Y1P12 XMQs and MS

(Total: 132 marks)

1. P1_Sample Q1 . 7 marks - Y1P12 Differentiation
2. P2_Sample Q14. 9 marks - Y1P12 Differentiation
3. P1_Specimen Q9 . 5 marks - Y1P12 Differentiation
4. P1_Specimen Q11. 10 marks - Y1P12 Differentiation
5. P2_Specimen Q8 . 7 marks - Y1P12 Differentiation
6. P1_2018 Q2 . 7 marks - Y1P12 Differentiation
7. P2_2019 Q13. 10 marks - Y1P12 Differentiation
8. P2_2021 Q5 . 7 marks - Y1P12 Differentiation
9. P1_2022 Q6 . 6 marks - Y1P4 Graphs and transformations
10. P2_2022 Q4 . 3 marks - Y1P12 Differentiation
11. P1(AS)_2018 Q8 . 9 marks - Y1P12 Differentiation
12. P1(AS)_2018 Q10. 4 marks - Y1P12 Differentiation
13. P1(AS)_2019 Q5 . 5 marks - Y1P12 Differentiation
14. P1(AS)_2020 Q1 . 5 marks - Y1P12 Differentiation
15. P1(AS)_2020 Q14. 9 marks - Y1P12 Differentiation
16. P1(AS)_2021 Q5 . 6 marks - Y1P12 Differentiation
17. P1(AS)_2021 Q16. 11 marks - Y1P12 Differentiation
18. P1(AS)_2022 Q12. 12 marks - Y1P12 Differentiation

Answer ALL questions. Write your answers in the spaces provided.

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Paper 1: Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	(i) $\frac{\mathrm{d}y}{\mathrm{d}x} = 12x^3 - 24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 36x^2 - 48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x = 2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x} = 12\times 2^3 - 24\times 2^2$	M1	1.1b
	Shows $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x = 2$ into their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 36\times 2^2 - 48\times 2$	M1	1.1b
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 48 > 0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
			(7 marks)

Notes:
(a)(i) **M1:** Differentiates to a cubic form **A1:** $\frac{\mathrm{d}y}{\mathrm{d}x} = 12x^3 - 24x^2$ **(a)(ii)** **A1ft:** Achieves a correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for their $\frac{\mathrm{d}y}{\mathrm{d}x} = 36x^2 - 48x$
(b) **M1:** Substitutes $x = 2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ **A1:** Shows $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and states "hence there is a stationary point" All aspects of the proof must be correct
(c) **M1:** Substitutes $x = 2$ into their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ Alternatively calculates the gradient of C either side of $x = 2$ **A1ft:** For a correct calculation, a valid reason and a correct conclusion. Follow through on an incorrect $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{dS}{dr}$	M1	3.4
	$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant	M1	2.1
	Radius $= 4.30$ cm	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2} \Rightarrow$ Height $= 8.60$ cm	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3$ cm and $h = 8.6$ cm the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
			9 marks

Notes:

(a)

B1: Uses the correct volume formula with V=500. Accept $500 = \pi r^2 h$

M1: Substitutes $h = \dfrac{500}{\pi r^2}$ or $rh = \dfrac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi rh$ to get S as a function of r

A1*: $S = 2\pi r^2 + \dfrac{1000}{r}$ Note that this is a given answer.

(b)

M1: Differentiates the given S to reach $\dfrac{dS}{dr} = Ar \pm Br^{-2}$

A1: $\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$ or exact equivalent

M1: Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant

A1: R = awrt 4.30cm

A1: H = awrt 8.60 cm

(c)

B1: Any valid reason. See scheme for alternatives

9. The curve C has equation

$$y = 2x^3 + 5$$

The curve C passes through the point $P(1, 7)$.

Use differentiation from first principles to find the value of the gradient of the tangent to C at P.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
9	Gradient of chord $= \dfrac{(2(x+h)^3+5)-(2x^3+5)}{x+h-h}$	B1	1.1b
		M1	2.1
	$(x+h)^3 = x^3+3x^2h+3xh^2+h^3$	B1	1.1b
	Gradient of chord $= \dfrac{(2(x^3+3x^2h+3xh^2+h^3)+5)-(2x^3+5)}{1+h-1}$		
	$= \dfrac{2x^3+6x^2h+6xh^2+2h^3+5-2x^3-5}{1+h-1}$		
	$= \dfrac{6x^2h+6xh^2+2h^3}{h}$		
	$= 6x^2+6xh+2h^2$	A1	1.1b
	$\dfrac{dy}{dx} = \lim_{h \to 0}\left(6x^2+6xh+2h^2\right) = 6x^2$ and so at *P*, $\dfrac{dy}{dx} = 6(1)^2 = 6$	A1	2.2a
		(5)	
9 Alt 1	Let a point *Q* have *x* coordinate $1+h$, so $y_Q = 2(1+h)^3+5$	B1	1.1b
	$\left\{P(1,7), Q(1+h, 2(1+h)^3+3) \Rightarrow\right\}$		
	Gradient *PQ* = $\dfrac{2(1+h)^3+5-7}{1+h-1}$	M1	2.1
	$(1+h)^3 = 1+3h+3h^2+h^3$	B1	1.1b
	Gradient *PQ* = $\dfrac{2(1+3h+3h^2+h^3)+5-7}{1+h-1}$		
	$= \dfrac{2+6h+6h^2+2h^3+5-7}{1+h-1}$		
	$= \dfrac{6h+6h^2+2h^3}{h}$		
	$= 6+6h+2h^2$	A1	1.1b
	$\dfrac{dy}{dx} = \lim_{h \to 0}\left(6+6h+2h^2\right) = 6$	A1	2.2a
		(5)	
			(5 marks)

