Fp1Ch4 XMQs and MS

(Total: 61 marks)

1. FP1_Specimen Q4 . 5 marks - FP1ch4 Inequalities
2. FP1_2020 Q6 . 10 marks - FP1ch4 Inequalities
3. FP1_2021 Q3 . 8 marks - FP1ch4 Inequalities
4. FP1_2022 Q7 . 8 marks - FP1ch4 Inequalities
5. FP1(AS)_2018 Q3 . 7 marks - FP1ch4 Inequalities
6. FP1(AS)_2019 Q2 . 6 marks - FP1ch4 Inequalities
7. FP1(AS)_2020 Q2 . 5 marks - FP1ch4 Inequalities
8. FP1(AS)_2021 Q1 . 6 marks - FP1ch4 Inequalities
9. FP1(AS)_2022 Q1 . 6 marks - FP1ch4 Inequalities

4. Giving your answer using set notation, use algebra to find the complete set of values of x for which

$$| x^2 - 2 | > 2x$$

(5)

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Question	Scheme	Marks	AOs
4	Solves $x^2 - 2x - 2 = 0$ or $x^2 + 2x - 2 = 0$	M1	1.1b
	Solves both $x^2 - 2x - 2 = 0$ and $x^2 + 2x - 2 = 0$	M1	3.1a
	$x = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$ and $x = \frac{-2 \pm \sqrt{12}}{2} = -1 \pm \sqrt{3}$	A1	1.1b
	Deduces the required roots are $x = 1 + \sqrt{3}$ and $x = -1 + \sqrt{3}$	M1	2.2a
	e.g. $\{x \in \mathbb{R}: x < -1 + \sqrt{3}\} \cup \{x \in \mathbb{R}: x > 1 + \sqrt{3}\}$	A1	2.5
		(5)	
		(5 marks)	

Notes:

M1: Solves either $x^2 - 2x - 2 = 0$ or $x^2 + 2x - 2 = 0$
M1: Complete strategy to identify and solve all relevant equations and gets two critical values
A1: Correct exact values for x, may be unsimplified
M1: Deduces that the larger roots are required in each case
A1: Correct set of values given and correct set notation form

6. A physics student is studying the movement of particles in an electric field. In one experiment, the distances in micrometres of two moving particles, A and B, from a fixed point O are modelled by

$$d_A = |5t - 31|$$
$$d_B = |3t^2 - 25t + 8|$$

respectively, where t is the time in seconds after motion begins.

(a) Use algebra to find the range of time for which particle A is further away from O than particle B is from O.

(8)

It was recorded that the distance of particle B from O was less than the distance of particle A from O for approximately 4 seconds.

(b) Use this information to assess the validity of the model.

(2)

Question	Scheme	Marks	AOs
6(a)	Establishes need for $\lvert 5t-31\rvert > \lvert 3t^2-25t+8\rvert$ and attempts to find all C.V.'s to form the critical region, e.g. via a sketch.	**M1**	3.1a
	$5t-31=3t^2-25t+8 \Rightarrow 3t^2-30t+39=0 \Rightarrow t=\ldots$ $t=5\pm 2\sqrt{3}$	**M1** **A1**	1.1b 3.4
	$-(5t-31)=3t^2-25t+8 \Rightarrow 3t^2-20t-23=0 \Rightarrow t=\ldots$ $t=(-1),\dfrac{23}{3}$	**M1** **A1**	2.1 3.4
	Selects "insides" $(-1<)\alpha<t<\beta,\ \gamma<\mathrm{t}<\delta$ where $\alpha<\beta<\gamma<\delta$	**M1**	2.2a
	$(-1<)0 \leqslant t<5-2\sqrt{3}$ or $\dfrac{23}{3}<t<5+2\sqrt{3}$	**A1**	1.1b
	Both regions, $0 \leqslant t<5-2\sqrt{3}$ and $\dfrac{23}{3}<t<5+2\sqrt{3}$	**A1**	2.3
	Establishes need for $\lvert 5t-31\rvert > \lvert 3t^2-25t+8\rvert$ and attempts to find all C.V.'s to form the critical region, e.g. via a sketch.	**M1**	3.1a
	$(5t-31)^2=(3t^2-25t+8)^2 \Rightarrow \ldots$ $9t^4-150t^3+648t^2-90t-897=0$	**M1** **A1**	1.1b 3.4
	Solves $9t^4-150t^3+648t^2-90t-897=0 \Rightarrow t=\ldots$ $t=5\pm 2\sqrt{3},\dfrac{23}{3},\{-1\}$	**M1** **A1**	2.1 3.4
	Selects "insides" $(-1<)\alpha<t<\beta,\ \gamma<\mathrm{t}<\delta$ where $\alpha<\beta<\gamma<\delta$	**M1**	2.2a
	$(-1<)0 \leqslant t<5-2\sqrt{3}$ or $\dfrac{23}{3}<t<5+2\sqrt{3}$ condone any variable	**A1**	1.1b
	Both regions, $0 \leqslant t<5-2\sqrt{3}$ and $\dfrac{23}{3}<t<5+2\sqrt{3}$ must be using t	**A1**	2.3
		(8)	
(b)	Time that *B* is closer to *O* than particle *A* is $5+2\sqrt{3}-\tfrac{23}{3}+5-2\sqrt{3}=\tfrac{7}{3}$ seconds.	**M1**	3.4
	This is considerably less than 4 seconds so the model does not seem appropriate.	**A1ft**	3.5a
		(2)	
			(10 marks)

