Fp1Ch3 XMQs and MS

(Total: 67 marks)

1. FP1_Sample Q7 . 8 marks - FP1ch3 Conic sections 2
2. FP1_Specimen Q1 . 4 marks - FP1ch3 Conic sections 2
3. FP1_Specimen Q9 . 13 marks - FP1ch3 Conic sections 2
4. FP1_2019 Q8 . 14 marks - FP1ch3 Conic sections 2
5. FP1_2020 Q5 . 7 marks - FP1ch3 Conic sections 2
6. FP1_2021 Q1 . 5 marks - FP1ch3 Conic sections 2
7. FP1_2021 Q5 . 9 marks - FP1ch2 Conic sections 1
8. FP1_2022 Q1 . 7 marks - FP1ch3 Conic sections 2

7. P and Q are two distinct points on the ellipse described by the equation $x^2 + 4y^2 = 4$

The line l passes through the point P and the point Q.

The tangent to the ellipse at P and the tangent to the ellipse at Q intersect at the point (r, s).

Show that an equation of the line l is

$$4sy + rx = 4$$

(8)

Question	Scheme	Marks	AOs
7	$x^2+4y^2=4 \Rightarrow 2x+8y\frac{\mathrm{d}y}{\mathrm{d}x}=0 \quad \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=...$	M1	3.1a
	Equation of tangent at $P(x_1,y_1)$ is $(y-y_1)\ =\ -\frac{x_1}{4y_1}(x-x_1)$	M1	3.1a
	$xx_1+4yy_1\ =\ x_1^2+4y_1^2\ =\ 4$ and at $Q(x_2,y_2)$: $xx_2+4yy_2=4$	A1	2.2a
	Intersect at (r,s) gives $rx_1+4sy_1\ =\ 4$ and $rx_2+4sy_2\ =\ 4$	B1	2.1
	Uses their previous results to find the gradient of the line *l*	M1	3.1a
	$\frac{y_2-y_1}{x_2-x_1}=\frac{-r}{4s}$	A1	1.1b
	Equation of *l* is $y-y_1=\frac{-r}{4s}(x-x_1)$	M1	2.1
	$4sy+rx\ =\ 4sy_1+rx_1\ =\ 4$ *	A1*	2.2a
		(8)	
			(8 marks)

Notes:

M1: Attempts to solve the problem by using differentiation to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Realise the need to form a general equation of the tangent at (x_1,y_1). May use alternative variables

A1: Deduces $x_1^2+4y_1^2=4$ to obtain a correct equation and deduces a correct second equation

B1: Uses (r, s) in both equations to form the two given equations or exact equivalents

M1: Uses their previous results to find the gradient of the line *l*

A1: $\frac{-r}{4s}$

M1: Formulates the line *l* with their $\frac{-r}{4s}$. Use of $y-y_1=m(x-x_1)$ or $y=mx+c$ with their gradient and an attempt to find c

A1*: Correct solution leading to $4sy+rx=4sy_1+rx_1$ with deduction that this equals 4 as (x_1,y_1) is on the ellipse. No errors seen

Answer ALL questions. Write your answers in the spaces provided.

1. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Show that an equation of the tangent to H at the point $P\,(a\sec\theta,\ b\tan\theta)$ is

$$ya\tan\theta = xb\sec\theta - ab$$

(4)

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9FM0/3A: Further Pure Mathematics 01 Mark scheme

Question	Scheme		Marks	AOs
1	$\frac{dx}{d\theta} = a\sec\theta\tan\theta,\ \frac{dy}{d\theta} = b\sec^2\theta$	$k\frac{x}{a^2} - n\frac{y}{b^2}\frac{dy}{dx} = 0$ where $k > 0$ and $n > 0$	M1	2.1
	$\frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}\left(= \frac{b\sec\theta}{a\tan\theta} = \frac{b}{a\sin\theta}\right)$	$\frac{dy}{dx} = \frac{b^2x}{a^2y} = \frac{b\sec\theta}{a\tan\theta}$	A1	1.1b
	$y - b\tan\theta = \left(\frac{b\sec\theta}{a\tan\theta}\right)(x - a\sec\theta)$		M1	1.1b
	$ya\tan\theta = xb\sec\theta - ab$*		A1*	2.1
			(4)	
			(4 marks)	