Question 9 Notes:	
B1:	$2(x+h)^3+5,$ seen or implied
M1:	Begins the proof by attempting to write the gradient of the chord in terms of *x* and *h*
B1:	$(x+h)^3 \to x^3+3x^2h+3xh^2+h^3,$ by expanding brackets or by using a correct binomial expansion
M1:	Correct process to obtain the gradient of the chord as $\alpha x^2+\beta xh+\gamma h^2$, $\alpha,\beta,\gamma \neq 0$
A1:	Correctly shows that the gradient of the chord is $6x^2+6xh+2h^2$ and applies a limiting argument to deduce when $y=2x^3+5,\ \frac{\mathrm{d}y}{\mathrm{d}x}=6x^2.$ E.g. $\lim_{h\to 0}\left(6x^2+6xh+2h^2\right)=6x^2.$ Finally, deduces that at the point *P*, $\frac{\mathrm{d}y}{\mathrm{d}x}=6.$
	Note: δx can be used in place of *h*
Alt 1	
B1:	Writes down the *y* coordinate of a point close to *P*. E.g. For a point *Q* with $x=1+h,\ \{y_Q\}=2(1+h)^3+5$
M1:	Begins the proof by attempting to write the gradient of the chord *PQ* in terms of *h*
B1:	$(1+h)^3 \to 1+3h+3h^2+h^3,$ by expanding brackets or by using a correct binomial expansion
M1:	Correct process to obtain the gradient of the chord *PQ* as $\alpha+\beta h+\gamma h^2$, $\alpha,\beta,\gamma \neq 0$
A1:	Correctly shows that the gradient of *PQ* is $6+6h+2h^2$ and applies a limiting argument to deduce that at the point P on $y=2x^3+5,\ \frac{\mathrm{d}y}{\mathrm{d}x}=6.$ E.g. $\lim_{h\to 0}\left(6+6h+2h^2\right)=6$
	Note: For Alt 1, δx can be used in place of *h*

11.

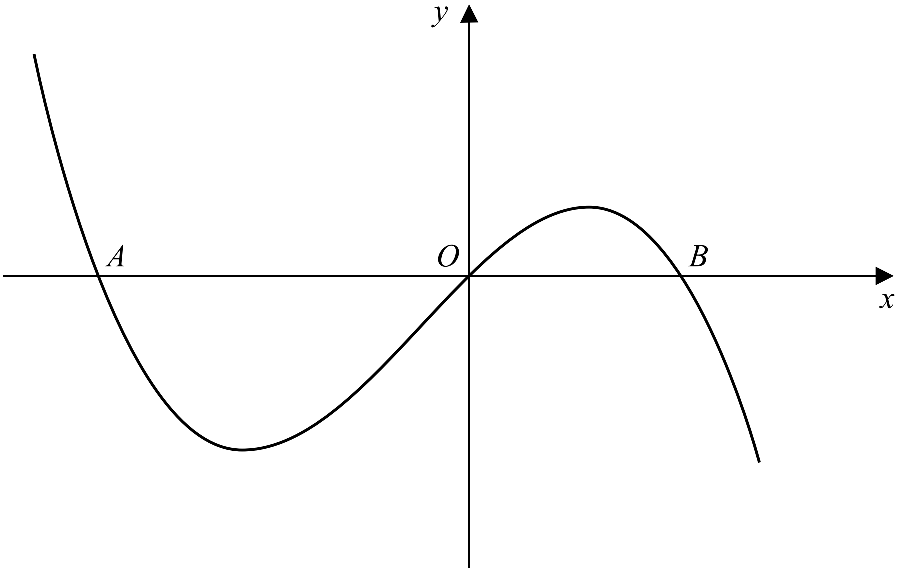

Figure 5

Figure 5 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.
The curve C crosses the x-axis at the origin, O, and at the points A and B as shown in Figure 5.

Given that

$$\mathrm{f}'(x) = k - 4x - 3x^2$$

where k is constant,

(a) show that C has a point of inflection at $x = -\frac{2}{3}$ **(3)**

Given also that the distance $AB = 4\sqrt{2}$

(b) find, showing your working, the integer value of k. **(7)**

Question	Scheme	Marks	AOs
11 (a)	$f'(x) = k - 4x - 3x^2$		
	$f''(x) = -4 - 6x = 0$	M1	1.1b
	<u>Criteria 1</u> **Either** $f''(x) = -4 - 6x = 0 \Rightarrow x = \frac{4}{-6} \Rightarrow x = -\frac{2}{3}$ **or** $f''\left(-\frac{2}{3}\right) = -4 - 6\left(-\frac{2}{3}\right) = 0$ **<u>Criteria 2</u>** **Either** • $f''(-0.7) = -4 - 6(-0.7) = 0.2 > 0$ $f''(-0.6) = -4 - 6(-0.6) = -0.4 < 0$ **or** • $f'''\left(-\frac{2}{3}\right) = -6 \neq 0$		
	At least one of Criteria 1 or Criteria 2	B1	2.4
	Both Criteria 1 and Criteria 2 **and** concludes *C* has a point of inflection at $x = -\frac{2}{3}$	A1	2.1
		(3)	
(b)	$f'(x) = k - 4x - 3x^2$, $AB = 4\sqrt{2}$		
	$f(x) = kx - 2x^2 - x^3 \{+c\}$	M1 A1	1.1b 1.1b
	$f(0) = 0$ or $(0,0) \Rightarrow c = 0 \Rightarrow f(x) = kx - 2x^2 - x^3$ $\{f(x) = 0 \Rightarrow\}\ f(x) = x(k - 2x - x^2) = 0 \Rightarrow \{x = 0,\}\ k - 2x - x^2 = 0$	A1	2.2a
	$\{x^2 + 2x - k = 0\} \Rightarrow (x+1)^2 - 1 - k = 0$, $x = ...$	M1	2.1
	$\Rightarrow x = -1 \pm \sqrt{k+1}$	A1	1.1b
	$AB = \left(-1 + \sqrt{k+1}\right) - \left(-1 - \sqrt{k+1}\right) = 4\sqrt{2} \Rightarrow k = ...$	M1	2.1
	So, $2\sqrt{k+1} = 4\sqrt{2} \Rightarrow k = 7$	A1	1.1b
		(7)	
			(10 marks)

Question 11 Notes:	
(a)	
M1:	E.g. • attempts to find $f''\left(-\frac{2}{3}\right)$ • finds $f''(x)$ and sets the result equal to 0
B1:	See scheme
A1:	See scheme
(b)	
M1:	Integrates $f'(x)$ to give $f(x) = \pm kx \pm \alpha x^2 \pm \beta x^3$, $\alpha, \beta \neq 0$ with or without the constant of integration
A1:	$f(x) = kx - 2x^2 - x^3$, with or without the constant of integration
A1:	Finds $f(x) = kx - 2x^2 - x^3 + c$, and makes some reference to $y = f(x)$ passing through the origin to deduce $c = 0$. Proceeds to produce the result $k - 2x - x^2 = 0$ or $x^2 + 2x - k = 0$
M1:	Uses a valid method to solve the quadratic equation to give *x* in terms of *k*
A1	Correct roots for *x* in terms of *k*. i.e. $x = -1 \pm \sqrt{k+1}$
M1:	Applies $AB = 4\sqrt{2}$ on $x = -1 \pm \sqrt{k+1}$ in a complete method to find $k = ...$
A1:	Finds $k = 7$ from correct solution only

8.

