Fp1Ch2 XMQs and MS

(Total: 86 marks)

1. FP1_Sample   Q5 . 9 marks - FP1ch2 Conic sections 1
2. FP1_2019   Q4 . 8 marks - FP1ch2 Conic sections 1
3. FP1_2020   Q7 . 14 marks - FP1ch2 Conic sections 1
4. FP1_2022   Q5 . 9 marks - FP1ch2 Conic sections 1
5. FP1(AS)_2018 Q5 . 10 marks - FP1ch2 Conic sections 1
6. FP1(AS)_2019 Q5 . 10 marks - FP1ch2 Conic sections 1
7. FP1(AS)_2020 Q4 . 7 marks - FP1ch2 Conic sections 1
8. FP1(AS)_2021 Q5 . 10 marks - FP1ch2 Conic sections 1
9. FP1(AS)_2022 Q4 . 9 marks - FP1ch2 Conic sections 1

5. The normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ passes through the parabola again at the point $Q(aq^2, 2aq)$.

The line OP is perpendicular to the line OQ, where O is the origin.

Prove that $p^2 = 2$

(9)

Question	Scheme	Marks	AOs
5	$y^2 = 4ax \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$	M1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{y} \Rightarrow$ Gradient of normal is $\frac{-y}{2a} = -p$	A1	1.1b
	Equation of normal is : $y - 2ap = -p(x - ap^2)$	M1	1.1b
	Normal passes through $Q(aq^2, 2aq)$ so $2aq + apq^2 = 2ap + ap^3$	M1	3.1a
	Grad OP × Grad $OQ = -1 \Rightarrow \frac{2ap}{ap^2}\frac{2aq}{aq^2} = -1$	M1	2.1
	$q = \frac{-4}{p}$	A1	1.1b
	$2a\left(\frac{-4}{p}\right) + ap\left(\frac{16}{p^2}\right) = 2ap + ap^3 \Rightarrow p^4 + 2p^2 - 8 = 0$	M1	2.1
	$(p^2 - 2)(p^2 + 4) = 0 \Rightarrow p^2 = \ldots$	M1	1.1b
	Hence (as $p^2 + 4 \neq 0$), $p^2 = 2$*	A1*	1.1b
		(9)	
	Alternative 1 First three marks as above and then as follows	M1 A1 M1	2.1 1.1b 1.1b
	Solves $y^2 = 4ax$ and their normal simultaneously to find, in terms of a and p, either $x_Q\left(= ap^2 + 4a + \frac{4a}{p^2}\right)$ or $y_Q\left(= -2ap - \frac{4a}{p}\right)$	M1	3.1a
	Finds the second coordinate of Q in terms of a and p	M1	1.1b
	Both $x_Q = ap^2 + 4a + \frac{4a}{p^2}$ and $y_Q = -2ap - \frac{4a}{p}$	A1	1.1b
	Grad OP × Grad $OQ = -1 \Rightarrow \frac{2ap}{ap^2} \times \frac{-2ap - \frac{4a}{p}}{ap^2 + 4a + \frac{4a}{p^2}} = -1$	M1	2.1
	Simplifies expression and solves: $4p^2 + 8 = p^4 + 4p^2 + 4$ $\Rightarrow p^4 - 4 = 0 \Rightarrow (p^2 - 2)(p^2 + 2) = 0 \Rightarrow p^2 = \ldots$	M1	2.1
	Hence (as $p^2 + 2 \neq 0$), $p^2 = 2$*	A1*	1.1b
		(9)	

Question	Scheme	Marks	AOs
5	**Alternative 2** First three marks as above and then as follows	M1 A1 M1	2.1 1.1b 1.1b
	Solves $y^2 = 4ax$ and their normal simultaneously to find, in terms of a and p, either $x_Q\left(= ap^2 + 4a + \frac{4a}{p^2}\right)$ or $y_Q\left(= -2ap - \frac{4a}{p}\right)$	M1	3.1a
	Forms a relationship between p and q from their first coordinate: **either** $y_Q = 2a\left(-p - \frac{2}{p}\right) \Rightarrow q = -p - \frac{2}{p}$ **or** $x_Q = a\left(p + \frac{2}{p}\right)^2 \Rightarrow q = \pm\left(p + \frac{2}{p}\right)$	M1	2.1
	$q = -p - \frac{2}{p}$ (if x coordinate used the correct root must be clearly identified before this mark is awarded)	A1	1.1b
	Grad OP × Grad $OQ = -1 \Rightarrow \frac{2ap}{ap^2} \times \frac{2aq}{aq^2} = -1 \left(\Rightarrow q = -\frac{4}{p}\right)$	M1	2.1
	Sets $q = -p - \frac{2}{p} = -\frac{4}{p}$ and solves to give $p^2 = ...$	M1	1.1b
	Hence $\left(\text{as } q = p + \frac{2}{p} = -\frac{4}{p} \text{ gives no solution}\right)$, $p^2 = 2$ (only)*	A1*	1.1b
		(9)	
		(9 marks)	