Notes:

M1: Differentiates in an attempt to find $\frac{dy}{dx}$

either differentiates x and y and divides $\frac{dy}{d\theta}$ by $\frac{dx}{d\theta}$

or achieves $k\frac{x}{a^2} - n\frac{y}{b^2}\frac{dy}{dx} = 0$ where $k > 0$ and $n > 0$

A1: Correct expression for $\frac{dy}{dx}$

M1: Uses $y - b\tan\theta =$ 'their $\frac{dy}{dx}$' $(x - a\sec\theta)$

A1*: Uses correct algebra and trig identities to achieve the correct equation of the tangent.

9.

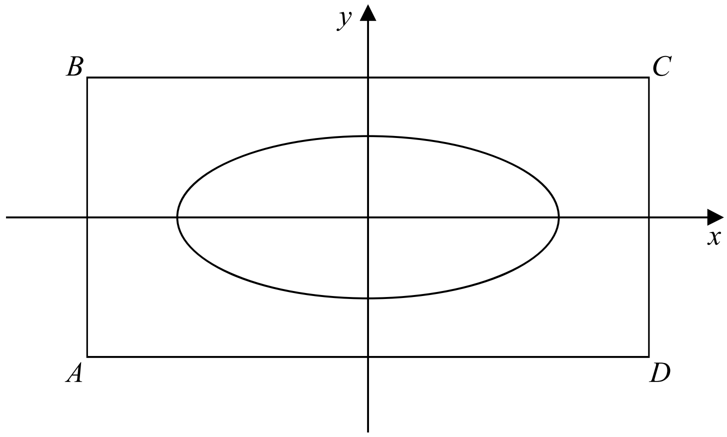

Figure 1

Figure 1 shows the plan for a rectangular garden *ABCD*.
In the middle of the garden is a large pond that may be modelled as an ellipse.
The length of the major axis of the ellipse is twice the length of the minor axis of the ellipse. The line *AB* and the line *CD* are modelled as the directrices of the ellipse.
The ellipse and the rectangle *ABCD* lie in the same horizontal plane.

Given that the length of the garden, *AD*, is $\frac{16}{3}\sqrt{3}$ metres,

(a) find an equation of the ellipse.

(6)

Two water features, modelled as particles, are to be placed in the pond. The sum of the horizontal distances from the water features to any point on the edge of the pond is constant.

(b) Find the coordinates of the points at which the water features are to be placed, according to the model.

(2)

Gnomes, modelled as particles, are to be placed on the edge of the pond. Each gnome will be exactly 2 m from a water feature.

(c) Find all the possible coordinates for the gnomes.

(5)