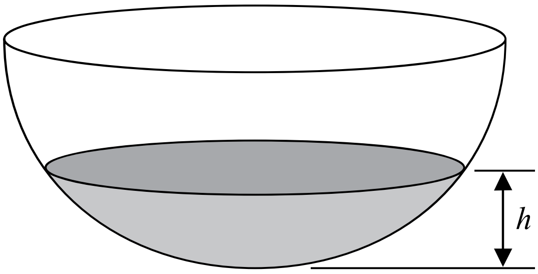

Figure 3

A bowl is modelled as a hemispherical shell as shown in Figure 3.

Initially the bowl is empty and water begins to flow into the bowl.

When the depth of the water is h cm, the volume of water, V cm^3, according to the model is given by

$$V = \frac{1}{3}\pi h^2(75 - h), \qquad 0 \leqslant h \leqslant 24$$

The flow of water into the bowl is at a constant rate of 160π cm^3 s^{-1} for $0 \leqslant h \leqslant 12$

(a) Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 10$

(5)

Given that the flow of water into the bowl is increased to a constant rate of 300π cm^3 s^{-1} for $12 < h \leqslant 24$

(b) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 20$

(2)

Question	Scheme	Marks	AOs
8(a)	$\frac{dV}{dt} = 160\pi$, $V = \frac{1}{3}\pi h^2(75 - h) = 25\pi h^2 - \frac{1}{3}\pi h^3$		
	$\frac{dV}{dh} = 50\pi h - \pi h^2$	M1	1.1b
		A1	1.1b
	$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\} \left(50\pi h - \pi h^2\right)\frac{dh}{dt} = 160\pi$	M1	3.1a
	When $h = 10$, $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\} \frac{160\pi}{50\pi(10) - \pi(10)^2}$ $\left\{= \frac{160\pi}{400\pi}\right\}$	dM1	3.4
	$\frac{dh}{dt} = 0.4$ (cm s^{-1})	A1	1.1b
		(5)	
(b)	$\frac{dh}{dt} = \frac{300\pi}{50\pi(20) - \pi(20)^2}$	M1	3.4
	$\frac{dh}{dt} = 0.5$ (cm s^{-1})	A1	1.1b
		(2)	
			(7 marks)

Question 8 Notes:

(a)

M1: Differentiates V with respect to h to give $\pm\alpha h \pm \beta h^2$, $\alpha \neq 0$, $\beta \neq 0$

A1: $50\pi h - \pi h^2$

M1: Attempts to solve the problem by applying a complete method of $\left(\text{their } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 160\pi$

M1: Depends on the previous M mark.

Substitutes $h = 10$ into their model for $\frac{dh}{dt}$ which is in the form $\frac{160\pi}{\left(\text{their } \frac{dV}{dh}\right)}$

A1: Obtains the correct answer 0.4

(b)

M1: Realises that rate for of 160π $\text{cm}^3\,\text{s}^{-1}$ for $0 \leqslant h \leqslant 12$ has no effect when the rate is increased to 300π $\text{cm}^3\,\text{s}^{-1}$ for $12 < h \leqslant 24$ and so substitutes $h = 20$ into their model for $\frac{dh}{dt}$ which is in the form $\frac{300\pi}{\left(\text{their } \frac{dV}{dh}\right)}$

A1: Obtains the correct answer 0.5

2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
2(a)	(i) $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2}=2+6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x=4$ into their $\frac{dy}{dx}=2\times4-2-12\times4^{-\frac{1}{2}}=...$	M1	1.1b
	Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x=4$ into their $\frac{d^2y}{dx^2}=2+6\times4^{-\frac{3}{2}}=(2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2}=2.75>0$ and states "hence minimum"	A1ft	2.2a
		(2)	
		(7 marks)	

(a)(i)

M1: Differentiates to $\frac{dy}{dx}=Ax+B+Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x=4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4}=...$

Alternatively substitutes $x=4$ into an equation resulting from $\frac{dy}{dx}=0$ Eg. $\frac{36}{x}=(x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓, QED for a minimal conclusion

Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe

Alt Shows that $x=4$ is a root of the resulting equation and states "hence there is a stationary point"
All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x=4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as when $x=4\Rightarrow\frac{d^2y}{dx^2}>0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x=4$ or calculates the value of y either side of $x=4$.

A1ft: For **a correct calculation**, **a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{d^2y}{dx^2}$ left and right of $x=4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".
Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

13.

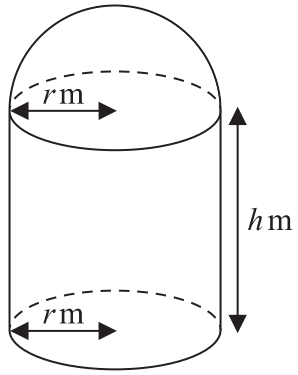

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\,\text{m}^3$.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

Question	Scheme	Marks	AOs
13 (a)	States or uses $6=\pi r^2 h+\frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow\ h=\frac{6}{\pi r^2}-\frac{2}{3}r,\ \pi h=\frac{6}{r^2}-\frac{2}{3}\pi r,\ \pi rh=\frac{6}{r}-\frac{2}{3}\pi r^2,\ rh=\frac{6}{\pi r}-\frac{2}{3}r^2$		
	$A=\pi r^2+2\pi rh+2\pi r^2\{\Rightarrow A=3\pi r^2+2\pi rh\}$		
	$A=2\pi r^2+2\pi r\left(\frac{6}{\pi r^2}-\frac{2}{3}r\right)+\pi r^2$	M1	3.1a
		A1	1.1b
	$A=3\pi r^2+\frac{12}{r}-\frac{4}{3}\pi r^2\ \Rightarrow\ A=\frac{12}{r}+\frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A=12r^{-1}+\frac{5}{3}\pi r^2\Rightarrow\right\}\ \frac{\mathrm{d}A}{\mathrm{d}r}=-12r^{-2}+\frac{10}{3}\pi r$	M1	3.4
		A1	1.1b
	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r}=0\Rightarrow\right\}\ -\frac{12}{r^2}+\frac{10}{3}\pi r=0\Rightarrow -36+10\pi r^3=0\Rightarrow r^{\pm 3}=...\left\{=\frac{18}{5\pi}\right\}$	M1	2.1
	$r=1.046447736...\Rightarrow r=1.05$ (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for r	**(4)**	
(c)	$A_{\min}=\frac{12}{(1.046...)}+\frac{5}{3}\pi(1.046...)^2$	M1	3.4
	$\{A_{\min}=17.20...\ \Rightarrow\}\ A=17\ (\mathrm{m}^2)$ or $A=$ awrt $17\ (\mathrm{m}^2)$	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question 13