Notes:

(a)
M1: Sets problem up as an inequalities problem, and forms complete strategy to solve – must see attempt at all critical values and some attempt to form at least one range from them. May be scored if algebra not used. This mark is for showing an overall awareness of the problem.

M1: Attempts to find C.V.'s for the "positives". Any valid method using algebra. Must see an attempt to find a 3TQ (oe), but allow answers from calculator once a 3TQ =0 is seen.

A1: Correct C.V.'s, both required.

M1: Attempts to find the other C.V.'s (same conditions as above)

A1: Correct C.V.'s. Need not see the negative value stated as $t > 0$ is required. (If both given, they must be correct)

M1: Selects correct critical regions, shows the idea the "insides" are needed. $(-1<)\alpha < t < \beta, \gamma < \text{t} < \delta$ where $\alpha < \beta < \gamma < \delta$ are their four critical values, possibly truncated at 0 as long as no more than 1 is negative.

A1: One correct interval. Allow with loose or strict inequalities. Allow this mark if $-1 < t < 5-2\sqrt{3}$ is given. Allow any variable for this mark.

A1: Fully correct solution. Must start at zero for the leftmost interval but accept < or $\leqslant$ here. Must be using *t*

(a) Alternative

M1: Sets problem up as an inequalities problem, and forms complete strategy to solve – must see attempt at all critical values and some attempt to form at least one ranges from them. May be scored if algebra not used. This mark is for showing an overall awareness of the problem.

M1: Attempts to find C.V.'s by squaring both sides and forming a quartic equation.

A1: Correct quartic equation

M1: Attempts to solve their quartic equation

A1: All 4 correct exact C.V.'s. Need not see the negative value stated as $t > 0$ is required.

M1: Selects correct critical regions, shows the idea the "insides" are needed. $(-1<)\alpha < t < \beta, \gamma < \text{t} < \delta$ where $\alpha < \beta < \gamma < \delta$ are their four critical values, possibly truncated at 0 as long as no more than 1 is negative.

A1: One correct interval. Allow with loose or strict inequalities. Allow this mark if $-1 < t < 5-2\sqrt{3}$ is given. Allow any variable for this mark.

A1: Fully correct solution. Must start at zero for the leftmost interval but accept < or $\leqslant$ here. Must be using *t*

(b)

M1: Uses their result from (a) to determine how long particle *B* is closer to *O* than particle *A* is.

A1: Draws a suitable conclusion for their answer to (a) – if correct in (a) it is that the model is not very suitable. Must not have a negative time.

3.

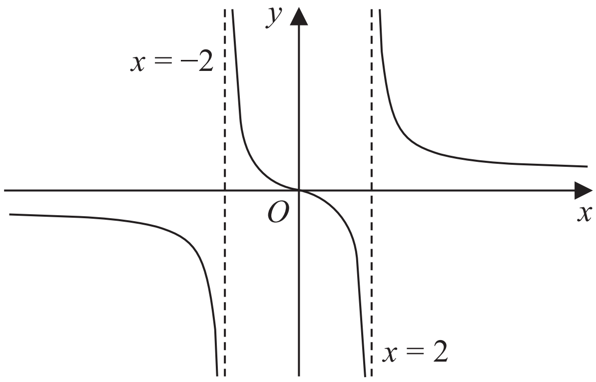


Figure 1

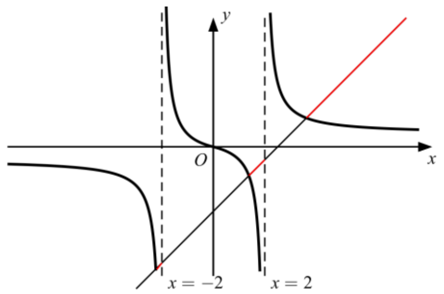
Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{|x| - 2}$$