Notes:

(a)

M1: Begins proof by differentiating and using the perpendicularity condition at point P in order to find the equation of the normal

A1: Correct gradient of normal, $-p$ only

M1: Use of $y - y_1 = m(x - x_1)$. Accept use of $y = mx + c$ and then substitute to find c

M1: Substitute coordinates of Q into their equation to find an equation relating p and q

M1: Use of $m_1 m_2 = -1$ with OP and OQ to form a second equation relating p and q

A1: $q = \frac{-4}{p}$ only

M1: Solves the simultaneous equations and cancels a from their results to obtain a quadratic equation in p^2 only

M1: Attempts to solve their quadratic in p^2. Usual rules

A1*: Correct solution leading to given answer stated. No errors seen

Question 5 notes continued:

Alternative 1:

M1A1M1: As main scheme

M1: Solves $y^2 = 4ax$ and their normal simultaneously to find one of the coordinates for Q in terms of a and p as shown

M1: Finds the second coordinate of Q in terms of a and p

A1: Both coordinates correct in terms of a and p

M1: Use of $m_1m_2 = -1$ with OP and OQ. i.e. $\frac{2ap}{ap^2} \times \frac{\text{their } y_Q}{\text{their } x_Q} = -1$ with coordinates of P and their expressions for x_Q and y_Q

M1: Cancels the a's, simplifies to a quadratic in p^2 and solves the quadratic. Usual rules

A1*: Correct solution leading to the given answer stated. No errors seen

Alternative 2:

M1A1M1: As main scheme

M1: Solves $y^2 = 4ax$ and their normal simultaneously to find one of the coordinates for Q in terms of a and p as shown

M1: Uses their coordinate to form a relationship between p and q. Allow $q = \left(p + \frac{2}{p}\right)$ for this mark

A1: For $q = -p - \frac{2}{p}$. If the x coordinate was used to find q then consideration of the negative root is needed for this mark. Allow for $q = \pm\left(p + \frac{2}{p}\right)$

M1: Use of $m_1m_2 = -1$ with OP and OQ to form a second equation relating p and q only

M1: Equates expressions for q and attempts to solve to give $p^2 = ...$

A1*: Correct solution leading to the given answer stated. No errors seen. If x coordinate used, invalid solution must be rejected

4. The parabola C has equation

$$y^2 = 16x$$

The distinct points $P(p^2, 4p)$ and $Q(q^2, 4q)$ lie on C, where $p \neq 0$, $q \neq 0$

The tangent to C at P and the tangent to C at Q meet at the point $R(-28, 6)$.

Show that the area of triangle PQR is 1331

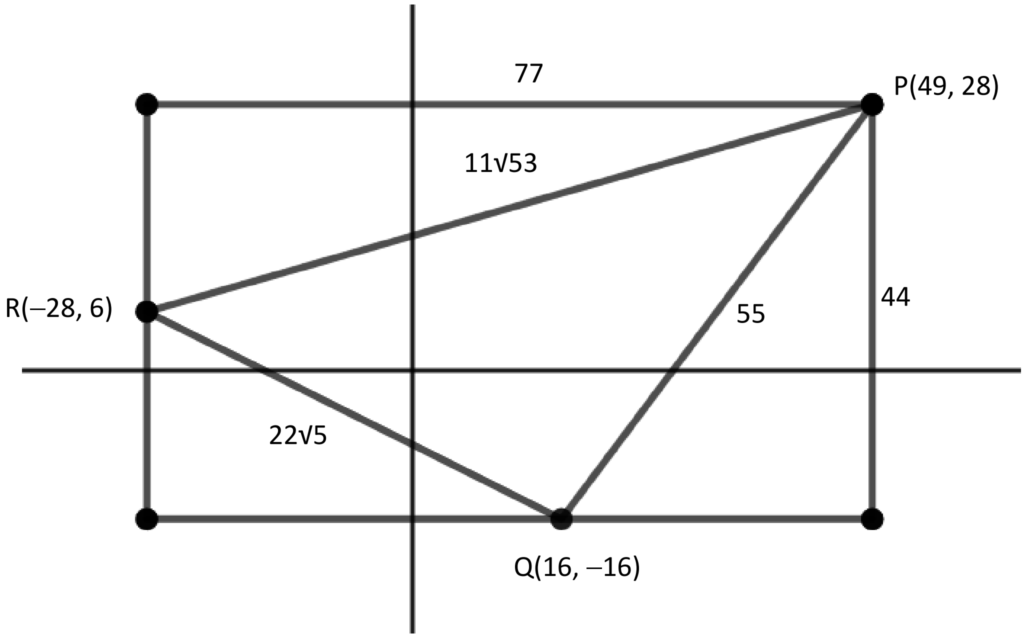
(8)