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h=...$ or $\pi h=...$ or $\pi rh=...$ or $rh=...$, where $'...'=\mathrm{f}(r)$ into an expression for the surface area which is of the form $A=\lambda\pi r^2+\mu\pi rh;\ \lambda,\mu\neq 0$
A1:	Obtains correct simplified or un-simplified $\{A=\}\ 2\pi r^2+2\pi r\left(\frac{6}{\pi r^2}-\frac{2}{3}r\right)+\pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
Note:	Condone the lack of $A=...$ or $S=...$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r}+\mu r^2$ to give $\alpha r^{-2}+\beta r$; $\lambda,\mu,\alpha,\beta\neq 0$
A1:	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r}=\right\}\ -12r^{-2}+\frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ and rearranges to give $r^{\pm 3}=k,\ k\neq 0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36+10\pi r^3=0\rightarrow r=\left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r=$ awrt 1.05 (ignoring units) or $r=$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r=1.05$ (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r=\left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

<table>
<tr><th colspan="2">Notes for Question 13 Continued</th></tr>
<tr><td>Note:</td><td>Give final M0 A0 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464$</td></tr>
<tr><td>Note:</td><td>Give final M1 A1 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464... \Rightarrow r = 1.0464...$</td></tr>
<tr><td>(c)</td><td></td></tr>
<tr><td>M1:</td><td>Substitutes their $r = 1.046...$, found from solving $\frac{dA}{dr}=0$ in part (b), into the model
with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$</td></tr>
<tr><td>Note:</td><td>Give M0 for substituting their r which has been found from solving $\frac{d^2A}{dr^2}=0$ or from using $\frac{d^2A}{dr^2}$
into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$</td></tr>
<tr><td>A1ft:</td><td>$\{A=\}\,17$ or $\{A=\}$ awrt 17 (ignoring units)</td></tr>
<tr><td>Note:</td><td>You can only follow through on values of r for $0.6 \le$ their $r \le 1.3$ (and where their r has been found from solving $\frac{dA}{dr}=0$ in part (b))

<table>
<tr><th>r</th><th>A</th><th>A
(nearest integer)</th></tr>
<tr><td>0.6</td><td>21.88495…</td><td>awrt 22</td></tr>
<tr><td>0.7</td><td>19.70849…</td><td>awrt 20</td></tr>
<tr><td>0.8</td><td>18.35103…</td><td>awrt 18</td></tr>
<tr><td>0.9</td><td>17.57448…</td><td>awrt 18</td></tr>
<tr><td>1.0</td><td>17.23598…</td><td>awrt 17</td></tr>
<tr><td>1.1</td><td>17.24463…</td><td>awrt 17</td></tr>
<tr><td>1.2</td><td>17.53982…</td><td>awrt 18</td></tr>
<tr><td>1.3</td><td>18.07958…</td><td>awrt 18</td></tr>
<tr><td>1.05</td><td>17.20124…</td><td>awrt 17</td></tr>
<tr><td>1.04644…</td><td>17.20105…</td><td>awrt 17</td></tr>
</table></td></tr>
<tr><td>Note:</td><td>Give M1 A1 for $A=17\ (\text{m}^2)$ or $A=$ awrt $17\ (\text{m}^2)$ from no working</td></tr>
</table>

5. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
5(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = ...$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = ...$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0$, $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = ...$ $\left(\frac{dy}{dx}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0$, $\left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
			(7 marks)

Notes

(a)(i)
M1: $x^n \to x^{n-1}$ for at least one power of x
A1: $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$
(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}=20x^3-72x^2+84x-32$

(b)(i)

M1: Substitutes x = 1 into their $\frac{dy}{dx}$

A1: Obtains $\frac{dy}{dx}=0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{dy}{dx}=0$ and then shows $\frac{dy}{dx}=0$

Alternative:

M1: Attempts to solve $\frac{dy}{dx}=0$ by factorisation. This may be by using the factor of (x – 1) or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x=\frac{8}{5}$ and x = 1 seen as the roots.

A1: Obtains x = 1 and makes a conclusion as above

(b)(ii)

M1: Considers the value of the second derivative either side of x = 1. Do not be too concerned with the interval for the method mark.

(NB $\frac{d^2y}{dx^2}=(x-1)(60x-84)$ so may use this factorised form when considering x < 1, x > 1 for sign change of second derivative)

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "> 0, < 0". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where x < 1.4

Alternative 1 for (b)(ii)

M1: Shows that second derivative at x = 1 is zero and **then finds the third derivative at x = 1**

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "≠ 0" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{d^3y}{dx^3}\right)_{x=1}=-24$

Alternative 2 for (b)(ii)

M1: Considers the value of the first derivative either side of x = 1. Do not be too concerned with the interval for the method mark.

A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"< 0, < 0". If values are given they should be correct (but be generous with accuracy). The interval must be where x < 1.4

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f′(x)	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
f″(x)	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
f′(x)	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
f″(x)	-1.8	-2.4	-1.8	0	3	7.2	12.6

6.

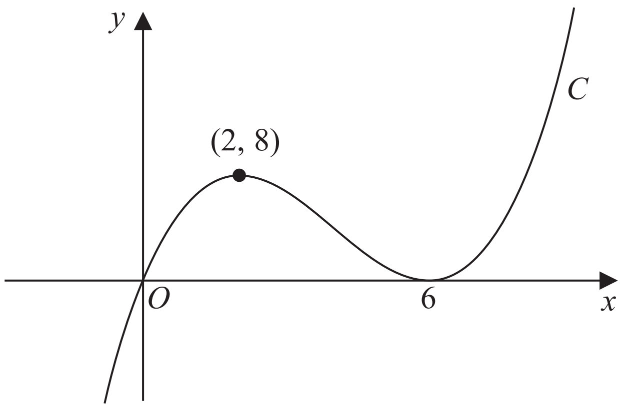

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
6 (a)	$2 < x < 6$	B1	1.1b
		(1)	
(b)	States either $k > 8$ or $k < 0$	M1	3.1a
	States e.g. $\{k : k > 8\} \cup \{k : k < 0\}$	A1	2.5
		(2)	
(c)	**Please see notes for alternatives**		
	States $y = ax(x-6)^2$ or $f(x) = ax(x-6)^2$	M1	1.1b
	Substitutes $(2,8)$ into $y = ax(x-6)^2$ and attempts to find a	dM1	3.1a
	$y = \frac{1}{4}x(x-6)^2$ or $f(x) = \frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
			(6 marks)