Use algebra to determine the values of x for which

$$2x - 5 > \frac{x}{|x| - 2}$$

(8)

Question	Scheme	Marks	AOs
3	For $x<0$ need $2x-5>\frac{x}{-x-2}$ and for $x\geqslant 0$ need $2x-5>\frac{x}{x-2}$ and goes on to find the critical values for each.	**M1**	3.1a
	For $x\geqslant 0$: $2x-5=\frac{x}{x-2}\Rightarrow 2x^2-10x+10=0\Rightarrow x=...$	**M1**	1.1b
	$x=\frac{5\pm\sqrt{5}}{2}$ (oe) awrt 3.62 and awrt 1.38	**A1**	1.1b
	For $x<0$: $2x-5=\frac{x}{-x-2}\Rightarrow -2x^2+10=0\Rightarrow x=...$	**M1**	1.1b
	$x=-\sqrt{5}$ only ($\sqrt{5}$ must be rejected at some stage)	**A1**	2.3
	 Uses graph or other means to identify correct regions. Asymptotes must have been considered, but may miss the region near $x=-2$ So e.g. "$-\sqrt{5}<x<-2$" or "$\frac{5-\sqrt{5}}{2}<x<2$" or "$x>\frac{5+\sqrt{5}}{2}$"	**M1**	3.1a
	Inequality holds when $-\sqrt{5}<x<-2$ or $\frac{5-\sqrt{5}}{2}<x<2$ or $x>\frac{5+\sqrt{5}}{2}$ Accept equivalent notation, e.g $\left(-\sqrt{5},-2\right)\cup\left(\frac{5-\sqrt{5}}{2},2\right)\cup\left(\frac{5+\sqrt{5}}{2},\infty\right)$	**A1ft** **A1**	2.2a 2.5
		(8)	
			(8 marks)

Notes:

M1: Considers the two cases of $x<0$ and $x\geqslant 0$ to find critical values. Don't be concerned which side the $x=0$ case is considered part of. Allow if "=" used when considering C.V.s. This mark is for the overall strategy, so both cases must be considered, or equivalent complete longer methods.

M1: Correct method for intersection of line and curve for x positive.

A1: Line and curve intersect at $x=\frac{5\pm\sqrt{5}}{2}$

M1: Correct method for intersection of line and curve for x negative.

A1: Line and curve intersect at $x=-\sqrt{5}$ Must have rejected the positive value for this mark (though may be done later)

M1: Uses the graph (or other method) to identify at least one correct region, which must include consideration of the vertical asymptotes. Implied by two correct intervals being given for their critical values. Allow if $y=2x-5$ is added to the sketch and at least two (not necessarily correct) intervals produced as long as the points $x=\pm 2$ are excluded.

A1ft: At least one correct interval identified following through their solutions (as long as it is sensible).

A1: Fully correct solution, all three intervals given – accept alternative notations, may be just listed (no need for unions shown).

Multiplying both sides by $(x-2)^2$ or $(|x|-2)^2$ can score a maximum of M0 M1 A1 M0 A1 M1 A1ft A0

M0 M1: for multiplying through by $(x-2)^2$

$$(2x-5)(x-2)^2 > x(x-2)$$

$$(x-2)\left[(2x-5)(x-2)-x\right] > 0 \text{ leading to a value for } x$$

$$(x-2)(x^2-5x+5) > 0$$

A1: Line and curve intersect at $x = \dfrac{5 \pm \sqrt{5}}{2}$

M0A0: Not finding the point of intersection for negative x

M1 A1ft: for either "$x > \dfrac{5+\sqrt{5}}{2}$" or "$\dfrac{5-\sqrt{5}}{2} < x < 2$"

A0:

If they multiply through by $(-x-2)^2$ the other marks can be scored

7.

