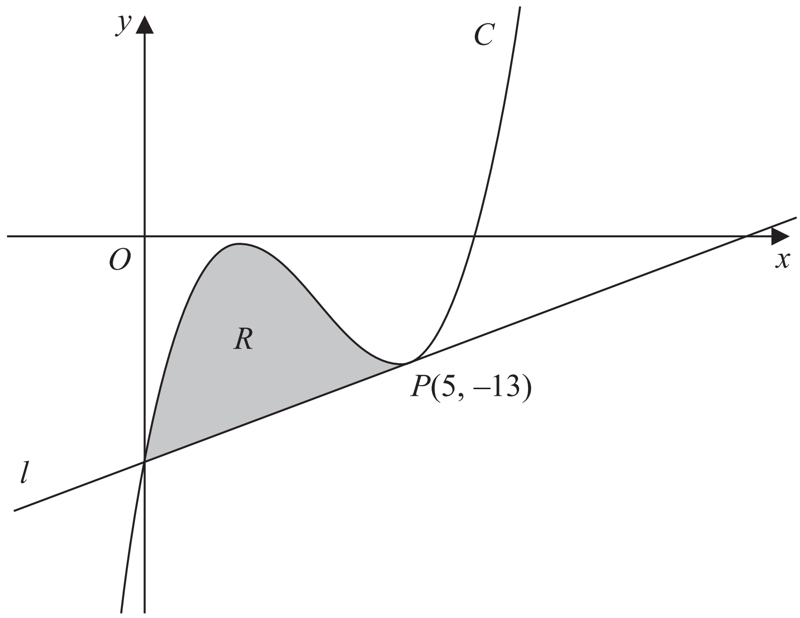


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

Question	Scheme	Marks	AOs
7(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \dfrac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\dfrac{dy}{dx}\right)_{x=5} = 3\times 5^2 - 20\times 5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both *C* and *l* pass through (0, −23) and so *C* meets *l* again on the *y*-axis	B1	2.2a
		(1)	
(c)	$\pm\int \left(x^3 - 10x^2 + 27x - 23 - (2x - 23)\right) dx$ $= \pm\left(\dfrac{x^4}{4} - \dfrac{10}{3}x^3 + \dfrac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\dfrac{x^4}{4} - \dfrac{10}{3}x^3 + \dfrac{25}{2}x^2\right]_0^5$ $= \left(\dfrac{625}{4} - \dfrac{1250}{3} + \dfrac{625}{2}\right)(-0)$	dM1	2.1
	$= \dfrac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative:		
	$\pm\int \left(x^3 - 10x^2 + 27x - 23\right) dx$ $= \pm\left(\dfrac{x^4}{4} - \dfrac{10}{3}x^3 + \dfrac{27}{2}x^2 - 23x\right)$	M1 A1	1.1b 1.1b
	$\left[\dfrac{x^4}{4} - \dfrac{10}{3}x^3 + \dfrac{27}{2}x^2 - 23x\right]_0^5 + \dfrac{1}{2}\times 5(23 + 13)$ $= -\dfrac{455}{12} + 90$	dM1	2.1
	$= \dfrac{625}{12}$	A1	1.1b
			(9 marks)

Notes
(a) B1: Correct derivative M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{dy}{dx}$ M1: Fully correct straight line method using (5, – 13) and their $\frac{dy}{dx}$ at $x = 5$ A1: cao. Must see the full equation in the required form. (b) B1: Makes a suitable deduction. Alternative via equating *l* and *C* and factorising e.g. $x^3 - 10x^2 + 27x - 23 = 2x - 23$ $x^3 - 10x^2 + 25x = 0$ $x\left(x^2 - 10x + 25\right) = 0 \Rightarrow x = 0$ So they meet on the *y*-axis (c) M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±" *C* – *l*" A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a)) If they attempt as 2 separate integrals e.g. $\int\left(x^3 - 10x^2 + 27x - 23\right)\,dx - \int\left(2x - 23\right)\,dx$ then award this mark for the correct integration of the curve as in the alternative. If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for ±" *C* – *l*" dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.** A1: Correct exact value **Alternative:** M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±*C* A1: Correct integration for ±*C* dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the *x*-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction. May see the trapezium area attempted as $\int\left(2x - 23\right)\,dx$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation. **Depends on the first method mark.** A1: Correct exact value Note if they do *l* – *C* rather than *C* – *l* and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with *l* – *C* leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks. If the answer is left as $-\frac{625}{12}$ then score A0

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

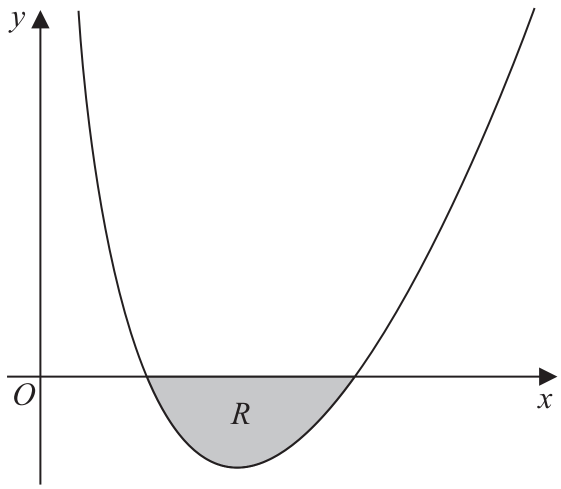


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

Find the exact area of R, writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(6)

