Y1M11 XMQs and MS

(Total: 52 marks)

1. P3_Sample   Q6 . 6 marks - Y1M11 Variable acceleration
2. P32(AS)_2018 Q8 . 10 marks - Y1M11 Variable acceleration
3. P32(AS)_2019 Q3 . 8 marks - Y1M11 Variable acceleration
4. P32(AS)_2020 Q3 . 9 marks - Y1M11 Variable acceleration
5. P32(AS)_2021 Q2 . 10 marks - Y1M11 Variable acceleration
6. P32(AS)_2022 Q3 . 9 marks - Y1M11 Variable acceleration

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

6. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

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Question	Scheme	Marks	AOs
6	Integrate **a** w.r.t. time	M1	1.1a
	$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of **C**)	A1	1.1b
	$\mathbf{v} = \dfrac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1**ft**	1.1b
			(6 marks)

Notes:

1st M1: for integrating **a** w.r.t. time (powers of t increasing by 1)
1st A1: for a correct **v** expression without **C**
2nd A1: for a correct **v** expression including **C**
2nd M1: for putting $t = 4$ into their **v** expression
3rd M1: for finding magnitude of their **v**
3rd A1: **ft** for 100 m s^{-1}, follow through on an incorrect **v**

8. A particle, P, moves along the x-axis. At time t seconds, $t \geqslant 0$, the displacement, x metres, of P from the origin O, is given by $x = \frac{1}{2}t^2(t^2 - 2t + 1)$

(a) Find the times when P is instantaneously at rest.

(5)

(b) Find the total distance travelled by P in the time interval $0 \leqslant t \leqslant 2$

(3)

(c) Show that P will never move along the negative x-axis.

(2)

Question	Scheme	Marks	AOs
8(a)	Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure)	M1	1.1a
	$v = 2t^3 - 3t^2 + t$	A1	1.1b
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t - 1)(t - 1) = 0$	**DM**1	1.1b
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1	1.1b
	All three	A1	1.1b
		(5)	
(b)	Find x when $t = 0$, $\frac{1}{2}$, 1 and 2 : $(0, \frac{1}{32}, 0, 2)$	M1	2.1
	Distance = $\frac{1}{32} + \frac{1}{32} + 2$	M1	2.1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1	1.1b
		(3)	
(c)	$x = \frac{1}{2}t^2(t - 1)^2$	M1	3.1a
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso	2.4
		(2)	
			(10 marks)

Notes:

(a)

M1: Must have 3 terms and at least two powers going down by 1

A1: A correct expression

DM1: Dependent on first M, for equating to zero and attempting to solve a <u>cubic</u>

A1: Any two of the three values (Two correct answers can imply a correct method)

A1: The third value

(b)

M1: For attempting to find the values of x (at least two) at their t values found in (a) or at $t = 2$ or equivalent e.g. they may integrate their v and sub in at least two of their t values

M1: Using a correct strategy to combine their distances (must have at least 3 distances)

A1: $2\frac{1}{16}$ (m) oe or 2.06 or better
(c) **M1:** Identify strategy to solve the problem such as: (i) writing x as $\frac{1}{2} \times$ perfect square (ii) or using x values identified in (b). (iii) or using calculus i.e. identifying min points on x–t graph. (iv) or using x-t graph. **A1 cso :** Fully correct explanation to show that $x \geq 0$ i.e. never negative

3. A particle, P, moves along a straight line such that at time t seconds, $t \geqslant 0$, the velocity of P, $v\,\text{m}\,\text{s}^{-1}$, is modelled as

$$v = 12 + 4t - t^2$$

Find

(a) the magnitude of the acceleration of P when P is at instantaneous rest,

(5)

(b) the distance travelled by P in the interval $0 \leqslant t \leqslant 3$

(3)

Question	Scheme	Marks	AOs	Notes
3(a)	$v = 12 + 4t - t^2 = 0$ and solving	M1	3.1a	Equating v to 0 and solving the quadratic If no evidence of solving, and at least one answer wrong, M0
	$t = 6$ (or -2)	A1	1.1b	6 but allow -2 as well at this stage
	Differentiate v wrt t	M1	1.1a	For differentiation (both powers decreasing by 1)
	$(a = \frac{dv}{dt} =)\ 4 - 2t$	A1	1.1b	Cao; only need RHS
	When $t = 6$, $a = -8$; Magnitude is 8 (m s^{-2})	A1	1.1b	Substitute in $t = 6$ and get 8 (m s^{-2}) as the answer . Must be **positive.** (A0 if two answers given)
		(5)		
(b)	Integrate v wrt t	M1	3.1a	For integration (at least two powers increasing by 1)
	$(s =)12t + 2t^2 - \frac{1}{3}t^3(+C)$	A1	1.1b	Correct expression (ignore C) only need RHS Must be used in part (b)
	$t = 3$ => distance $= 45$ (m)	A1	1.1b	Correct distance. Ignore units
		(3)		
		(8 marks)		

3. A particle P moves along a straight line such that at time t seconds, $t \geqslant 0$, after leaving the point O on the line, the velocity, $v\,\text{m}\,\text{s}^{-1}$, of P is modelled as

$$v = (7 - 2t)(t + 2)$$

(a) Find the value of t at the instant when P stops accelerating.

(4)

(b) Find the distance of P from O at the instant when P changes its direction of motion.

(5)

In this question, solutions relying on calculator technology are not acceptable.