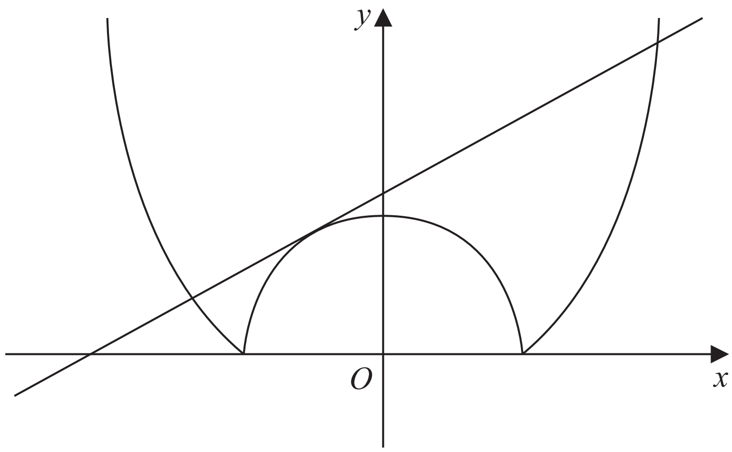


Figure 1

Figure 1 shows a sketch of the curve with equation $y = |x^2 - 8|$ and a sketch of the straight line with equation $y = mx + c$, where m and c are positive constants.

The equation

$$|x^2 - 8| = mx + c$$

has exactly 3 roots, as shown in Figure 1.

(a) Show that

$$m^2 - 4c + 32 = 0$$

(2)

Given that $c = 3m$

(b) determine the value of m and the value of c

(3)

(c) Hence solve

$$|x^2 - 8| \geqslant mx + c$$

(3)

Question	Scheme	Marks	AOs
7(a)	Considers $x^2 - 8 = -(mx + c) \Rightarrow x^2 + mx - 8 + c = 0$ and sets the discriminant = 0 $\{m^2 - 4(-8 + c) = 0\}$	M1	3.1a
	$m^2 - 4c + 32 = 0$ *	A1*	2.1
		(2)	
(b)	$c = 3m \Rightarrow m^2 - 4[3m] + 32 = 0 \Rightarrow m = \ldots (4,8)$ or $\Rightarrow \left(\frac{c}{3}\right)^2 - 4c + 32 = 0 \Rightarrow c = \ldots (12,24)$	M1	3.1a
	$m = "4" \Rightarrow c = \ldots$ or $c = "12" \Rightarrow m = \ldots$	M1	1.1b
	Deduces that $m = 4$ and $c = 12$ and no other values for *m* and *c*	A1	2.2a
		(3)	
(c)	Solves $x^2 - 8 = 'm'x + 'c'$ and $x^2 - 8 = -('m'x + 'c')$ $x^2 - 8 = 4x + 12$ and $x^2 - 8 = -(4x + 12)$	M1	2.1
	$x = 2 \pm \sqrt{24}$ o.e. and $x = -2$ (follow through $m = 8, c = 24 \Rightarrow x = 4 \pm 4\sqrt{3}, x = -4$)	A1ft	1.1b
	$x \leqslant 2 - 2\sqrt{6}, x \geqslant 2 + 2\sqrt{6}, x = -2$ (oe notation)	A1	2.2a
		(3)	
		(8 marks)	

Notes:

(a) If both case are attempted mark for the correct one.

M1: Considers $x^2 - 8 = -(mx + c)$ collects terms, finds the discriminant and sets = 0. Must see a correct equation (without modulus) initially, though allow if subsequent slips rearranging occur.

A1*: Correct result with no incorrect working seen.

(b)

M1: Substitutes $c = 3m$ into the equation (or their equation as long as it came from an attempt at using the correct equation in (a)) and solves the resulting 3TQ to find a value for *m* or *c*.

M1: Finds the corresponding value of *c* (or *m*) or solved the other 3TQ to get values for *c* (or *m*)

A1: Deduces the correct values for *m* and *c*. If two sets of values are stated this mark is not achieved until the extra set $m = 8$ and $c = 24$are rejected (correct reason for rejection is not needed).

(c)

M1: Correct method to find all the critical values (so solves both equations). Allow if both cases are included and more than three critical values are found. Allow if relevant work was seen in (b).

A1ft: Correct three critical values **only**. May be implied by their final answer. **Follow through on *m* = 8 and *c* = 24 only.** Allow if the −2 was seen in (b).