uestion	Scheme	Marks	AOs
4	$y^2 = 16x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 16 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{y} = \frac{8}{4p}$ Requires $\alpha y\frac{\mathrm{d}y}{\mathrm{d}x} = \beta \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(p\,or\,q)$ $y^2 = 16x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2x^{-\frac{1}{2}} = 2\left(p^2\right)^{-\frac{1}{2}}$ Requires $\frac{\mathrm{d}y}{\mathrm{d}x} = \alpha x^{-\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(p\,or\,q)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}p}\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{4}{2p}$ Requires $\frac{\mathrm{d}y}{\mathrm{d}x} = their\frac{\mathrm{d}y}{\mathrm{d}p} \div their\frac{\mathrm{d}x}{\mathrm{d}p} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(p\,or\,q)$	M1	3.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{4p} \Rightarrow y - 4p = \frac{2}{p}\left(x - p^2\right)$ or $y - 4q = \frac{2}{q}\left(x - q^2\right)$	M1 A1	3.1a 1.1b
	Using $x = -28$ and $y = 6$, $6p = -56 + 2p^2 \Rightarrow p = ...$	M1	3.1a
	Alternative for 3rd Method mark: $py = 2x + 2p^2,\ qy = 2x + 2q^2 \Rightarrow x = pq, y = 2(p+q)$ Using $x = -28$ and $y = 6 \Rightarrow p(\text{or } q) = ...$		
	p (or q) $= -4, 7$	A1	1.1b
	$(16, -16),\ (49, 28)$	A1	2.2a
	Way 1 $\frac{1}{2}\begin{vmatrix} -28 & 16 & 49 & -28 \\ 6 & -16 & 28 & 6 \end{vmatrix} = \frac{1}{2}\left\lvert 448 + 448 + 294 - 96 + 784 + 784 \right\rvert$ **Way 2** $77 \times 44 - \frac{1}{2} \times 44 \times 22 - \frac{1}{2} \times 77 \times 22 - \frac{1}{2} \times 44 \times 33$ **Way 3** $\frac{1}{2}22\sqrt{5} \times 11\sqrt{53}\sin\left(\cos^{-1}\left(\frac{\left(11\sqrt{53}\right)^2 + \left(22\sqrt{5}\right)^2 - 55^2}{2 \times 11\sqrt{53} \times 22\sqrt{5}}\right)\right)$ NB angle at R is 42.5 (1dp) **Way 4** $\frac{1}{2}55 \times 11\sqrt{53}\sin\left(\cos^{-1}\left(\frac{\left(11\sqrt{53}\right)^2 + 55^2 - \left(22\sqrt{5}\right)^2}{2 \times 11\sqrt{53} \times 55}\right)\right)$ NB angle at P is 37.2 (1dp) **Way 5** $\frac{1}{2}55 \times 22\sqrt{5}\sin\left(\cos^{-1}\left(\frac{\left(22\sqrt{5}\right)^2 + 55^2 - \left(11\sqrt{53}\right)^2}{2 \times 22\sqrt{5} \times 55}\right)\right)$ NB angle at Q is 100.3 (1dp)	M1	3.1a

	Way 6 $S=\frac{55+22\sqrt{5}+11\sqrt{53}}{2}\Rightarrow A=\sqrt{S(S-55)\left(S-22\sqrt{5}\right)\left(S-11\sqrt{53}\right)}$		
	Way 7 Line PR $y-28=\frac{28-6}{49+28}(x-49), x=16\Rightarrow y=\frac{130}{7}$ $A=\frac{1}{2}\times\frac{242}{7}(28+16)+\frac{1}{2}\times\frac{242}{7}(49-16)$		
	Way 8 $\frac{1}{2}\lvert RP\times QP\rvert=\frac{1}{2}\left\lvert\begin{pmatrix}77\\22\end{pmatrix}\times\begin{pmatrix}33\\44\end{pmatrix}\right\rvert=\frac{1}{2}(2662)$ For such methods, a minimum of e.g. $\frac{1}{2}(2662)$ must be seen		
	= 1331 (units^2)*	A1*	1.1b
		(8)	
			(8 marks)