Question	Scheme	Marks	AOs
8	$y=\dfrac{(x-2)(x-4)}{4\sqrt{x}}=\dfrac{x^2-6x+8}{4\sqrt{x}}=\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\displaystyle\int \frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\,dx=\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}\,(+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 4 and applies to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$	M1	2.2a
	$\left(\dfrac{32}{10}-8+8\right)-\left(\dfrac{2}{5}\sqrt{2}-2\sqrt{2}+4\sqrt{2}\right)=\dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}$ Area R $=\dfrac{12}{5}\sqrt{2}-\dfrac{16}{5}\ \left(\text{or } \dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}\right)$	A1	2.1
		(6)	
			(6 marks)

Notes:

M1: Correct attempt to write $\dfrac{(x-2)(x-4)}{4\sqrt{x}}$ as a sum of terms with **indices**.

Look for at least two different terms with the correct index e.g. two of $x^{\frac{3}{2}}$, $x^{\frac{1}{2}}$, $x^{-\frac{1}{2}}$ which have come from the correct places.

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

A1: $\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ which can be left unsimplified e.g. $\dfrac{1}{4}x^{2-\frac{1}{2}}-\dfrac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(x^{\frac{3}{2}}-6x^{\frac{1}{2}}+8x^{-\frac{1}{2}}\right)$

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

dM1: Integrates $x^n \to x^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $x^{\frac{3}{2}} \to x^{\frac{5}{2}},\quad x^{\frac{1}{2}} \to x^{\frac{3}{2}},\quad x^{-\frac{1}{2}} \to x^{\frac{1}{2}}$

It is dependent upon the first M so at least two terms must have had a correct index.

A1: $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$. Allow unsimplified e.g. $\dfrac{1}{4}\times\dfrac{2}{5}x^{\frac{3}{2}+1}-\dfrac{1}{2}\times\dfrac{2}{3}x^{\frac{1}{2}+1}-\dfrac{2}{3}x^{\frac{1}{2}+1}+2\times2x^{\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(\dfrac{2}{5}x^{\frac{5}{2}}-4x^{\frac{3}{2}}+16x^{\frac{1}{2}}\right)(+c)$.

M1: Substitutes the limits 4 and 2 to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$ and subtracts either way round.

There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\dfrac{1}{10}\times4^{\frac{5}{2}}-4^{\frac{3}{2}}+4\times4^{\frac{1}{2}}-\dfrac{1}{10}\times2^{\frac{5}{2}}-2^{\frac{3}{2}}+4\times2^{\frac{1}{2}}$

This is an independent mark but the limits must be applied to an expression that is not y so they may even have differentiated.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw

See overleaf for integration by parts and integration by substitution.

Integration by parts:

$\int \frac{(x-2)(x-4)}{4\sqrt{x}}\,dx = \int \frac{1}{4}(x-2)(x-4)x^{-\frac{1}{2}}\,dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx$	M1 A1	1.1b 1.1b
$\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\,dx$ $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ Or e.g. $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	dM1 A1	3.1a 1.1b
Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	M1	2.2a
$0 - \frac{16}{3} + \frac{128}{15} - \left(0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}\right)$ Area $R = \frac{12}{5}\sqrt{2} - \frac{16}{5}$ $\left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies integration by parts and reaches the form $\alpha(x-2)(x-4)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ $\alpha, p \neq 0$

oe e.g. $\alpha\left(x^2 - 6x + 8\right)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ $\alpha, p \neq 0$

A1: Correct first application of parts in any form

dM1: Attempts their $\int (px+q)x^{\frac{1}{2}}\,dx$ by expanding and integrating or may attempt parts again.

E.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \int \left(2x^{\frac{3}{2}} - 6x^{\frac{1}{2}}\right)dx = ...$ or e.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}}(2x-6) - \frac{4}{3}\int x^{\frac{3}{2}}\,dx$

If they expand then at least one term requires $x^n \to x^{n+1}$ or if parts is attempted again, the structure must be correct.

A1: Fully correct integration in any form

M1: Substitutes the limits 4 and 2 to their $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ and subtracts

either way round. There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $0 - \frac{16}{3} + \frac{128}{15} - 0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}$

This is an independent mark but the limits must be applied to a "changed" function.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents

Attempts at integration by parts "the other way round" should be sent to review.

Integration by substitution example:

$u=\sqrt{x}\left(x=u^2\right)\Rightarrow\int\frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ $=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}2u\mathrm{d}u$	M1 A1	1.1b 1.1b
$=\frac{1}{2}\int\left(u^4-6u^2+8\right)\mathrm{d}u=\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$	dM1 A1	3.1a 1.1b
Deduces limits of integral are $\sqrt{2}$ and 2 and applies to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$	M1	2.2a
$\frac{1}{2}\left(\frac{32}{5}-16+16-\left(\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)\right)$ Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\left(\text{or } \frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies the substitution e.g. $u=\sqrt{x}$ and attempts $k\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$

A1: Fully correct integral in terms of u in any form e.g. $\frac{1}{2}\int\left(u^2-2\right)\left(u^2-4\right)\mathrm{d}u$

dM1: Expands the bracket and integrates $u^n\rightarrow u^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $u^4\rightarrow u^5,\ u^2\rightarrow u^3,\ k\rightarrow ku$

A1: $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$. Allow unsimplified.