Question	Scheme	Marks	AOs
9(a)	$\frac{a}{e} = \frac{8}{3}\sqrt{3}$	B1	3.4
	Uses $a = 2b$ and $b^2 = a^2(1 - e^2)$ Either $b^2 = 4b^2(1 - e^2)$ or $\frac{a^2}{4} = a^2(1 - e^2)$	M1	3.1a
	Either $\frac{1}{4} = 1 - e^2$ then $e = \frac{\sqrt{3}}{2}$ so $a = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{8}{3}\sqrt{3}\right)$ or $e = \frac{3}{8\sqrt{3}} = \frac{a\sqrt{3}}{8}$ then $\frac{1}{4} = \left(1 - \frac{3a^2}{64}\right)$ leading to $a = \ldots$	M1	2.1
	$a = 4$	A1	1.1b
	$b = \frac{a}{2} = 2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	M1	1.1b
	$\frac{x^2}{16} + \frac{y^2}{4} = 1$	A1	1.1b
		(6)	
(b)	Foci $(\pm ae, 0)$, $x = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$	M1	3.1b
	Water features at $(2\sqrt{3}, 0)$ and $(-2\sqrt{3}, 0)$	A1	3.4
		(2)	
(c)	Uses $PS = ePN$, leading to $2 = \frac{\sqrt{3}}{2}PN$	M1	3.4
	$PN = \frac{4}{\sqrt{3}} = \frac{4}{3}\sqrt{3}$	A1	1.1b
	$x = \frac{8}{3}\sqrt{3} - \frac{4}{3}\sqrt{3} = \frac{4}{3}\sqrt{3}$	M1	1.1b
	$\frac{\left(\frac{4}{3}\sqrt{3}\right)^2}{16} + \frac{y^2}{4} = 1$ leading to a value for y	M1	1.1b
	Uses symmetry to find all 4 points $\left(\frac{4}{3}\sqrt{3}, \frac{2}{3}\sqrt{6}\right)$, $\left(\frac{4}{3}\sqrt{3}, -\frac{2}{3}\sqrt{6}\right)$, $\left(-\frac{4}{3}\sqrt{3}, \frac{2}{3}\sqrt{6}\right)$ and $\left(-\frac{4}{3}\sqrt{3}, -\frac{2}{3}\sqrt{6}\right)$	A1	3.4
		(5)	

	Alternative 9(c) Solves $\left(x - '2\sqrt{3}'\right)^2 + y^2 = 4$ or $\left(x + '2\sqrt{3}'\right)^2 + y^2 = 4$ with the equation of their ellipse and follow through on their foci to find a value of *x* or *y*	M1	3.4
	Any correct value of *x* or *y*	A1	1.1b
	Uses symmetry to find another value of *x* or *y* or Solves $\left(x - '2\sqrt{3}'\right)^2 + y^2 = 4$ and $\left(x + '2\sqrt{3}'\right)^2 + y^2 = 4$ with the equation of their ellipse and follow through on their foci to find a value of *x* or *y*	M1	1.1b
	Finds a complete point	M1	1.1b
	Finds all 4 points $\left(\frac{4}{3}\sqrt{3}, \frac{2}{3}\sqrt{6}\right), \left(\frac{4}{3}\sqrt{3}, -\frac{2}{3}\sqrt{6}\right), \left(-\frac{4}{3}\sqrt{3}, \frac{2}{3}\sqrt{6}\right)$ and $\left(-\frac{4}{3}\sqrt{3}, -\frac{2}{3}\sqrt{6}\right)$	A1	3.4
		(5)	
			(13 marks)

Notes:

(a)
B1: Using half the length equals the *x* coordinate of the directrix
M1: Uses $a = 2b$ and $b^2 = a^2(1 - e^2)$
M1: For a complete method to find a value for a
A1: For $a = 4$
M1: Finding the value for *b* and substituting the values of *a* and *b* into the equation of an ellipse
A1: Correct equation of the ellipse, must square out *a* and *b*.

(b)
M1: For realising that the foci for the ellipse are required and finds *x*-coordinate of focus $x = ae$
A1: Finds both coordinates for the water features

(c)
M1: Uses focus directrix property with *PS* = 2 and their value for e
A1: Correct distance for *PN*
M1: Using *x* = directrix – *PN*
M1: Substitutes value for *x* into their equation of the ellipse to find a value for *y*
A1: All 4 correct points

(c) Alternative
M1: Solves simultaneously their equation of the ellipse and a circle with centre (‘their foci’, 0) and radius 2. Finds at least one value for *x* or *y*
A1: A correct value of $x = \pm\frac{4}{3}\sqrt{3}$ or $y = \pm\frac{2}{3}\sqrt{6}$
M1: Or uses symmetry to find another values of *x* or *y*. Or solves simultaneously their equation of the ellipse and both circles with centre (‘their foci’, 0) and radius 2. Finds at least one value of *x* or *y*.
M1: Finds a complete coordinate
A1: All 4 correct points

8. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

The line l_1 is the tangent to H at the point $P(4\cosh\theta, 3\sinh\theta)$.