A1cao: Deduces the correct region. Accept any correct notation. Accept with "and" or "or", but not with /\

3. Use algebra to find the values of x for which

$$\frac{x}{x^2 - 2x - 3} \leqslant \frac{1}{x + 3}$$

(7)

Question	Scheme	Marks	AOs
3	$\dfrac{x}{x^2-2x-3} \le \dfrac{1}{x+3}$		
	$\dfrac{x(x+3)-(x^2-2x-3)}{(x^2-2x-3)(x+3)} \le 0$ or $x(x-3)(x+1)(x+3)^2-(x-3)^2(x+1)^2(x+3) \le 0$ or $x(x^2-2x-3)(x+3)^2-(x^2-2x-3)^2(x+3) \le 0$	M1	2.1
	$\dfrac{5x+3}{(x-3)(x+3)(x+1)}\ \{\le 0\}$ or $(x-3)(x+1)(x+3)(5x+3)\ \{\le 0\}$	M1	1.1b
		A1	1.1b
	All three critical values $-3,\ 3,\ -1$	B1	1.1b
	Critical value $-\dfrac{3}{5}$	B1ft	1.1b
	$\{x\in\mathbb{R}: -3<x<-1\}\cup\left\{x\in\mathbb{R}: -\dfrac{3}{5}\le x<3\right\}$	M1	2.2a
		A1	2.5
		(7)	
		(7 marks)	

Notes

M1:	Gathers terms on one side and puts over a common denominator, **or** multiplies by $(x+1)^2(x-3)^2(x+3)^2$ (or by the equivalent $(x^2-2x-3)^2(x+3)^2$) and gathers terms onto one side
M1:	Expands and simplifies fully the numerator **or** takes out a factor of $(x-3)(x+1)(x+3)$ (or the equivalent $(x^2-2x-3)(x+3)$) and then simplifies fully their remaining factor
A1:	$\dfrac{5x+3}{(x-3)(x+3)(x+1)}$ or $(x-3)(x+1)(x+3)(5x+3)$
B1:	Correct critical values of -3, 3 and -1 which can be implied, e.g. from their inequalities
B1ft:	Correct critical value of $-\dfrac{3}{5}$ which can be implied, e.g. from their inequalities
Note:	**B1ft:** You can follow through their fourth factor which is in the form $(ax+b)$, $a,b\ne 0$ to give C.V. $=-\dfrac{b}{a}$, if their fourth factor is not any of either $(x-3)$, $(x+3)$ or $(x+1)$
M1:	Deduces that 2 "inside" inequalities are required with critical values in ascending order
A1:	Exactly 2 correct intervals, condoning omission of the union symbol
Note:	Also accept, e.g. • $-3<x<-1,\ -\dfrac{3}{5}\le x<3$ • $(-3,\ -1),\ \left[-\dfrac{3}{5},\ 3\right)$ • $-1>x>-3,\ 3>x\ge -\dfrac{3}{5}$

Notes Continued	
Note:	Give 1st A0 for $(x^2-2x-3)(x+3)(5x+3)\ \{\le 0\}$ with no other working seen
Note:	Give 1st A1 (implied) for $(x^2-2x-3)(x+3)(5x+3)\ \{\le 0\}$ with $x=3,\ x=-1$ stated
Note:	Give 1st A0 for $\frac{5x+3}{(x^2-2x-3)(x+3)}\ \{\le 0\}$ with no other working seen
Note:	Give 1st A1 (implied) for $\frac{5x+3}{(x^2-2x-3)(x+3)}\ \{\le 0\}$ with $x=3,\ x=-1$ stated
Note:	Give 1st A0 for $\frac{5x+3}{x^3+x^2-9x-9}\ \{\le 0\}$ with no other working seen
Note:	Give 1st A1 (implied) for $\frac{5x+3}{x^3+x^2-9x-9}\ \{\le 0\}$ with $x=3,\ x=-1,\ x=-3$ stated
Note:	Allow special case final M1 for any of • $-3 < x < -1$ (condoning closed inequalities or a mixture of open and closed inequalities) • $-\frac{3}{5} \le x < 3$ (condoning closed inequalities or a mixture of open and closed inequalities) but ***do not allow*** M1 for any of • e.g. $-3 < x < -1,\ -1 < x \le -\frac{3}{5}$ ("continuing inequalities") • e.g. $-3 < x < 1,\ -\frac{3}{5} \le x < 3$ ("overlapping inequalities")
	<u>Alternative Method</u> $x(x-3)(x+1)(x+3)^2 \le (x-3)^2(x+1)^2(x+3)$ $x^5+4x^4-6x^3-36x^2-27x \le x^5-x^4-14x^3+6x^2+45x+27$ $5x^4+8x^3-42x^2-72x-27 \le 0$
Note:	$5x^4+8x^3-42x^2-72x-27 \le 0$ without any other working is M1M0A0
Note:	$5x^4+8x^3-42x^2-72x-27 \le 0 \Rightarrow x = -3, -1, 3$ is M1M1A1B1
Note:	$5x^4+8x^3-42x^2-72x-27 \le 0 \Rightarrow x = -3, -1, 3, -\frac{3}{5}$ is M1M1A1B1B1