M1: Substitutes the limits 2 and $\sqrt{2}$ to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$ and subtracts either way round.

There is no requirement to evaluate but 2 and $\sqrt{2}$ must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{2}\left(\frac{32}{5}-16+16-\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)$

Alternatively reverses the substitution and applies the limits 4 and 2 with the same conditions.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw.

There may be other substitutions seen and the same marking principles apply.

Answer ALL questions. Write your answers in the spaces provided.

1. Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

AS Paper 1 June Pure Mathematics Mark Scheme

Very brief explanation of the AO's

AO	It is awarded for.....
1.1	Select or carry out routine procedures, recall facts, definitions
2.1	Constructing an argument
2.2	Making a deduction
2.3	Assessing the validity of an argument / Identifying errors in a solution
2.4	Explaining their reasoning
2.5	Using mathematical language/notation correctly
3.1	Translating a problem into maths
3.2	Interpreting solutions or limitations to their problem
3.3	Modelling a problem
3.4	Using a model
3.5	Evaluating the outcome/ refining a model

Question	Scheme	Marks	AOs
1	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int\left(\frac{2}{3}x^3-6\sqrt{x}+1\right)\mathrm{d}x=\frac{2}{3}\times\frac{x^4}{4}+...+x$	A1	1.1b
	$=...-6\frac{x^{\frac{3}{2}}}{3/2}+....$	A1	1.1b
	$=\ \frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$	A1	1.1b
			(4 marks)

Notes

M1: Allow for raising power by one. $x^n \rightarrow x^{n+1}$
Award for any correct power including sight of $1x$

A1: Correct two '**non fractional power'** terms (may be un-simplified at this stage)

A1: Correct '**fractional power'** term (may be un-simplified at this stage)

A1: Completely correct, simplified and including constant of integration seen on one line.
Simplification is expected for full marks.

Accept correct exact equivalent expressions such as $\frac{x^4}{6}-4x\sqrt{x}+1x^1+c$

Accept $\frac{x^4-24x^{\frac{3}{2}}+6x}{6}+c$

Remember to isw after a correct answer.

Condone poor notation. Eg answer given as $\int\frac{1}{6}x^4-4x^{\frac{3}{2}}+x+c$

15.

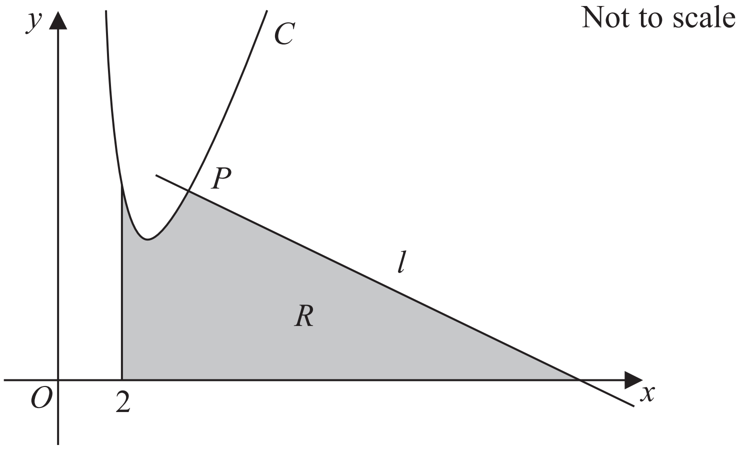


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(10)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 5 8 3 4 6 A 0 4 4 4 8

Question	Scheme	Marks	AOs
15.	For the complete strategy of finding where the normal cuts the x-axis. Key points that must be seen are • Attempt at differentiation • Attempt at using a changed gradient to find equation of normal • Correct attempt to find where normal cuts the x - axis	M1	3.1a
	$y=\frac{32}{x^2}+3x-8\Rightarrow\frac{dy}{dx}=-\frac{64}{x^3}+3$	M1 A1	1.1b 1.1b
	For a correct method of attempting to find Either the equation of the normal: this requires substituting $x=4$ in their $\frac{dy}{dx}=-\frac{64}{x^3}+3=(2)$, then using the perpendicular gradient rule to find the equation of normal $y-6="-\frac{1}{2}"(x-4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0-6="-\frac{1}{2}"(x-4)\Rightarrow x=...$ or an attempt using just gradients $"-\frac{1}{2}"=\frac{6}{a-4}\Rightarrow a=...$	dM1	2.1
	Normal cuts the x-axis at $x=16$	A1	1.1b
	For the complete strategy of finding the values of the two key areas. Points that must be seen are • There must be an attempt to find the area under the curve by integrating between 2 and 4 • There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-4)\times 6$ or $\int_4^{"16"}"\left(-\frac{1}{2}x+8\right)"dx$ The "16" cannot have just been made up.	M1	3.1a
	$\int\frac{32}{x^2}+3x-8\,dx=-\frac{32}{x}+\frac{3}{2}x^2-8x$	M1 A1	1.1b 1.1b
	Area under curve = $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$	dM1	1.1b
	Total area =10 + 36 =46 *	A1*	2.1
		(10)	
			(10 marks)

(a)

The first 5 marks are for finding the normal to the curve cuts the x - axis

M1: For the complete strategy of finding where the normal cuts the x- axis. See scheme

M1: Differentiates with at least one index reduced by one

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{64}{x^3}+3$

dM1: Method of finding

either the equation of the normal at (4, 6) .

or where the equation of the normal at (4, 6) cuts the x - axis

See scheme. It is dependent upon having gained the M mark for differentiation.