The line l_1 meets the x-axis at the point A.

The line l_2 is the tangent to H at the point $(4, 0)$.

The lines l_1 and l_2 meet at the point B and the midpoint of AB is the point M.

(a) Show that, as θ varies, a Cartesian equation for the locus of M is

$$y^2 = \frac{9(4 - x)}{4x} \qquad p < x < q$$

where p and q are values to be determined.

(11)

Let S be the focus of H that lies on the positive x-axis.

(b) Show that the distance from M to S is greater than 1

(3)

Question	Scheme	Marks	AOs
8(a)	$\frac{x^2}{16}-\frac{y^2}{9}=1\Rightarrow\frac{x}{8}-\frac{2yy'}{9}=0\Rightarrow y'=\frac{9x}{16y}=\frac{36\cosh\theta}{48\sinh\theta}$ or $x=4\cosh\theta, y=3\sinh\theta\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3\cosh\theta}{4\sinh\theta}$	M1	3.1a
	$y-3\sinh\theta=\frac{3\cosh\theta}{4\sinh\theta}(x-4\cosh\theta)$	M1	3.1a
	$y=0\Rightarrow x=\frac{4}{\cosh\theta}$	A1	2.2a
	line l_2 has equation $x = 4$	B1	2.2a
	$x=4\Rightarrow y-3\sinh\theta=\frac{3\cosh\theta}{4\sinh\theta}(4-4\cosh\theta)$	M1	2.1
	$y=\frac{3\cosh\theta-3}{\sinh\theta}$	A1	2.2a
	M is $\left(\frac{1}{2}\left(4+\frac{4}{\cosh\theta}\right),\frac{1}{2}\left(\frac{3\cosh\theta-3}{\sinh\theta}\right)\right)$	M1	1.1b
	$x=2+\frac{2}{\cosh\theta}\Rightarrow\cosh\theta=\frac{2}{x-2}$ $\Rightarrow y^2=\frac{9(\cosh\theta-1)^2}{4\sinh^2\theta}=\frac{9\left(\frac{2}{x-2}-1\right)^2}{4\left(\left(\frac{2}{x-2}\right)^2-1\right)}$	M1	3.1a
	$=\frac{9\left(\frac{2}{x-2}-1\right)^2}{4\left(\frac{2}{x-2}-1\right)\left(\frac{2}{x-2}+1\right)}=\frac{9\left(\frac{2}{x-2}-1\right)}{4\left(\frac{2}{x-2}+1\right)}=\frac{9(4-x)}{4x}$ *	A1*	1.1b
	Alternative for M1A1: $y^2=\frac{9(\cosh\theta-1)^2}{4\sinh^2\theta}=\frac{9(\cosh\theta-1)^2}{4(\cosh\theta-1)(\cosh\theta+1)}=\frac{9(\cosh\theta-1)}{4(\cosh\theta+1)}$ $\frac{9(4-x)}{4x}=\frac{9\left(4-2-\frac{2}{\cosh\theta}\right)}{8+\frac{8}{\cosh\theta}}=\frac{9(\cosh\theta-1)}{4(\cosh\theta+1)}\Rightarrow y^2=\frac{9(4-x)}{4x}$		
	$p = 2$ **or** $q = 4$	M1	3.1a
	$p = 2$ **and** $q = 4$	A1	1.1b
		(11)	

(b)	$b^2 = a^2\left(e^2 - 1\right) \Rightarrow 9 = 16\left(e^2 - 1\right) \Rightarrow e = \frac{5}{4}$ Focus is at $x = ae = 4 \times \frac{5}{4} = 5$	M1	1.1b
	$d >$ "5" $- 4 = \ldots$	M1	3.1a
	$d > 1$*	A1*	1.1b
		(3)	
(14 marks)			