Notes: Watch for answers written by the question. If they are beside the question and in the answer space, the one in the answer space takes precedence

(a)

B1: Deduces $2 < x < 6$ o.e. such as $x > 2, x < 6$ $x > 2$ and $x < 6$ $\{x : x > 2\} \cap \{x : x < 6\}$ $x \in (2,6)$

Condone attempts in which set notation is incorrectly attempted but correct values can be seen or implied E.g. $\{x > 2\} \cap \{x < 6\}$ $\{x > 2, x < 6\}$. Allow just the open interval $(2,6)$

Do not allow for incorrect inequalities such as e.g. $x > 2$ or $x < 6$, $\{x : x > 2\} \cup \{x : x < 6\}$ $x \in [2,6]$

(b)

M1: Establishes a correct method by finding one of the (correct) inequalities

States either $k > 8$ (condone $k \geqslant 8$) or $k < 0$ (condone $k \leqslant 0$)

Condone for this mark $y \leftrightarrow k$ or $f(x) \leftrightarrow k$ and $8 < k < 0$

A1: Fully correct solution in the form $\{k : k > 8\} \cup \{k : k < 0\}$ or $\{k | k > 8\} \cup \{k | k < 0\}$ either way around but condone $\{k < 0\} \cup \{k > 8\}$, $\{k : k < 0 \cup k > 8\}$, $\{k < 0 \cup k > 8\}$. It is not necessary to mention $\mathbb{R}$, e.g. $\{k : k \in \mathbb{R}, k > 8\} \cup \{k : k \in \mathbb{R}, k < 0\}$ Look for $\{\ \}$ and $\cup$

Do not allow solutions not in set notation such as $k < 0$ or $k > 8$.

(c)

M1: Realises that the equation of C is of the form $y = ax(x-6)^2$. Condone with $a = 1$ for this mark.

So award for sight of $ax(x-6)^2$ even with $a = 1$

dM1: Substitutes $(2,8)$ into the form $y = ax(x-6)^2$ and attempts to find the value for a.

It is dependent upon having an equation, which the ($y =$..) may be implied, of the correct form.

A1: Uses all of the information to form a correct **equation** for C $y = \frac{1}{4}x(x-6)^2$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x(x-6)^2$ but not $C = \frac{1}{4}x(x-6)^2$.

Allow this to be written down for all 3 marks

Examples of alternative methods

....

Alternative I part (c):
Using the form $y = ax^3 + bx^2 + cx$ and setting up then solving simultaneous equations.
There are various versions of this but can be marked similarly

M1: Realises that the equation of C is of the form $y = ax^3 + bx^2 + cx$ and forms two equations in a, b and c. Condone with $a = 1$ for this mark.
Note that the form $y = ax^3 + bx^2 + cx + d$ is M0 until d is set equal to 0.
There are four equations that could be formed, only two are necessary for this mark.
Condone slips
Using $(6, 0)$ $\Rightarrow 216a+36b+6c=0$
Using $(2, 8)$ $\Rightarrow 8a+4b+2c=8$
Using $\frac{dy}{dx}=0$ at $x=2$ $\Rightarrow 12a+4b+c=0$
Using $\frac{dy}{dx}=0$ at $x=6 \Rightarrow 108a+12b+c=0$

dM1: Forms and solves three different equations, one of which must be using (2, 8) to find values for a, b and c. A calculator can be used to solve the equations

A1: Uses all of the information to form a correct equation for C $y = \frac{1}{4}x^3 - 3x^2 + 9x$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x^3 - 3x^2 + 9x$

....

Alternative II part (c)
Using the gradient and integrating

M1: Realises that the gradient of C is zero at 2 and 6 so sets $f'(x) = k(x-2)(x-6)$ oe **and** attempts to integrate. Condone with $k = 1$

dM1: Substitutes $x = 2, y = 8$ into $f(x) = k\left(...x^3 + ...x + ...\right)$ and finds a value for k

A1: Uses all of the information to form a correct equation for C $y = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$

....

4. Given that

$$y = 2x^2$$

use differentiation from first principles to show that

$$\frac{dy}{dx} = 4x$$

(3)

Question	Scheme	Marks	AOs
4	$\dfrac{2(x+h)^2-2x^2}{h}=...$	M1	2.1
	$\dfrac{2(x+h)^2-2x^2}{h}=\dfrac{4xh+2h^2}{h}$	A1	1.1b
	$\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}(4x+2h)=4x$ *	A1*	2.5
		(3)	
			(3 marks)
Notes:			

Throughout the question allow the use of δx for h or any other letter e.g. α if used consistently.
If δx is used then you can condone e.g. $\delta^2 x$ for δx^2 as well as condoning e.g. poorly formed δ's

M1: Begins the process by writing down the gradient of the chord and attempts to expand the correct bracket – you can condone "poor" squaring e.g. $(x+h)^2=x^2+h^2$.

Note that $\dfrac{2(x-h)^2-2x^2}{-h}=...$ is also a possible approach.

A1: Reaches a correct fraction oe with the x^2 terms cancelled out.

E.g. $\dfrac{4xh+2h^2}{h}$, $\dfrac{\cancel{2x^2}+4xh+2h^2-\cancel{2x^2}}{h}$, $4x+2h$

A1*: Completes the process by applying a limiting argument and deduces that $\dfrac{dy}{dx}=4x$ with no errors seen. The "$\dfrac{dy}{dx}=$" doesn't have to appear but there must be something equivalent e.g. "$f'(x)=$" or "Gradient =" which can appear anywhere in their working. If $f'(x)$ is used then there is no requirement to see $f(x)$ defined first. Condone e.g. $\dfrac{dy}{dx}\to 4x$ or $f'(x)\to 4x$.

Condone missing brackets so allow e.g. $\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}4x+2h=4x$

Do not allow $h = 0$ if there is never a reference to $h\to 0$

e.g. $\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}4x+2(0)=4x$ is acceptable

but e.g. $\dfrac{dy}{dx}=\dfrac{4xh+2h^2}{h}=4x+2h=4x+2(0)=4x$ is not if there is no $h\to 0$ seen.

The $h\to 0$ does not need to be present throughout the proof e.g. on every line.

They must reach $4x+2h$ at the end and not $\dfrac{4xh+2h^2}{h}$ (without the h's cancelled) to complete the limiting argument.

8. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(ii) the minimum cost of the journey.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(b) Prove by using $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

(c) State one limitation of this model.