A1: Normal cuts the x-axis at $x=16$

The next 5 marks are for finding the area *R*

M1: For the complete strategy of finding the values of two key areas. See scheme

M1: Integrates $\int \frac{32}{x^2}+3x-8\,\mathrm{d}x$ raising the power of at least one index

A1: $\int \frac{32}{x^2}+3x-8\,\mathrm{d}x=-\frac{32}{x}+\frac{3}{2}x^2-8x$ which may be unsimplified

dM1: Area $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$

It is dependent upon having scored the M mark for integration, for substituting in both 4 and 2 and subtracting either way around. The above line shows the minimum allowed working for a correct answer.

A1*: Shows that the area under curve = 46. No errors or omissions are allowed

....

A number of candidates are equating the line and the curve (or subtracting the line from the curve) The last 5 marks are scored as follows.

M1: For the complete strategy of finding the values of the two key areas. Points that must be seen are

- There must be an attempt to find the area BETWEEN the line and the curve either way around by integrating between 2 and 4
- There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-2)\times\left(-\frac{1}{2}\times2+8\right)$ or via integration $\int_2^{16}\left("-\frac{1}{2}x+8"\right)\mathrm{d}x$

M1: Integrates $\int\left("-\frac{1}{2}x+8"\right)-\left(\frac{32}{x^2}+3x-8\right)\mathrm{d}x$ either way around and raises the power of at least one index by one

A1: $\pm\left(-\frac{32}{x}+\frac{7}{4}x^2-16x\right)$ must be correct

dM1: Area = $\int_2^4\left("-\frac{1}{2}x+8"\right)-\left(\frac{32}{x^2}+3x-8\right)\mathrm{d}x=$ ......either way around

A1: Area = $49-3=46$

NB: Watch for candidates who calculate the area under the curve between 2 and 4 = 10 and subtract this from the large triangle = 56. They will lose both the strategy mark and the answer mark.

NB. Watch for students who use their calculators to do the majority of the work. Please send these items to review

3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)

Question	Scheme	Marks	AOs
3(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int\left(\frac{4}{x^3}+kx\right)\mathrm{d}x = -\frac{2}{x^2}+\frac{1}{2}kx^2+c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2}+\frac{1}{2}kx^2\right]_{0.5}^{2} = \left(-\frac{2}{2^2}+\frac{1}{2}k\times4\right)-\left(-\frac{2}{(0.5)^2}+\frac{1}{2}k\times(0.5)^2\right)=8$	M1	1.1b
	$7.5+\frac{15}{8}k=8 \Rightarrow k=...$	dM1	1.1b
	$k=\frac{4}{15}$ oe	A1	1.1b
		(3)	
			(6 marks)

Notes

Mark parts (a) and (b) as one

(a)

M1: For $x^n \to x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2.
Condone "unprocessed" values here. Eg. x^{-3+1} and x^{1+1}

A1: Either term correct (un simplified).

Accept $4\times\frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ **with** the indices processed.

A1: Correct (and simplified) with $+c$.

Ignore spurious notation e.g. answer appearing with an $\int$ sign or with $\mathrm{d}x$ on the end.

Accept $-\frac{2}{x^2}+\frac{1}{2}kx^2+c$ or exact simplified equivalent such as $-2x^{-2}+k\frac{x^2}{2}+c$

(b)

M1: For substituting both limits into their $-\frac{2}{x^2}+\frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits.

dM1: For solving a **linear** equation in k. It is dependent upon the previous M only
Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k =$

A1: $k=\frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n}=\frac{4}{15}$

Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267
Please remember to isw after a correct answer

13.

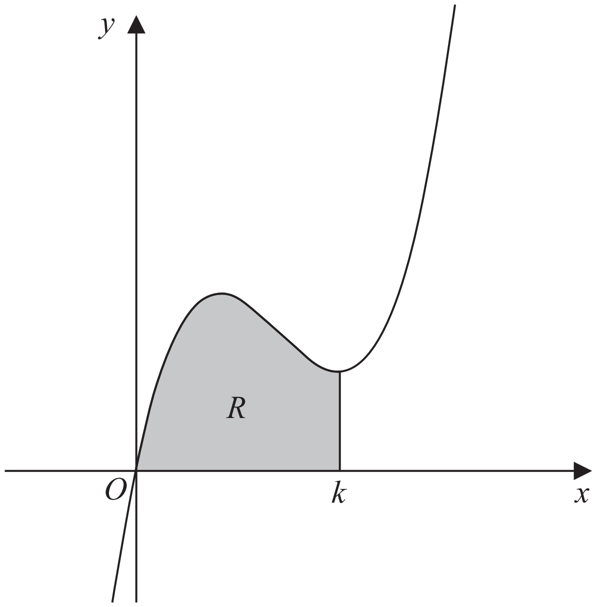


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

	Scheme	Marks	AOs
13.	The overall method of finding the x coordinate of A.	M1	3.1a
	$y = 2x^3 - 17x^2 + 40x \Rightarrow \frac{dy}{dx} = 6x^2 - 34x + 40$	B1	1.1b
	$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 34x + 40 = 0 \Rightarrow 2(3x-5)(x-4) = 0 \Rightarrow x = \ldots$	M1	1.1b
	Chooses $x = 4$ ~~$x = \frac{5}{3}$~~	A1	3.2a
	$\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$	B1	1.1b
	Area $= \frac{1}{2}(4)^4 - \frac{17}{3}(4)^3 + 20(4)^2$	M1	1.1b
	$= \frac{256}{3}$ *	A1*	2.1
		(7)	