Notes

(a)

M1: Attempts to solve the problem by using differentiation to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. Allow this mark for $\frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow \alpha x - \beta y y' = 0 \Rightarrow y' = \ldots$ or an attempt to differentiate x and y wrt θ and then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}x}{\mathrm{d}\theta} = \ldots$

M1: Correct straight line method using the coordinates of P and their gradient in terms of θ

Allow the results for the first 2 M marks to be "quoted", but any statements must be correct to score the marks.

A1: Uses $y = 0$ to deduce the correct coordinates (or value of x) for the point A. Allow in any form, simplified or unsimplified (e.g. unsimplified: $x = 4\cosh\theta - 4\tanh\theta\sinh\theta$)

B1: Deduces that the equation of l_2 is $x = 4$ (may be implied by $x = 4$ used to find y coordinate of B)

M1: Realises that $x = 4$ is all that is needed for the second line and substitutes this into the first line in order to find the point B

A1: Deduces the correct coordinates or y value for B

(e.g. unsimplified $y = \frac{3}{\tanh\theta} - \frac{3\cosh\theta}{\tanh\theta} + 3\sinh\theta$)

M1: Uses a correct method for the midpoint of AB (coordinates must be the right way round). This may be seen as the coordinates written separately e.g. $x = \ldots, y = \ldots$

M1: Having found the midpoint, identifies a correct strategy that will enable a Cartesian equation to be found. E.g. find $\cosh\theta$ in terms of x and substitutes into y or y^2 to obtain an equation in terms of y and x only. Mark positively here, so allow the mark if the candidate makes progress in eliminating θ even if there are slips in the working.

A1*: Obtains the printed answer with no errors

Alternative for the previous 2 marks: Substitutes the coordinates of their midpoint into both sides of the given equation in an attempt to show they are equal. Again mark positively but having made the substitution, some progress needs to be made in showing that both sides are equal. **For this method there must be a minimal conclusion for the A1 e.g. tick, hence true etc.**

Note that these 2 marks can also be attempted by expressing the midpoint in terms of exponentials – if you are in doubt whether to award marks seek advice from your Team Leader.

M1: For $p = 2$ **or** $q = 4$

A1: For $p = 2$ **and** $q = 4$

(b)
M1: A complete method for finding the x coordinate of the focus using a correct eccentricity formula to find a value for e and then calculating 4e
M1: Completes the problem by subtracting 4 from the x coordinate of the focus
A1*: Correct answer
If you come across correct attempts using Pythagoras to prove the result send to review.

5. The ellipse E has equation

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

The points S and S' are the foci of E.

(a) Find the coordinates of S and S'

(3)

(b) Show that for any point P on E, the triangle PSS' has constant perimeter and determine its value.

(4)