Notes

M1: Attempts to solve the problem by using differentiation to obtain an expression for $\frac{dy}{dx}$ in terms of p or q.
See scheme for requirements for this mark depending on the method chosen.
(Can be implied by a correct expression)
M1: Correct straight line method to find the equation of the tangent using P or Q.
If using $y = mx + c$, must reach as far as $c = \ldots$
A1: Obtains a correct general tangent at P or Q or both
Note that if a correct tangent equation is quoted, the first 3 marks are available
M1: Uses $x = -28$ and $y = 6$ **with the values correctly placed** in one of their tangent equations **and** attempts to solve the resulting 3TQ to obtain 2 values for p (or q).
An **alternative approach** for this mark is to obtain equations for both tangents and solve simultaneously to obtain the coordinates for the intersection and then to use $x = -28$ and $y = 6$ to find values for p and q. Note that a calculator may be used for the simultaneous equations but answers must be correct for their equations if no working is shown.
A1: Correct values
A1: Deduces the correct coordinates of P and Q
M1: Completes the problem by using a suitable complete correct method for finding the area of PQR – See examples – there will be others – in general, score M1 for a correct triangle area method for their values
A1*: Correct area. Allow this mark even if the candidate reverts to decimals within their solution, providing all the working is correct.



Generally, using midpoints of sides is unlikely to be successful, however, the line from R to the midpoint of PQ is horizontal so this is a correct approach:

Midpoint: $\left(\frac{49+16}{2},\frac{28-16}{2}\right)=\left(\frac{65}{2},\,6\right)\Rightarrow \text{Area}=\frac{1}{2}\left(\frac{65}{2}+28\right)\times 44=1331$

7. The points $P(9p^2, 18p)$ and $Q(9q^2, 18q)$, $p \neq q$, lie on the parabola C with equation

$$y^2 = 36x$$

The line l passes through the points P and Q

(a) Show that an equation for the line l is

$$(p + q)y = 2(x + 9pq)$$

(3)

The normal to C at P and the normal to C at Q meet at the point A.

(b) Show that the coordinates of A are

$$\left(9(p^2 + q^2 + pq + 2),\ -9pq(p + q)\right)$$

(7)

Given that the points P and Q vary such that l always passes through the point (12, 0)

(c) find, in the form $y^2 = \mathrm{f}(x)$, an equation for the locus of A, giving $\mathrm{f}(x)$ in simplest form.

(4)

Question	Scheme	Marks	AOs
7(a)	Gradient of $PQ = \frac{18q-18p}{9q^2-9p^2} = \frac{2}{p+q}$ $18p = 9p^2m + c$, $18q = 9q^2m + c \Rightarrow 18p - 18q = 9p^2m - 9q^2m$ $\Rightarrow m = \frac{2}{p+q}$	B1	2.2a
	Equation of l is $y-18p = "\frac{2}{p+q}"(x-9p^2)$ $18p = 9p^2\left(\frac{2}{p+q}\right) + c$ $\Rightarrow c = \ldots$	M1	1.1b
	Leading to $(p+q)y = 2(x+9pq)$ *	A1*	2.1
		(3)	
(b)	Complete method to find equation of both normals and attempts to solve simultaneously	M1	3.1a
	E.g. $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 36 \Rightarrow m_T = \frac{36}{36p} \Rightarrow m_N = -p$	B1	1.1b
	Normal at P is $y-18p = -p(x-9p^2)$ or normal at Q is $y-18q = -q(x-9q^2)$ (oe)	M1	2.1
	Both normals correct $y-18p = -p(x-9p^2)$ or $y = -px+9p^3+18p$ (o.e.) $y-18q = -q(x-9q^2)$ or $y = -qx+9q^3+18q$ (o.e.)	A1	2.2a
	E.g. $18p - px + 9p^3 - 18q = -qx + 9q^3 \Rightarrow x = \ldots$	M1	1.1b
	Need to show that $\left(9p^3 - 9q^3 + 18p - 18q\right) = \left(9p^2 + q^2 + pq + 2\right)\left(p - q\right)$ or $p^3 - q^3 = \left(p^2 + pq + q^2\right)\left(p - q\right)$ Leading to $x_A = 9(p^2+q^2+pq+2)$ *	A1*	2.2a
	$y = -9p(p^2+q^2+pq+2)+9p^3+18p = -9p^2q-9pq^2$ Leading to $y_A = -9pq(p+q)$ *	A1*	2.2a
		(7)	
(c)	(12,0) on $l \Rightarrow pq = -\frac{4}{3}$ (oe)	B1	3.1a
	Hence $x_A = 9\left(p^2+q^2+\frac{2}{3}\right)$ and $y_A = 12(p+q)$	M1	1.1b
	$y^2 = 144(p^2+q^2+2pq) = 144\left(\frac{x}{9}-\frac{2}{3}+2\left(-\frac{4}{3}\right)\right)$	M1	3.1a
	$y^2 = 16\left(x-30\right)$ or $y^2 = 16x-480$	A1	1.1b
		(4)	