(7 marks)

Notes

M1: An overall problem -solving method mark to find the minimum point. To score this you need to see

- an attempt to differentiate with at least **two correct terms**
- an attempt to set their $\frac{dy}{dx} = 0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution

B1: $\left(\frac{dy}{dx} =\right) 6x^2 - 34x + 40$ which may be unsimplified

M1: Sets their $\frac{dy}{dx} = 0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic.
If $\frac{dy}{dx}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x - \frac{5}{3}\right)$

A1: Chooses $x = 4$ This may be awarded from the upper limit in their integral

B1: $\int 2x^3 - 17x^2 + 40x \, dx = \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]$ which may be unsimplified

M1: Correct attempt at area. There may be slips on the integration but expect **two correct terms**
The upper limit used must be their larger solution of $\frac{dy}{dx} = 0$ and the lower limit used must be 0.
So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value.
Expect to see embedded or calculated values.
Don't accept $\int_0^4 2x^3 - 17x^2 + 40x \, dx = \frac{256}{3}$ without seeing the integration and the embedded or calculated values

A1*: Area $= \frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer.

Notes on Question 13 continue
For correct notation expect to see • $\frac{dy}{dx}$ or $\frac{d}{dx}$ used correctly at least once. If $f(x)$ is used accept $f'(x)$. Condone y' • $\int 2x^3 - 17x^2 + 40x \, dx$ used correctly at least once with or without the limits.

7. Given that k is a positive constant and $\displaystyle\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$$

(4)

Question	Scheme	Marks	AOs
7 (a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}}+3\right)\mathrm{d}x = 5\sqrt{x}+3x$	A1	1.1b
	$\left[5\sqrt{x}+3x\right]_1^k = 4 \Rightarrow 5\sqrt{k}+3k-8=4$	dM1	1.1b
	$3k+5\sqrt{k}-12=0$ *	A1*	2.1
		(4)	
(b)	$3k+5\sqrt{k}-12=0 \Rightarrow \left(3\sqrt{k}-4\right)\left(\sqrt{k}+3\right)=0$	M1	3.1a
	$\sqrt{k}=\frac{4}{3},(-3)$	A1	1.1b
	$\sqrt{k}=\ldots \Rightarrow k=\ldots$ oe	dM1	1.1b
	$k=\frac{16}{9},\cancel{9}$	A1	2.3
		(4)	
			(8 marks)

Notes

(a)

M1: For $x^n \to x^{n+1}$ on correct indices. This can be implied by the sight of either $x^{\frac{1}{2}}$ or x

A1: $5\sqrt{x}+3x$ or $5x^{\frac{1}{2}}+3x$ but may be unsimplified. Also allow with + c and condone any spurious notation.

dM1: Uses both limits, subtracts, and sets equal to 4. They cannot proceed to the given answer without a line of working showing this.

A1*: Fully correct proof with no errors (bracketing or otherwise) leading to given answer.

(b)

M1: For a correct method of solving. This could be as the scheme, treating as a quadratic in $\sqrt{k}$ and using allowable method to solve including factorisation, formula etc.

Allow values for $\sqrt{k}$ to be just written down, e.g. allow $\sqrt{k}=\pm\frac{4}{3},(\pm 3)$

Alternatively score for rearranging to $5\sqrt{k}=12-3k$ and then squaring to get $\ldots k=(12-3k)^2$

A1: $\sqrt{k}=\frac{4}{3},(-3)$

Or in the alt method it is for reaching a correct 3TQ equation $9k^2-97k+144=0$

dM1: For solving to find at least one value for k. It is dependent upon the first M mark.
In the main method it is scored for squaring their value(s) of $\sqrt{k}$
In the alternative scored for solving their 3TQ by an appropriate method

A1: Full and rigorous method leading to $k=\frac{16}{9}$ only. The 9 must be rejected.

10. $$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.
(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.
(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.
(4)

Question	Scheme	Marks	AOs
10 (a)	$g(5) = 2\times5^3 + 5^2 - 41\times5 - 70 = ...$	M1	1.1a
	$g(5) = 0 \Rightarrow (x-5)$ is a factor, hence $g(x)$ is divisible by $(x-5)$.	A1	2.4
		(2)	
(b)	$2x^3 + x^2 - 41x - 70 = (x-5)(2x^2 ... x \pm 14)$	M1	1.1b
	$= (x-5)(2x^2 + 11x + 14)$	A1	1.1b
	Attempts to factorise quadratic factor	dM1	1.1b
	$(g(x)) = (x-5)(2x+7)(x+2)$	A1	1.1b
		(4)	
(c)	$\int 2x^3 + x^2 - 41x - 70 \, dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$	M1 A1	1.1b 1.1b
	Deduces the need to use $\int_{-2}^{5} g(x)dx$ $-\frac{1525}{3} - \frac{190}{3}$	M1	2.2a
	Area = $571\frac{2}{3}$	A1	2.1
		(4)	
			(10 marks)

Notes

(a)