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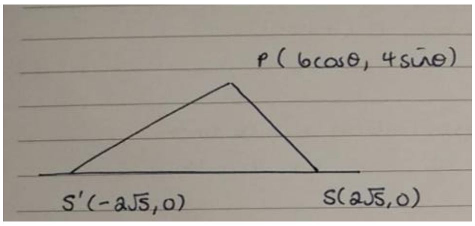
Question	Scheme	Marks	AOs
5(a)	$b^2 = a^2(1-e^2) \Rightarrow 16 = 36(1-e^2) \Rightarrow e = ...$	**M1**	1.1b
	$e^2 = \frac{20}{36}$ or $\frac{5}{9}$ or $e = \frac{\sqrt{5}}{3}$	**A1**	1.1b
	Foci are $\left(\pm 2\sqrt{5}, 0\right)$	**A1**	1.1b
		(3)	
(b)	Perimeter $= PS + PS' + SS'$ where $PS + PS' = e(PM + PM') = ...$	**M1**	3.1a
	$= e \times \frac{2a}{e} = ...$	**M1**	2.2a
	$... + 2 \times 2\sqrt{5}$	**B1ft**	1.1b
	$= 12 + 4\sqrt{5}$ hence perimeter is constant for any *P* on *E*.*	**A1***	2.1
	 Perimeter $= PS + PS' + SS'$ $PS = \sqrt{\left(2\sqrt{5} - 6\cos q\right)^2 + \left(4\sin q\right)^2} = ...\left\{6 - 2\sqrt{5}\cos q\right\}$ $PS¢ = \sqrt{\left(2\sqrt{5} + 6\cos q\right)^2 + \left(4\sin q\right)^2} = ...\left\{6 + 2\sqrt{5}\cos q\right\}$	**M1**	3.1a
	$PS + PS¢ = \left(6 - 2\sqrt{5}\cos q\right) + \left(6 + 2\sqrt{5}\cos q\right) = B$	**M1**	2.2a
	$... + 2 \times 2\sqrt{5}$	**B1ft**	1.1b
	$= 12 + 4\sqrt{5}$ hence perimeter is constant for any *P* on *E*.*	**A1***	2.1
		(4)	
		(7 marks)	

Notes:

(a)

M1: Uses $b^2 = a^2(1-e^2)$ with $a = 6$ and $b = 4$ to find a value for e^2 or e.

A1: Correct value for e or e^2

A1: Correct foci

(b)

M1: Forms a complete strategy to find the perimeter using general *P* and applies the focus directrix property to the sides *PS* and *PS'*

M1: Deduces the length of the two sides adjacent to *P* is a constant

B1ft: Uses SS' is twice their *ae* from (a)

A1*: Finds the value and makes conclusion that perimeter is constant for any *P* on *E*

Alternative

M1: Forms a complete strategy to find the perimeter using general *P*. Finds the lengths of PS and PS' using Pythagoras theorem and the general coordinate $(6\cos\theta, 4\sin\theta)$

M1: Deduces the length of the two sides adjacent to *P* is a constant, using trig identities.

B1ft: Uses SS' is twice their *ae* from (a)

A1*: Finds the value and makes conclusion that perimeter is constant for any *P* on *E*

Note: Using the property that *PS* + *PS*' = 2a both method marks may be awarded as long as a reason is given e.g. definition/property of an ellipse

1. The ellipse E has equation

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

Find

(a) the coordinates of the foci of E,

(3)

(b) the equations of the directrices of E.

(2)

Question	Scheme	Marks	AOs
1(a)	Uses $b^2 = a^2\left(1-e^2\right)$ to find a value of e look for $20 = 36\left(1-e^2\right)$	**M1**	1.1b
	$e = \frac{2}{3} \Rightarrow$ foci are $\left(\pm 6 \times \text{"their } e\text{"}, 0\right)$	**dM1**	1.1b
	Foci are $\left(\pm 4, 0\right)$	**A1**	1.1b
		(3)	
	Alternative Sets up an equation such as $2\sqrt{p^2+b^2} = 2a$ where p is the x coordinate of the foci $2\sqrt{p^2+20} = 12$	**M1**	1.1b
	Solves to find the value of p	**dM1**	1.1b
	Foci are $\left(\pm 4, 0\right)$	**A1**	1.1b
		(3)	
(b)	Directrices are $x = (\pm)\frac{6}{\text{their } e}$	**M1**	1.1b
	$x = \pm 9$ only	**A1**	1.1b
		(2)	
		(5 marks)	

Notes:

(a)

M1: Uses $b^2 = a^2(1-e^2)$ to obtain a value of e (allow if $-\frac{2}{3}$ also given)

dM1: Uses $a = 6$ and their value of e with $0 < e < 1$, to find at least one focus using $\left((\pm)ae, 0\right)$

A1: Correct foci – both required, including y coordinates.