(1)

Question	Scheme	Marks	AOs
8 (a)(i)	$C = \frac{1500}{v} + \frac{2v}{11} + 60 \Rightarrow \frac{dC}{dv} = -\frac{1500}{v^2} + \frac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\frac{dC}{dv} = 0 \Rightarrow v^2 = 8250$	M1	1.1b
	$\Rightarrow v = \sqrt{8250} \Rightarrow v = 90.8\ (\text{km h}^{-1})$	A1	1.1b
(ii)	For substituting their $v = 90.8$ in $C = \frac{1500}{v} + \frac{2v}{11} + 60$	M1	3.4
	Minimum cost =awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\frac{d^2C}{dv^2} = +\frac{3000}{v^3}$ at $v = 90.8$	M1	1.1b
	$\frac{d^2C}{dv^2} = (+0.004) > 0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)(i)

M1: Attempts to differentiate (deals with the powers of v correctly).

Look for an expression for $\frac{dC}{dv}$ in the form $\frac{A}{v^2} + B$

A1: $\left(\frac{dC}{dv}\right) = -\frac{1500}{v^2} + \frac{2}{11}$

A number of students are solving part (a) numerically or graphically. Allow these students to pick up the M1 A1 here from part (b) when they attempt the second derivative.

M1: Sets $\frac{dC}{dv} = 0$ (which may be implied) and proceeds to an equation of the type $v^n = k, k > 0$

Allow here equations of the type $\frac{1}{v^n} = k, k > 0$

A1: $v = \sqrt{8250}$ or $5\sqrt{330}$ awrt 90.8 (km h^{-1}) . Don't be concerned by incorrect / lack of units.

As this is a speed withhold this mark for answers such as $v = \pm\sqrt{8250}$

***** Condone $\frac{dC}{dv}$ appearing as $\frac{dy}{dx}$ or perhaps not appearing at all. Just look for the rhs.

(a)(ii)

M1: For a correct method of finding C = from their solution to $\frac{\mathrm{d}C}{\mathrm{d}v} = 0$.

Do not accept attempts using negative values of v.
Award if you see $v = .., C = ...$ where the v used is their solution to (a)(i). You do not need to check this calculation.
A1ft: Minimum cost = awrt (£) 93. Condone the omission of units
Follow through on sensible values of v. $60 < v < 110$

v	C
60	95.9
65	94.9
70	94.2
75	93.6
80	93.3
85	93.1
90	93.0
95	93.1
100	93.2
105	93.4
110	93.6

(b)

M1: Finds $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ **(**following through on their $\frac{\mathrm{d}C}{\mathrm{d}v}$ which must be of equivalent difficulty) and attempts to find its value / sign at their v

Allow a substitution of their answer to (a) (i) in their $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$

Allow an explanation into the sign of $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ from its terms (as $v > 0$)

A1ft: $\frac{\mathrm{d}^2C}{\mathrm{d}v^2} = +0.004 > 0$ hence minimum (cost). Alternatively $\frac{\mathrm{d}^2C}{\mathrm{d}v^2} = +\frac{3000}{v^3} > 0$ as $v > 0$

Requires a correct calculation or expression, a correct statement and a correct conclusion.

Follow through on their v ($v > 0$) and their $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$

* Condone $\frac{\mathrm{d}^2C}{\mathrm{d}v^2}$ appearing as $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ or not appearing at all for the M1 but for the A1 the correct notation must be used (accept notation C'').

(c)
B1: Gives a limitation of the given model, for example

- It would be impossible to drive at this speed over the whole journey
- The traffic would mean that you cannot drive at a constant speed

Any statement that implies that the speed could not be constant is acceptable. Do not accept/ignore irrelevant statements such as "air resistance" etc

10. Prove, from first principles, that the derivative of x^3 is $3x^2$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
10	Considers $\frac{(x+h)^3 - x^3}{h}$	B1	2.1
	Expands $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	M1	1.1b
	so gradient (of chord) $= \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$	A1	1.1b
	States as $h \to 0$, $3x^2 + 3xh + h^2 \to 3x^2$ so derivative $= 3x^2$ *	A1*	2.5
		(4 marks)	

Note: On e pen this is set up as B1 M1 M1 A1. We are scoring it B1 M1 A1 A1

B1: Gives the correct fraction for the gradient of the chord either $\frac{(x+h)^3 - x^3}{h}$ or $\frac{(x+\delta x)^3 - x^3}{\delta x}$

It may also be awarded for $\frac{(x+h)^3 - x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3 + ... + h^3$
This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2 + 3xh + h^2$ or exact un simplified equivalent such as $3x^2 + 2xh + xh + h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $f'(x), \frac{dy}{dx}, y'$ should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $f'(x) \underset{\lim h \to 0}{=} \frac{(x+h)^3 - x^3}{h} = 3x^2 + 3xh + h^2 = 3x^2$
- Gradient of chord $= 3x^2 + 3xh + h^2$ As $h \to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $f'(x) \underset{\lim h \to 0}{=} 3x^2 + 3xh + h^2 = 3x^2$
- Gradient of **chord** $= 3x^2 + 3xh + h^2$ when $h \to 0$ gradient of **curve** $= 3x^2$
- Do not allow $h = 0$ alone without limit being considered somewhere:
  so don't accept $h = 0 \Rightarrow f'(x) = 3x^2 + 3x \times 0 + 0^2 = 3x^2$

....

Alternative: B1: Considers $\frac{(x+h)^3 - (x-h)^3}{2h}$ M1: As above A1: $\frac{6x^2h^2 + 2h^3}{2h} = 3x^2 + h^2$

5. A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

Question	Scheme	Marks	AOs
5(a)	$x^n \to x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) = 6x - \frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \frac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$	A1	2.5
		(2)	
			(5 marks)

Notes

(a)

M1: $x^n \to x^{n-1}$ for any correct index of x. Score for $x^2 \to x$ or $x^{-1} \to x^{-2}$

Allow for unprocessed indices. $x^2 \to x^{2-1}$ oe

A1: Sight of either $6x$ or $-\frac{24}{x^2}$ which may be un simplified.

Condone an additional term e.g. + 2 for this mark

The indices now must have been processed

A1: $\frac{dy}{dx} = 6x - \frac{24}{x^2}$ or exact simplified equivalent. Eg accept $\frac{dy}{dx} = 6x^1 - 24x^{-2}$

You do not need to see the $\frac{dy}{dx}$ and you should isw after a correct simplified answer.

(b)

M1: Sets an allowable $\frac{dy}{dx}$**...**0 and proceeds to x... via an allowable intermediate equation

$\frac{dy}{dx}$ must be in the form $Ax + Bx^{-2}$ where $A, B \neq 0$

and the intermediate equation must be of the form $x^p ... q$ oe

Do not be concerned by either the processing, an equality or a different inequality.