2. A student was set the following problem.

Use algebra to find the set of values of x for which

$$\frac{x}{x-24} > \frac{1}{x+11}$$

The student's attempt at a solution is written below.

$$x(x-24)(x+11)^2 > (x+11)(x-24)^2$$

$$x(x-24)(x+11)^2 - (x+11)(x-24)^2 > 0$$

$$(x-24)(x+11)\left[x(x+11) - x - 24\right] > 0 \qquad \text{Line 3}$$

$$(x-24)(x+11)\left[x^2 + 10x - 24\right] > 0$$

$$(x-24)(x+11)(x+12)(x-2) > 0$$

$$x = 24,\ x = -11,\ x = -12,\ x = 2$$

$$\{x \in \mathbb{R} : -12 < x < -11\} \cup \{x \in \mathbb{R} : 2 < x < 24\} \qquad \text{Line 7}$$

There are errors in the student's solution.

(a) Identify the error made

(i) in line 3

(ii) in line 7

(2)

(b) Find a correct solution to this problem.

(4)

Question	Scheme	Marks	AOs
2 (a)(i)	Line 3: Allow any of either • bracketing error • -24 should be 24 in the square brackets • $x(x+11)-x-24$ should be $x(x+11)-(x-24)$ • $x(x+11)-x-24$ should be $x(x+11)-x+24$	B1	2.3
(a)(ii)	Line 7: Allow any of either • should be $\{x\in\mathbb{R}: x<-12 \text{ or } -11<x<2 \text{ or } x>24\}$ • they have found the regions where the inequality is <0 • they have reversed the inequality	B1	2.3
		(2)	
(b) **Way 1**	$(x-24)(x+11)[x(x+11)-(x-24)]>0$ $(x-24)(x+11)[x^2+10x+24]>0$	M1	1.1b
	$(x-24)(x+11)(x+6)(x+4)>0$ Critical values $x=-11, -6, -4, 24$	A1	1.1b
	$\{x\in\mathbb{R}: x<-11\}\cup\{x\in\mathbb{R}: -6<x<-4\}\cup\{x\in\mathbb{R}: x>24\}$	M1 A1	2.2a 2.5
		(4)	
(b) **Way 2**	$\frac{x}{x-24}>\frac{1}{x+11} \Rightarrow \frac{x}{x-24}-\frac{1}{x+11}>0 \Rightarrow \frac{x(x+11)-(x-24)}{(x-24)(x+11)}>0$	M1	1.1b
	$\Rightarrow \frac{x^2+10x+24}{(x-24)(x+11)}>0 \Rightarrow \frac{(x+6)(x+4)}{(x-24)(x+11)}>0$ Critical values $x=-11, -6, -4, 24$	A1	1.1b
	$\{x\in\mathbb{R}: x<-11\}\cup\{x\in\mathbb{R}: -6<x<-4\}\cup\{x\in\mathbb{R}: x>24\}$	M1 A1	2.2a 2.5
		(4)	
(b) **Way 3**	Considering $x<-11$ $\frac{x}{x-24}>\frac{1}{x+11} \Rightarrow x^2+11x>x-24 \Rightarrow x^2+10x+24>0$ gives $x<-6$ or $x>-4$. Hence $x<-11$	M1	1.1b
	Considering $-11<x<24$ $\frac{x}{x-24}>\frac{1}{x+11} \Rightarrow x^2+11x<x-24 \Rightarrow x^2+10x+24<0$ gives $-6<x<-4$. Hence $-6<x<-4$ Considering $x>24$ $\frac{x}{x-24}>\frac{1}{x+11} \Rightarrow x^2+11x>x-24 \Rightarrow x^2+10x+24>0$ gives $x<-6$ or $x>-4$. Hence $x>24$	A1	1.1b
	Overall, $\{x\in\mathbb{R}: x<-11\}\cup\{x\in\mathbb{R}: -6<x<-4\}\cup\{x\in\mathbb{R}: x>24\}$	M1 A1	2.2a 2.5
		(4)	
			(6 marks)