Alternative

M1: Sets up an equation using total distance from foci to point on ellipse = 2a

dM1: Solves to find a value for the x coordinate of the foci

A1: Correct foci – both required, including y coordinates.

(b)

M1: Uses $x = (\pm)\frac{a}{e}$ with $a = 6$ and their e to attempt directrices.

A1: Correct directrices, both required and no other lines

5. The parabola C has equation

$$y^2 = 32x$$

and the hyperbola H has equation

$$\frac{x^2}{36} - \frac{y^2}{9} = 1$$

(a) Write down the equations of the asymptotes of H.

(1)

The line l_1 is normal to C and parallel to the asymptote of C with positive gradient.

The line l_2 is normal to C and parallel to the asymptote of C with negative gradient.

(b) Determine

(i) an equation for l_1

(ii) an equation for l_2

(4)

The lines l_1 and l_2 meet H at the points P and Q respectively.

(c) Find the area of the triangle OPQ, where O is the origin.

(4)

Question	Scheme	Marks	AOs
5(a)	Equations of asymptotes of H are $y=\pm\frac{1}{2}x$ oe e.g $y=\pm\frac{3}{6}x$ or $\frac{x}{6}=\pm\frac{y}{3}$	**B1**	1.1b
		(1)	
(b)	For parabola $2y\frac{dy}{dx}=32\Rightarrow\frac{dy}{dx}=...$ or $y=\sqrt{32x}\Rightarrow\frac{dy}{dx}=...x^{-\frac{1}{2}}$ or $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2a}{2at}=...$	**M1**	2.1
	Finds the gradient of the normal using $m_N=\frac{-1}{\text{their}\frac{dy}{dx}}$ $m_N=-\frac{y}{16}$ or $-t$ or $-\frac{\sqrt{x}}{2\sqrt{2}}$ so $m_N=(\pm)\frac{1}{2}\Rightarrow y=(\pm)8, x=2$	**M1**	3.1a
	Finds the equation of either l_1 or l_2 $y-"8"="\text{their } m_N"(x-"2")$ or $y-"-8"="\text{their } m_N"(x-"2")$	**M1**	1.1b
	l_1 is $y+8=\frac{1}{2}(x-2)$ and l_2 is $y-8=-\frac{1}{2}(x-2)$ oe $y=\frac{1}{2}x-9$ and $y=-\frac{1}{2}x+9$	**A1**	1.1b
		(4)	
(c)	Meet $H\Rightarrow\frac{x^2}{36}-\frac{\left(\pm\left(\frac{1}{2}x-9\right)\right)^2}{9}=1\Rightarrow\frac{x^2}{36}-\frac{\frac{1}{4}x^2-9x+81}{9}=1\Rightarrow x=...$ or $\frac{(18\pm2y)^2}{36}-\frac{y^2}{9}=1\Rightarrow\frac{81\pm18y+y^2}{9}-\frac{y^2}{9}=1\Rightarrow y=...$	**M1**	2.1
	One correct point of intersection $(10,\pm4)$	**A1**	2.2a
	Area OPQ is $\frac{1}{2}\times10\times(4-(-4))=...$ $-\frac{1}{2}\begin{vmatrix}10 & 4 & 0\\ 10 & -4 & 0\end{vmatrix}=-\frac{1}{2}\lvert-40-40\rvert$ $-\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 10 & 4 & 1\\ 10 & -4 & 1\end{vmatrix}=-\frac{1}{2}\lvert0-0+[\,10\times-4\;-\;10\times4\,]\rvert$	**dM1**	1.1b
	$=40$	**A1**	1.1b
		(4)	