M1: Attempts to calculate $g(5)$ Attempted division by $(x-5)$ is M0

Look for evidence of embedded values or two correct terms of $g(5) = 250 + 25 - 205 - 70 = ...$

A1: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,

$g(5) = 0 \Rightarrow (x-5)$ is a factor, hence divisible by $(x-5)$

$g(5) = 0 \Rightarrow (x-5)$ is a factor ✓

Do not allow if candidate states

$f(5) = 0 \Rightarrow (x-5)$ is a factor, hence divisible by $(x-5)$ **(It is not f)**

$g(x) = 0 \Rightarrow (x-5)$ is a factor **(It is not g(*x*) and there is no conclusion)**

This may be seen in a preamble before finding $g(5) = 0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to find the quadratic factor by inspection (correct coefficients of first term and ± last term) or by division (correct coefficients of first term and ± second term). Allow this to be scored from division in part (a)

A1: $(2x^2 + 11x + 14)$ You may not see the $(x-5)$ which can be condoned

dM1: Correct attempt to factorise their $(2x^2 + 11x + 14)$

A1: $(g(x)=)\ (x-5)(2x+7)(x+2)$ or $(g(x)=)\ (x-5)(x+3.5)(2x+4)$
It is for the product of factors and not just a statement of the three factors
Attempts with calculators via the three roots are likely to score 0 marks. The question was "Hence" so the two M's must be awarded.

(c)

M1: For $x^n \to x^{n+1}$ for any of the terms in x for $g(x)$ so
$2x^3 \to ...x^4,\ x^2 \to ...x^3,\ -41x \to ...x^2,\ -70 \to ...x$

A1: $\int 2x^3 + x^2 - 41x - 70\,dx = \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x$ which may be left unsimplified (ignore any reference to $+C$)

M1: Deduces the need to use $\int_{-2}^{5} g(x)dx$.

This may be awarded from the limits on their integral (either way round) or from embedded values which can be subtracted either way round.

A1: For clear work showing all algebraic steps leading to area = $571\frac{2}{3}$ oe

So allow $\int_{-2}^{5} 2x^3 + x^2 - 41x - 70\,dx = \left[\frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x\right]_{-2}^{5} = -\frac{1715}{3} \Rightarrow$ area $= \frac{1715}{3}$ for 4 marks

Condone spurious notation, as long as the algebraic steps are correct. If they find $\int_{-2}^{5} g(x)dx$ then withhold the final mark if they just write a positive value to this integral since $\int_{-2}^{5} g(x)dx = -\frac{1715}{3}$

Note $\int_{-2}^{5} 2x^3 + x^2 - 41x - 70\,dx \Rightarrow \frac{1715}{3}$ with no algebraic integration seen scores M0A0M1A0

3. Find

$$\int \frac{3x^4 - 4}{2x^3}\,dx$$

writing your answer in simplest form.

(4)

Question	Scheme	Marks	AOs
3	$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3}{2}x - 2x^{-3}\,dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2}\times\frac{x^2}{2} - 2\times\frac{x^{-2}}{-2} \quad (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c \quad$ o.e	A1	1.1b
		(4)	
		(4 marks)	

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,dx$ o.e such as $\frac{1}{2}\int\left(3x - 4x^{-3}\right)\,dx$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index.
Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$

9. Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

9	$\displaystyle\int_k^9 \frac{6}{\sqrt{x}}\,\mathrm{d}x = \left[ax^{\frac{1}{2}}\right]_k^9 = 20 \Rightarrow 36 - 12\sqrt{k} = 20$	M1 A1	1.1b 1.1b
	Correct method of solving Eg. $36 - 12\sqrt{k} = 20 \Rightarrow k =$	dM1	3.1a
	$\Rightarrow k = \dfrac{16}{9}$ oe	A1	1.1b
		(4)	
			(4 marks)

Notes:

M1: For setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$

A1: A correct equation involving *p* Eg. $36 - 12\sqrt{k} = 20$

dM1: For a whole strategy to find *k*. In the scheme it is awarded for setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$, using both limits and proceeding using correct index work to find *k*. It cannot be scored if $k^{\frac{1}{2}} < 0$