Notes for Question 2	
(a)(i)	
B1:	See scheme
Note:	Give B0 for contradictory reasons
(a)(ii)	**Way 1**
B1:	See scheme
Note:	Give B0 for contradictory reasons
Note:	Allow "Should be $x < -12$, $-11 < x < 2$, $x > 24$ "
Note:	Do not allow • "Should be $x < -12 \cap -11 < x < 2 \cap x > 24$ " • They have found where $x < 0$ and not where $x > 0$ • "There should be 3 inequalities and not 2 inequalities" • "The sign is the wrong way around"
(b)	**Way 1**
M1:	Uses brackets {to correct the error made on line 3}, forms a 3TQ and uses a correct method of solving a 3TQ to give $x = ...$
A1:	All four correct critical values for x
M1:	Deduces that the 2 "outsides" and the "middle interval" are required
A1:	Exactly 3 correct intervals. Their answer must be given in set notation. Accept equivalent set notation. E.g. Allow • $\{x \in \mathbb{R} : x < -11 \text{ or } -6 < x < -4 \text{ or } x > 24\}$ • $\{x < -11 \text{ or } -6 < x < -4 \text{ or } x > 24\}$ • $\{x < -11 \cup -6 < x < -4 \cup x > 24\}$ • $\mathbb{R} - ([-11, -6] \cup [-4, 24])$
Note:	Give final A0 for $\{x \in \mathbb{R} : x < -11\} \cap \{x \in \mathbb{R} : -6 < x < -4\} \cap \{x \in \mathbb{R} : x > 24\}$
Note:	Allow A1 for $\{x \in \mathbb{R} : x < -11, -6 < x < -4, x > 24\}$
(b)	**Way 2**
M1:	Gathers terms on one side and puts over a common denominator. Simplifies the numerator to $x(x+11)-(x-24)$ {and thereby corrects the error made in line 3}, forms a 3TQ and uses a correct method of solving a 3TQ to give $x = ...$
A1:	See Way 1
M1:	See Way 1
A1:	See Way 1
(b)	**Way 3**
M1:	Considers each of the intervals $x < -11$, $-11 < x < 24$, $x > 24$ separately and evaluates which parts (if any) of these regions satisfy the original inequality
A1:	Obtains a correct inequality statement for each of the intervals $x < -11$, $-11 < x < 24$, $x > 24$
M1:	See Way 1
A1:	See Way 1

2. Use algebra to determine the values of x for which

$$\frac{x+1}{2x^2+5x-3} > \frac{x}{4x^2-1}$$

(5)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
2	$\dfrac{x+1}{2x^2+5x-3} > \dfrac{x}{4x^2-1}$		
	$\dfrac{2x^2+3x+1-x^2-3x}{(2x-1)(2x+1)(x+3)} > 0$ or $(x+1)(2x-1)(2x+1)^2(x+3)-x(2x-1)(2x+1)(x+3)^2 > 0$	M1	2.1
	$\dfrac{x^2+1}{(2x-1)(2x+1)(x+3)} > 0$ or $(x+3)(2x-1)(2x+1)(x^2+1) > 0$	dM1	1.1b
	All three critical values $-3, -\dfrac{1}{2}, \dfrac{1}{2}$	A1	1.1b
	$\left\{x \in \mathbb{R} : -3 < x < -\dfrac{1}{2}\right\} \cup \left\{x \in \mathbb{R} : x > \dfrac{1}{2}\right\}$	dM1 A1	2.2a 2.5
		(5)	
			(5 marks)

Notes

M1: Gathers terms on one side and puts over a common denominator, or multiplies by $(2x + 1)^2(2x - 1)^2(x + 3)^2$ and gathers terms on one side

dM1: Expands and simplifies numerator or factorises into 4 factors. Depends on the previous method mark.

A1: Correct critical values and no "extras" but ignore any attempts to solve $x^2 + 1 = 0$ (correct or otherwise)

dM1: Deduces that 1 "inside" inequality and 1 "outside" inequality is required with critical values in ascending order. Depends on the previous method mark.

A1: Exactly 2 correct intervals, accepting equivalent notation

Special Case: Allow M1M0A0M0A0

$$\frac{x+1}{2x^2+5x-3} > \frac{x}{4x^2-1} \Rightarrow \frac{x+1}{(2x-1)(x+3)} > \frac{x}{(2x-1)(2x+1)} \Rightarrow \frac{x+1}{(x+3)} > \frac{x}{(2x+1)}$$

$\Rightarrow (x+1)(x+3)(2x+1)^2 > x(x+3)^2(2x+1)$ etc.

1. Use algebra to determine the values of x for which

$$x(x-1) > \frac{x-1}{x}$$

giving your answer in set notation.