(9 marks)
Notes:
(a) **B1:** Correct equations for the asymptotes of *H* seen or implied, any form and need not be simplified. **(b) Note M1 M1 A1 A1 on ePen** **M1:** A correct method to find the gradient of the parabola. **M1:** Finds the gradient of the normal and sets their normal gradient equal to their asymptote gradient to obtain at least one point on *C* where normal is parallel to an asymptote **M1:** Finds the equation of either l_1 or l_2 **A1:** Correct equation for each normal, $y+8=\frac{1}{2}(x-2)$ and $y-8=-\frac{1}{2}(x-2)$. Ignore labelling. **(c)** **M1:** Substitutes for *x* or *y* into the equation of the hyperbola and solves for their variable. **A1:** Achieves one correct coordinate $x=10$ and $y=\pm 4$ **dM1:** Dependent on previous method**.** Correct method for the area of their triangle e.g, $\frac{1}{2}\times$ their $10\times$ twice their 4 or equivalent determinant methods. **A1:** Area is 40. Correct answer only.

1. An ellipse has equation $\frac{x^2}{16} + \frac{y^2}{4} = 1$ and eccentricity e_1

A hyperbola has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and eccentricity e_2

Given that $e_1 \times e_2 = 1$

(a) show that $a^2 = 3b^2$

(4)

Given also that the coordinates of the foci of the ellipse are the same as the coordinates of the foci of the hyperbola,

(b) determine the equation of the hyperbola.

(3)

Question	Scheme	Marks	AOs
1(a)	$b^2 = a^2(1-e_1^2) \Rightarrow 4 = 16(1-e_1^2) \Rightarrow e_1^2 = \ldots$	M1	1.1b
	$e_1^2 = \frac{3}{4}$ or $e_1 = \frac{\sqrt{3}}{2}$	A1	1.1b
	E.g. $b^2 = a^2(e_2^2 - 1) = a^2\left(\frac{1}{e_1^2} - 1\right) = a^2\left(\frac{4}{3} - 1\right)$	dM1	2.1
	$\Rightarrow b^2 = \frac{1}{3}a^2 \Rightarrow a^2 = 3b^2$ * cso	A1*	1.lb
		(4)	
(b)	For the focus of the ellipse $(x =)4 \times '\frac{\sqrt{3}}{2}'$	M1	1.1b
	For focus of the hyperbola (x =) $a \times '\frac{2}{\sqrt{3}}' \Rightarrow 2\sqrt{3} = \frac{2a}{\sqrt{3}} \Rightarrow a = \ldots (= 3)$ $\Rightarrow b^2 = \frac{1}{3}a^2 = \ldots$	M1	3.1a
	$\frac{x^2}{9} - \frac{y^2}{3} = 1$ cso	A1	2.2a
		(3)	
			(7 marks)

Notes:

(a)

M1: Uses "$b^2 = a^2(1-e_1^2)$" with values for *a* and *b* to find a value for e_1 or e_1^2. They may just call it *e* and will likely use *a* and *b* before substituted, which is fine. The formula must be correct but allow slips with *a* and *b*.

A1: Correct exact value for e_1 or e_1^2. Note: allow M1A1 here if the relevant work is seen in (b).

dM1: Dependent on previous method mark. Uses $e_1 \times e_2 = 1$ with their e_1 or e_1^2 to find an expression between *a* and *b*. May find an expression for ${e_2}^{(2)}$ and apply $e_1 \times e_2 = 1$ directly or may first substitute as per scheme. Any full method.

SC: Allow M0A0dM1A0 if $b^2 = a^2(1-e_1)$ and $b^2 = a^2(e_2 - 1)$ are used in an otherwise correct process.

A1*: Achieves $a^2 = 3b^2$ with at least one intermediate unsimplified equation in *a* and *b* cso

(b)

M1: Uses/implies *x* coordinate of focus for the ellipse is $4 \times$ their e_1

M1: For a full process to find values for *a* and *b* or their squares. E.g. for focus of hyperbola *x* = $a \times$ their $e_2 = \frac{a}{e_1}$ sets equal to $4e_1$ and solves for *a* then attempting to use $a^2 = 3b^2$ to obtain b^2(or b). Other methods are possible.

A1: Deduces the correct equation for the hyperbola.