It may be implied by $x =$ awrt 1.59

A1: $x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$ oe such as $x > 4^{\frac{1}{3}}$ or $x \geqslant 2^{\frac{2}{3}}$

1. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

Question	Scheme	Marks	AOs
1	Attempts to differentiate $x^n \rightarrow x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x - 2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Attempts to differentiate $x^n \rightarrow x^{n-1}$ seen once. Score for $x^3 \rightarrow x^2$ or $\pm 4x \rightarrow 4$ or $+5 \rightarrow 0$

A1: $\left(\frac{dy}{dx} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{dy}{dx}$. The first M must have been awarded.

Score for sight of embedded values, or sight of "$\frac{dy}{dx}$ at $x = 2$ is" or a correct follow through.
Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x - 2)$
It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

14. A curve has equation $y = g(x)$.

Given that

- $g(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = g(x)$ passes through the origin
- the curve with equation $y = g(x)$ has a stationary point at (2, 9)

(a) find $g(x)$,

(7)

(b) prove that the stationary point at (2, 9) is a maximum.

(2)

Question	Scheme	Marks	AOs
14 (a)	Deduces $g(x)=ax^3+bx^2+ax$	B1	2.2a
	Uses $(2,9)\Rightarrow 9=8a+4b+2a$ $\Rightarrow 10a+4b=9$	M1 A1	2.1 1.1b
	Uses $g'(2)=0\Rightarrow 0=12a+4b+a$ $\Rightarrow 13a+4b=0$	M1 A1	2.1 1.1b
	Solves simultaneously $\Rightarrow a,b$	dM1	1.1b
	$g(x)=-3x^3+\frac{39}{4}x^2-3x$	A1	1.1b
		(7)	
(b)	Attempts $g''(x)=-18x+\frac{39}{2}$ and substitutes $x=2$	M1	1.1b
	$g''(2)=-\frac{33}{2}<0$ hence maximum	A1	2.4
		(2)	
			(9 marks)

Notes

(a)

B1: Uses the information given to deduce that $g(x)=ax^3+bx^2+ax$. (Seen or implied)

M1: Uses the fact that $(2,9)$ lies on the curve so uses $x=2, y=9$ within a cubic function

A1: For a simplified equation in just two variables. E.g. $10a+4b=9$

M1: Differentiates their cubic to a quadratic and uses the fact that $g'(2)=0$ to obtain an equation in a and b.

A1: For a different simplified equation in two variables E.g. $13a+4b=0$

dM1: Solves simultaneously $\Rightarrow a=...,b=...$ It is dependent upon the B and both M's

A1: $g(x)=-3x^3+\frac{39}{4}x^2-3x$

(b)

M1: Attempts $g''(x)=-18x+\frac{39}{2}$ and substitutes $x=2$. Award for second derivatives of the form $g''(x)=Ax+B$ with $x=2$ substituted in.
Alternatively attempts to find the value of their $g'(x)$ or $g(x)$ either side of $x=2$ (by substituting a value for x within 0.5 either side of 2)

A1: $g''(2)=-\frac{33}{2}<0$ hence maximum. (allow embedded values but they must refer to the sign or that it is less than zero)
If $g'(x)=-9x^2+\frac{39}{2}x-3$ or $g(x)=-3x^3+\frac{39}{4}x^2-3x$ is calculated either side of $x=2$ then the values must be correct or embedded correctly (you will need to check these) they need to compare $g'(x)>0$ to the left of $x=2$ and $g'(x)<0$ to the right of $x=2$ or $g(x)<9$ to the left and $g(x)>9$ to the right of $x=2$ hence maximum.

Note If they only sketch the cubic function $g(x)=-3x^3+\frac{39}{4}x^2-3x$ then award M1A0

5.

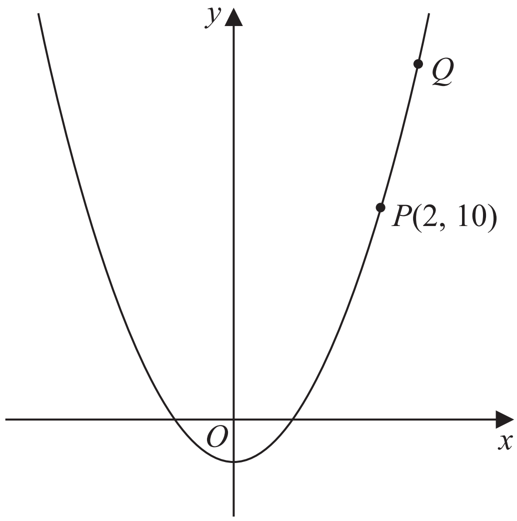

Figure 1

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line PQ, giving your answer in terms of h in simplest form.

(3)

(c) Explain briefly the relationship between part (b) and the answer to part (a).

(1)

Question	Scheme	Marks	AOs
5(a)	Attempts to find the value of $\frac{dy}{dx}$ at $x = 2$	M1	1.1b
	$\frac{dy}{dx} = 6x \Rightarrow$ gradient of tangent at P is 12	A1	1.1b
		(2)	
(b)	Gradient $PQ = \frac{3(2+h)^2 - 2 - 10}{(2+h)-2}$ oe	B1	1.1b
	$= \frac{3(2+h)^2 - 12}{(2+h)-2} = \frac{12h + 3h^2}{h}$	M1	1.1b
	$= 12 + 3h$	A1	2.1
		(3)	
(c)	Explains that as $h \to 0,\ 12 + 3h \to 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	
			(6 marks)

Notes

(a)

M1: Attempts to differentiate, allow $3x^2 - 2 \to ...x$ and substitutes $x = 2$ into their answer

A1: cso $\frac{dy}{dx} = 6x \Rightarrow$ gradient of tangent at P is 12

(b)

B1: Correct expression for the gradient of the chord seen or implied.