Question	Scheme	Marks	AOs
3(a)	$v = 3t - 2t^2 + 14$ and differentiate	M1	3.1a
	$a = \frac{dv}{dt} = 3 - 4t$ or $(7 - 2t) - 2(t + 2)$ using product rule	A1	1.1b
	$3 - 4t = 0$ and solve for t	M1	1.1b
	$t = \frac{3}{4}$ oe	A1	1.1b
		(4)	
3(b)	Solve problem using $v = 0$ to find a value of t $\left(t = \frac{7}{2}\right)$	M1	3.1a
	$v = 3t - 2t^2 + 14$ and integrate	M1	1.1b
	$s = \frac{3t^2}{2} - \frac{2t^3}{3} + 14t$	A1	1.1b
	Substitute $t = \frac{7}{2}$ into their s expression (M0 if using *suvat*)	M1	1.1b
	$s = \frac{931}{24} = 38\frac{19}{24} = 38.79166..(\text{m})$ Accept 39 or better	A1	1.1b
		(5)	
			(9 marks)

Notes:

(a)	M1	Multiply out and attempt to differentiate, with at least one power decreasing
	A1	Correct expression
	M1	Equate their a to 0 and solve for t
	A1	cao
(b)	M1	Uses $v = 0$ to obtain a value of t
	M1	Attempt to integrate, with at least one power increasing
	A1	Correct expression
	M1	Substitute in their value of t ,which must have come from using $v = 0$, into their s (must have integrated)
	A1	39 or better

2. A particle P moves along a straight line.

At time t seconds, the velocity $v\,\mathrm{m\,s^{-1}}$ of P is modelled as

$$v = 10t - t^2 - k \qquad t \geqslant 0$$

where k is a constant.

(a) Find the acceleration of P at time t seconds.

(2)

The particle P is instantaneously at rest when $t = 6$

(b) Find the other value of t when P is instantaneously at rest.

(4)

(c) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 6$

(4)

Question	Scheme	Marks	AOs
2(a)	Differentiate v w.r.t. t	M1	3.1a
	$a = \frac{dv}{dt} = 10 - 2t$ isw	A1	1.1b
		(2)	
2(b)	Solve problem using $v = 0$ when $t = 6$	M1	3.1a
	$0 = 10t - t^2 - 24$	A1	1.1b
	Solve quadratic oe to find other value of t	M1	1.1b
	$t = 4$	A1	1.1b
		(4)	
2(c)	Integrate v or $-v$ w.r.t. t	M1	3.1a
	$5t^2 - \frac{1}{3}t^3 - 24t$	A1	1.1b
	Total distance = $-\left[5t^2 - \frac{1}{3}t^3 - 24t\right]_0^4 + \left[5t^2 - \frac{1}{3}t^3 - 24t\right]_4^6$	M1	2.1
	$\frac{116}{3}$ (m)	A1	1.1b
		(4)	
			(10 marks)

Notes:

2a	M1	Differentiate, with both powers decreasing by 1
	A1	Correct expression
2b	M1	Put $t = 6$ **OR** use $(t-6)(t-x) = t^2 - 10t + k$ oe
	A1	Correct expression (unsimplified) for v **OR** $\boldsymbol{v} =$ $(t-6)(t-4)$
	M1	Put $v = 0$ to give quadratic in t and solve for other value of t
	A1	$t = 4$
2c	M1	Integrate, with at least two powers increasing by 1 (allow if only two terms integrated)
	A1	Correct expression
	M1	Complete method to find the total distance
	A1	Accept 39(m) or better

3. A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geqslant 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest.

(6)

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(3)

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Question	Scheme	Marks	AOs
3(a)	Differentiate s wrt t	M1	3.1a
	$(v =)\ t^2 - 5t + 6$	A1	1.1b
	Equate their v to 0 and solve	M1	1.1b
	$t = 2$ or 3	A1	1.1b
	$(a =)\ 2t - 5$	B1**ft**	2.1
	$a =$ 1 and -1 (m s^{-2}) isw (A0 if extras)	A1	1.1b
		(6)	
(b)	Attempt to find values of s for t = 2, 3 *and* 4 oe Correct values are $\left(s_2 = \frac{14}{3}, s_3 = \frac{9}{2} \text{ and } s_4 = \frac{16}{3} \right)$ Could be implied by correct values for: s_2 , $(s_3 - s_2)$ and $(s_4 - s_3)$ which are $\frac{14}{3}$, $(-\frac{1}{6})$ and $\frac{5}{6}$	**DM**1	1.1b
	Total distance travelled $= s_2 + (s_2 - s_3) + s_4 - s_3$ **OR** $s_2 - (s_3 - s_2) + s_4 - s_3$ **OR** $\left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_0^2 - \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_2^3 + \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_3^4$ **OR** $\frac{14}{3} - (-\frac{1}{6}) + \frac{5}{6}$ **OR** $s_2 + 2(s_2 - s_3) + s_4 - s_2$ $(= 2s_2 - 2s_3 + s_4)$ oe	M1	2.1
	$5\frac{2}{3}$ oe (m) Accept 5.7 or better	A1	1.1b
		(3)	
			(9 marks)

Notes:

3a	M1	Differentiate, with at least 2 powers decreasing by 1
	A1	Correct expression
	M1	Must have attempted to differentiate s to find v and be solving a 3 term quadratic
	A1	Both values needed
	B1**ft**	Follow their v (must be differentiating)

	A1	cao
3b	**DM** 1	This mark is dependent on the 2nd M1 in part (a) and their *t* values are between 0 and 4. Clear attempt to find all three *s* values (may integrate their *v* incorrectly) **N.B.** No penalty for extra values.
	M1	Complete method using their *s* values Do NOT condone sign errors.
	A1	Any equivalent fraction, 5.7 or better.
		S.C. Correct answer, with no working, scores all 3 marks, since $\int_0^4 \lvert t^2 - 5t + 6 \rvert \, dt$ entered on a calculator will give $\frac{17}{3}$