A1: $k = \dfrac{16}{9}$

14. A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = -3x^2 + 12x + 8$$

(a) Write $\mathrm{f}(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

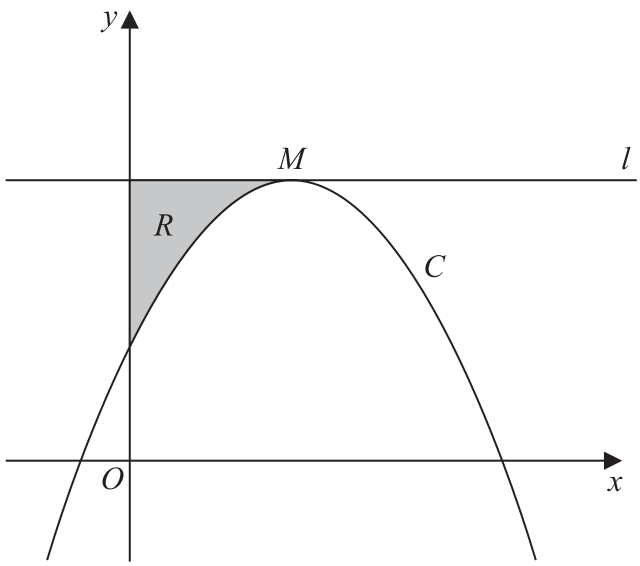


Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
14 (a)	$f(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + ...$	M1	1.1b
	$= -3(x-2)^2 + ...$	A1	1.1b
	$= -3(x-2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x$	M1 A1	1.1b 1.1b
	Method to find $R =$ their $2 \times 20 - \int_0^2 \left(-3x^2 + 12x + 8\right) dx$	M1	3.1a
	$R = 40 - \left[-2^3 + 24 + 16\right]$	dM1	1.1b
	$= 8$	A1	1.1b
		(5)	
			(10 marks)
Alt(c)	$\int 3x^2 - 12x + 12 \, dx = x^3 - 6x^2 + 12x$	M1 A1	1.1b 1.1b
	Method to find $R = \int_0^2 3x^2 - 12x + 12 \, dx$	M1	3.1a
	$R = 2^3 - 6 \times 2^2 + 12 \times 2$	dM1	1.1b
	$= 8$	A1	1.1b

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x \pm 2)^2 + ...$

Alternatively attempt to compare $-3x^2 + 12x + 8$ to $ax^2 + 2abx + ab^2 + c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2 + ...$ or via comparison finds $a = -3, b = -2$

A1: $-3(x-2)^2 + 20$

(b)

B1ft: One correct coordinate

B1ft: Correct coordinates. Allow as $x = ..., y = ...$
Follow through on their $\left(-b, c\right)$

(c)

M1: Attempts to integrate. Award for any correct index

A1: $\int -3x^2 + 12x + 8 \, \mathrm{d}x = -x^3 + 6x^2 + 8x \ (+c)$ (which may be unsimplified)

M1: Method to find area of R. Look for their $2 \times "20" - \int_0^{"2"} \mathrm{f}(x) \, \mathrm{d}x$

dM1: Correct application of limits on their integrated function. Their 2 must be used

A1: Shows that area of $R = 8$

1. Find

$$\int\left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right)\mathrm{d}x$$

giving your answer in simplest form.

(4)

Question	Scheme	Marks	AOs
1	$x^n \rightarrow x^{n+1}$	M1	1.1b
	$\int\left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right)\mathrm{d}x = \frac{8x^4}{4} ... + 5x$	A1	1.1b
	$= ... - 2 \times \frac{3}{2} x^{\frac{1}{2}} + ...$	A1	1.1b
	$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$	A1	1.1b
		(4)	
			(4 marks)

Notes

M1: For raising any correct power of x by 1 including $5 \rightarrow 5x$ (not for + c) Also allow eg $x^3 \rightarrow x^{3+1}$

A1: For 2 correct non-fractional power terms (allow unsimplified coefficients) and may appear on separate lines. The indices must be processed. The + c does not count as a correct term here. Condone the 1 appearing as a power or denominator such as $\frac{5x^1}{1}$ for this mark.

A1: For the correct fractional power term (allow unsimplified) Allow eg $+ - 2 \times 1.5\sqrt{x^1}$.

Also allow fractions within fractions for this mark such as $\frac{\frac{3}{2}}{\frac{1}{2}} x^{\frac{1}{2}}$

A1: All correct and simplified and on one line including + c. Allow $-3\sqrt{x}$ or $-\sqrt{9x}$ for $-3x^{\frac{1}{2}}$. Do not accept $+ - 3x^{\frac{1}{2}}$ for this mark.

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

Eg. $\int 2x^4 - 3x^{\frac{1}{2}} + 5x + c \ \mathrm{d}x$ or $2x^4 - 3x^{\frac{1}{2}} + 5x + c = 0$ with no correct expression seen earlier are both A0.

10.

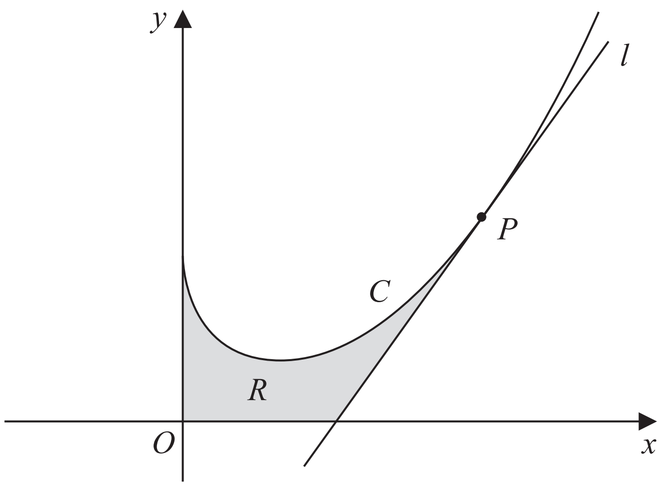


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

Question	Scheme	Marks	AOs
10(a)	$y=\frac{1}{3}x^2-2\sqrt{x}+3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{3}x-x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$x=4 \Rightarrow y=\frac{13}{3}$	B1	1.1b
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=4}=\frac{2}{3}\times4-4^{-\frac{1}{2}}\left(=\frac{13}{6}\right) \quad \therefore y-\frac{13}{3}=\frac{13}{6}(x-4)$	M1	2.1
	$13x-6y-26=0*$	A1*	1.1b
		(5)	
(b)	$\int\left(\frac{x^2}{3}-2\sqrt{x}+3\right)\mathrm{d}x=\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x(+c)$	M1 A1	1.1b 1.1b
	$y=0 \Rightarrow x=2$	B1	2.2a
	Area of R is $\left[\frac{x^3}{9}-\frac{4}{3}x^{\frac{3}{2}}+3x\right]_0^4-\frac{1}{2}\times(4-"2")\times"\frac{13}{3}"=\frac{76}{9}-\frac{13}{3}$	M1	3.1a
	$=\frac{37}{9}$	A1	1.1b
		(5)	
			(10 marks)

Notes

(a) **Calculators: If no algebraic differentiation seen then maximum in a) is M0A0B1M1A0***

M1: $x^n \to x^{n-1}$ seen at least once $...x^2 \to ...x^1$, $...x^{\frac{1}{2}} \to ...x^{-\frac{1}{2}}$, $3 \to 0$.