M1: Attempts $\frac{\delta y}{\delta x}$, condoning slips, and attempts to simplify the numerator. The denominator must be h

A1: cso $12 + 3h$

(c)

B1: Explains that as $h \to 0,\ 12 + 3h \to 12$ and states that the gradient of the chord tends to the gradient of the curve

16. The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is –3

(a) (i) show that the value of a is –2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write f(x) in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

(2)

Question	Scheme	Marks	AOs
16 (a) (i)	Uses $\frac{dy}{dx} = -3$ at $x = 2 \Rightarrow 12a + 60 - 39 = -3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a + 60 - 39 = -3 \Rightarrow 12a = -24 \Rightarrow a = -2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $10 = 8a + 60 - 78 + b$	M1	3.1a
	Subs $a = -2$ into $10 = 8a + 60 - 78 + b \Rightarrow b = 44$	A1	1.1b
		(4)	
(b)	$f(x) = -2x^3 + 15x^2 - 39x + 44 \Rightarrow f'(x) = -6x^2 + 30x - 39$	B1	1.1b
	Attempts to show that $-6x^2 + 30x - 39$ has no roots Eg. calculates $b^2 - 4ac = 30^2 - 4\times -6 \times -39 = -36$	M1	3.1a
	States that as $f'(x) \neq 0 \Rightarrow$ hence $f(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3 + 15x^2 - 39x + 44 \equiv (x-4)\left(-2x^2 + 7x - 11\right)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
			(11 marks)

Notes

(a)(i)

M1: Attempts to use $\frac{dy}{dx} = -3$ at $x = 2$ to form an equation in a. Condone slips but expect to see two of the powers reduced correctly

A1*: Correct differentiation with one correct intermediate step before $a = -2$

(a)(ii)

M1: Attempts to use the fact that $(2,10)$ lies on C by setting up an equation in a and b with $a = -2$ leading to $b = ...$

A1: $b = 44$

(b)

B1: $f'(x) = -6x^2 + 30x - 39$ oe

M1: Correct attempt to show that "$-6x^2 + 30x - 39$" has no roots.
This could involve an attempt at

- finding the numerical value of $b^2 - 4ac$
- finding the roots of $-6x^2 + 30x - 39$ using the quadratic formula (or their calculator)
- completing the square for $-6x^2 + 30x - 39$

A1*: A fully correct method with reason and conclusion. Eg as $b^2 - 4ac = -36 < 0, f'(x) \neq 0$ meaning that no stationary points exist

(c)

M1: For an attempt at division (seen or implied) Eg $-2x^3+15x^2-39x+b \equiv (x-4)\left(-2x^2 ... \pm \frac{b}{4}\right)$

A1: $(x-4)\left(-2x^2+7x-11\right)$ Sight of the quadratic with no incorrect working seen can score both marks.

(d)

See scheme. You can follow through on their value for b

12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is $355\,\text{cm}^3$

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

(4)

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

(4)

(c) Using $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}$ prove that the cost is minimised for the value of r found in part (b).

(2)

(d) Hence find the minimum value of C, giving your answer to the nearest integer.

(2)

Question	Scheme	Marks	AOs
12(a)	$V=\pi r^2h=355\Rightarrow h=\dfrac{355}{\pi r^2}$ $\left(\text{or } rh=\dfrac{355}{\pi r} \text{ or } \pi rh=\dfrac{355}{r}\right)$	B1	1.1b
	$C=0.04\left(\pi r^2+2\pi rh\right)+0.09\left(\pi r^2\right)$	M1	3.4
	$C=0.13\pi r^2+0.08\pi rh=0.13\pi r^2+0.08\pi r\left(\dfrac{355}{\pi r^2}\right)$	dM1	2.1
	$C=0.13\pi r^2+\dfrac{28.4}{r}$ *	A1*	1.1b
		(4)	
(b)	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=0.26\pi r-\dfrac{28.4}{r^2}$	M1 A1	3.4 1.1b
	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=0\Rightarrow r^3=\dfrac{28.4}{0.26\pi}\Rightarrow r=...$	M1	1.1b
	$r=\sqrt[3]{\dfrac{1420}{13\pi}}=3.26...$	A1	1.1b
		(4)	
(c)	$\left(\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right)\ 0.26\pi+\dfrac{56.8}{r^3}=0.26\pi+\dfrac{56.8}{\text{"}3.26\text{"}^3}$	M1	1.1b
	$\left(\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right)\ (2.45..)>0$ Hence minimum (cost)	A1	2.4
		(2)	
(d)	$C=0.13\pi\left(\text{"}3.26\text{"}\right)^2+\dfrac{28.4}{\text{"}3.26\text{"}}$	M1	3.4
	$(C=)13$	A1	1.1b
		(2)	
		(12 marks)	

Notes

(a)

B1: Correct expression for h or rh or πrh in terms of r. This may be implied by their later substitution.

M1: Scored for the sum of the three terms of the form $0.04...r^2$, $0.09...r^2$ and $0.04\times...rh$

The $0.04\times...rh$ may be implied by eg $0.04\times...r\times\dfrac{355}{\pi r^2}$ if h has already been replaced

dM1: Substitutes h or rh or πrh into their equation for C which must be of an allowable form (see above) to obtain an equation connecting C and r.
It is dependent on a correct expression for h or rh or πrh in terms of r

A1*: Achieves given answer with no errors. Allow Cost instead of C but they cannot just have an expression.
As a minimum you must see

- the separate equation for volume
- the two costs for the top and bottom separate before combining
- a substitution before seeing the $\frac{28.4}{r}$ term

Eg $355 = \pi r^2 h$ and $C = 0.04\pi r^2 + 0.09\pi r^2 + 0.04 \times 2\pi rh = 0.13\pi r^2 + 0.08\pi \times \left(\frac{355}{\pi r}\right)$

(b)

M1: Differentiates to obtain at least $r^{-1} \rightarrow r^{-2}$

A1: Correct derivative.

M1: Sets $\frac{dC}{dr} = 0$ and solves for r. There must have been some attempt at differentiation of the equation for C ($...r^2 \rightarrow ...r$ or $...r^{-1} \rightarrow ...r^{-2}$) Do not be concerned with the mechanics of their rearrangement and do not withhold this mark if their solution for r is negative

A1: Correct value for r. Allow exact value or awrt 3.26

(c)

M1: Finds $\frac{d^2C}{dr^2}$ at their (positive) r or considers the sign of $\frac{d^2C}{dr^2}$.

This mark can be scored as long as their second derivative is of the form $A + \frac{B}{r^3}$ where A and B are non zero

A1: Requires

- A correct $\frac{d^2C}{dr^2}$
- Either
  - deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$ (without evaluating). There must be some minimal explanation as to why it is positive.
  - substitute their positive r into $\frac{d^2C}{dr^2}$ without evaluating and deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$
  - evaluate $\frac{d^2C}{dr^2}$ (which must be awrt 2.5) and deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$

(d)

M1: Uses the model and their positive r found in (b) to find the minimum cost. Their r embedded in the expression is sufficient. May be seen in (b) but must be used in (d).

A1: $(C =)\ 13$ ignore units