(6)

Question	Scheme	Marks	AOs
1	$x(x-1) > \dfrac{x-1}{x}$		
	$\dfrac{x^2(x-1)-x-1}{x}>0$ or $x^3(x-1)-x(x-1)>0$	M1	2.1
	$\dfrac{(x-1)^2(x+1)}{x}>0$ or $x(x-1)^2(x+1)>0$	M1	1.1b
	Critical values 0 and 1	A1	1.1b
	All three critical values $-1,\ 0, 1$	A1	1.1b
	$\{x\in\mathbb{R}:x<-1\}\cup\{x\in\mathbb{R}:0<x<1\}\cup\{x\in\mathbb{R}:x>1\}$	M1 A1	2.2a 2.5
		(6)	
		(6 marks)	

Notes

M1: Gathers terms on one side and puts over a common denominator, or multiplies by x^2 and gathers terms on one side

M1: Factorises numerator into 3 factors or factorises into 4 factors

A1: Identifies the critical values 0 and 1

A1: All 3 correct critical values

M1: Deduces that 1 "inside" inequality and 2 "outside" inequalities are required with critical values in ascending order as shown

A1: Exactly 3 correct intervals using correct notation

Allow e.g. $\{x:x<-1\}\cup\{x:0<x<1\}\cup\{x:x>1\}$

1. Use algebra to find the set of values of x for which

$$x \geqslant \frac{2x + 15}{2x + 3}$$

(6)

Question	Scheme	Marks	AOs
1	$x=\frac{2x+15}{2x+3}\Rightarrow 2x^2+3x=2x+15\Rightarrow 2x^2+x-15=0\Rightarrow x=...$ Alternative 1: $(2x+3)^2x\geqslant(2x+3)(2x+15)\Rightarrow(2x+3)(2x^2+3x-2x-15)\geqslant 0$ $(2x+3)(x+3)(2x-5)\geqslant 0$ Alternative 2; $x-\frac{2x+15}{2x+3}\geqslant 0\Rightarrow\frac{x(2x+3)-2x-15}{2x+3}\geqslant 0\Rightarrow\frac{(x+3)(2x-5)}{2x+3}\geqslant 0$	**M1**	1.1b
	$\Rightarrow(x+3)(2x-5)=0\Rightarrow$ CVs are $-3,\ \frac{5}{2}$	**A1**	1.1b
	Also $2x+3=0\Rightarrow x=-\frac{3}{2}$ a CV	**B1**	2.3
	Hence from graph (oe) the solution set is $\left\{x\in\mathbb{R}:-3\leqslant x<-\frac{3}{2},x\geqslant\frac{5}{2}\right\}\ \left\{x:-3\leqslant x<-\frac{3}{2},x\geqslant\frac{5}{2}\right\}$	**M1** **A1** **A1**	1.1b 2.2a 2.5
		(6)	

(6 marks)

Notes:

M1: For a complete method to find the critical values other than $-\frac{3}{2}$.

Alternative 1: Multiplies by $(2x+3)^2$, collects terms onto one side and factorises into three brackets.

Alternative 2: Collects terms onto one side and combines into single fraction using a common denominator and factorises the numerator

A1: Correct critical values −3 and $\frac{5}{2}$

B1: For the critical value $-\frac{3}{2}$

M1: Selects the correct regions for their three CV's. Should include the right hand side open ended and another bounded region. CV's of a < b < c then must be of the form $a\leqslant x\leqslant b,\ x\geqslant c$ or $a<x<b,\ x>c$ the direction of the inequalities must be correct with or without strict inequalities.

A1: At least one correct interval identified. Alternatively allow for both intervals with correct end points but incorrect strict or inclusive inequalities

A1: Fully correct solution as a set – accept alternative set notations e.g. $\left[-3,-\frac{3}{2}\right)\cup\left[\frac{5}{2},\infty\right)$, but not just inequalities. Minimum use of set notation $-3\leqslant x<-\frac{3}{2}\cup x\geqslant\frac{5}{2}$

Note: Correct answer with no working scores M0 A0 but can score B1 M1 A1 A1

No working shown to factorise a cubic equation e.g.

$4x^3+8x^2-27x-45=(x+3)(2x+3)(2x-5)$ is M0 A0 but can still score B1 M1 A1 A1

A0 for $-3 \leqslant x < -\frac{3}{2} \cap x \geqslant \frac{5}{2}$ or $-3 \leqslant x < -\frac{3}{2}$ and $x \geqslant \frac{5}{2}$

Special case: If they have a repeated root final 3 marks M1 A1 A0 is possible e.g.