Also accept on sight of eg $...x^{\frac{1}{2}} \to ...x^{\frac{1}{2}-1}$

A1: $\frac{2}{3}x-x^{-\frac{1}{2}}$ or any unsimplified equivalent (indices must be processed) accept the use of $0.\dot{6}x$ but not rounded or ambiguous values eg $0.6x$ or eg $0.66....x$

B1: Correct y coordinate of P. May be seen embedded in an attempt of the equation of l

M1: Fully correct strategy for an equation for l. Look for $y-"\frac{13}{3}"="\frac{13}{6}"(x-4)$ where their $\frac{13}{6}$ is from differentiating the equation of the curve and substituting in $x=4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the y coordinate is from substituting $x=4$ into the given equation.
If they use $y=mx+c$ they must proceed as far as $c=...$ to score this mark.
Do not allow this mark if they use a perpendicular gradient.

A1*: Obtains the printed answer with no errors.

(b) **Calculators: If no algebraic integration seen then maximum in b) is M0A0B1M1A0**

M1: $x^n \to x^{n+1}$ seen at least once. Eg $...x^2 \to ...x^3$, $...x^{\frac{1}{2}} \to ...x^{\frac{3}{2}}$, $3 \to 3x^1$. Allow eg $...x^2 \to ...x^{2+1}$ The $+c$ is not a valid term for this mark.

A1: $\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x$ or any unsimplified equivalent (indices must be processed) accept the use of exact decimals for $\frac{1}{9}$ ($0.\dot{1}$) and $-\frac{4}{3}$ ($-1.\dot{3}$) but not rounded or ambiguous values.

B1: Deduces the correct value for x for the intersection of l with the x-axis. May be seen indicated on Figure 2.

M1: Fully correct strategy for the area. This needs to include

- a correct attempt at the area of the triangle using their values (**could use integration**)
- a correct attempt at the area under the curve using 0 and 4 in their integrated expression
- the two values subtracted.

Be aware of those who mix up using the y-coordinate of P and the gradient at P which is M0. The values embedded in an expression is sufficient to score this mark.

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.111…. isw after a correct answer

----

Be aware of other strategies to find the area *R*

eg Finding the area under the curve between 0 and 2 and then the difference between the curve and the straight line between 2 and 4:

$$\int_0^2 \frac{x^2}{3} - 2\sqrt{x} + 3 \, dx + \int_2^4 \frac{x^2}{3} - 2\sqrt{x} - \frac{13}{6}x + \frac{22}{3} \, dx$$

M1 $x^n \to x^{n+1}$ seen at least once on either integral (or on the equation of the line $y = \frac{1}{3}x + 3$)

A1 for correct integration of **either** integral $\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x$ or $\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} - \frac{13}{12}x^2 + \frac{22}{3}x$ (may be unsimplified/uncollected terms but the indices must be processed with/without the $+C$)

B1 Correct value for x can be seen from the top of the first integral (or bottom value of the second integral)

M1 Correct strategy for the area eg.

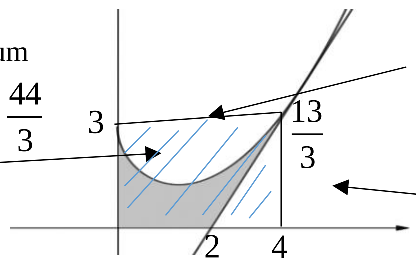
$$\left[\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x\right]_0^2 + \left[\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} - \frac{13}{12}x^2 + \frac{22}{3}x\right]_2^4 = \frac{62}{9} - \frac{4}{3}(2)^{\frac{3}{2}} + \frac{76}{9} - \frac{101}{9} + \frac{4}{3}(2)^{\frac{3}{2}}$$

A1: $\frac{37}{9}$ or exact equivalent eg $4\frac{1}{9}$ or $4.\dot{1}$ but not 4.1 or 4.111….

You could also see use of the area of a trapezium and/or the use of the line $y = \frac{1}{3}x + 3$ to find other areas which could be combined or used as part of the strategy to find *R*. Ignore areas which are not used. The marks should still be able to be applied as per the scheme

Area of large trapezium

$= \frac{1}{2} \times \left(3 + \frac{13}{3}\right) \times 4 = \frac{44}{3}$



$y = \frac{1}{3}x + 3$

Area of triangle and the y coordinate of point P are both $\frac{13}{3}$

Give BOD if in doubt of intention

Area of trapezium – (Area between $y = \frac{1}{3}x + 3$ and curve C + area of triangle)

$$= \frac{44}{3} - \frac{56}{9} - \frac{13}{3} = \frac{37}{9}$$