(14 marks)
Notes:
(a) **B1:** Deduces gradient is $\frac{2}{p+q}$. May be implied by correct simplification of equation if the unsimplified form is used to start with. **M1:** Correct method for the equation of the line (gradient need not be simplified/correct for this method, as long as it is clearly an attempt at the gradient). **A1*:** Completes to the correct equation with no errors seen.
(b) **M1:** A correct overall method – must find both normals and attempt to solve simultaneously. They do not need to reach x = or y = as long as they have eliminated one variable. **B1:** Correct gradient of normal found from any correct method or just stated. **M1:** A full correct method to find the equation of at least one of the normals with justification of the gradient shown. **A1:** Deduces equation of the second normal – so both correct. **M1:** Solves the two normal equations simultaneously leading to either x = … or y = … **A1*:** Need to show that $(9p^3 - 9q^3 + 18p - 18q) = (9p^2 + q^2 + pq + 2)(p - q)$ leading to correct x coordinate with no errors seen. This could by long division or factorising. **A1*:** Correct y coordinate with no errors seen.
(c) **B1:** Uses the condition on l to establish the relationship between p and q **M1:** Uses their relationship between p and q to simplify the expressions **M1:** Any complete method for relating x and y independently of p and q **A1:** $y^2 = 16(x-30)$ or $y^2 = 16x - 480$

5. The rectangular hyperbola H has equation $xy = 36$

(a) Use calculus to show that the equation of the tangent to H at the point $P\left(6t, \frac{6}{t}\right)$ is

$$yt^2 + x = 12t$$

(3)

The point $Q\left(12t, \frac{3}{t}\right)$ also lies on H.

(b) Find the equation of the tangent to H at the point Q.

(2)

The tangent at P and the tangent at Q meet at the point R.

(c) Show that as t varies the locus of R is also a rectangular hyperbola.

(4)

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Question	Scheme	Marks	AOs
5 (a)	$y + x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} = \frac{-\frac{6}{t}}{6t} = -\frac{1}{t^2}$ or $y = \frac{36}{x} \Rightarrow \frac{dy}{dx} = -\frac{36}{x^2} = -\frac{36}{(6t)^2} = -\frac{1}{t^2}$ or $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-6t^{-2}}{6} = -\frac{1}{t^2}$	M1	1.1b
	$y - \frac{6}{t} = "-\frac{1}{t^2}"(x - 6t)$	M1	1.1b
	$yt^2 + x = 12t$*	A1 *	2.1
		(3)	
(b)	$\frac{dy}{dx} = -\frac{y}{x} = \frac{-\frac{3}{t}}{12t} = -\frac{1}{4t^2}$ and $y - \frac{3}{t} = '-\frac{1}{4t^2}'(x - 12t)$	M1	1.1b
	$y - \frac{3}{t} = -\frac{1}{4t^2}(x - 12t)$ o.e such as $4yt^2 + x = 24t$	A1	1.1b
		(2)	
(c)	E.g. $\left.\begin{array}{r}4yt^2 + x = 24t\\ yt^2 + x = 12t\end{array}\right\} 3yt^2 = 12t \Rightarrow y = \ldots$ and $x = 12t - yt^2 = \ldots$	M1	2.1
	$x = 8t$ and $y = \frac{4}{t}$	A1	1.1b
	$xy = \ldots$	dM1	1.1b
	$xy = 32$ hence **rectangular hyperbola**	A1	2.4
		(4)	
			(9 marks)

Notes:

(a)
M1: Differentiates implicitly, directly or parametrically to find the gradient at the point *P* in terms of *t*. Allow slips in coefficients, as long as method is clear.
M1: Finds the equation of the tangent at the point *P* using their gradient (not reciprocal etc). If using $y = mx + c$ must proceed to find *c* and substitute back in to equation.
A1*: The correct equation for the tangent at the point *P* from correct working.

(b)
M1: Finds the new gradient (any method as above) and proceeds to find the equation of the tangent at the point *Q*. Alternatively replaces *t* by 2*t* in the answer to (a).
A1: Correct equation - any form, need not be simplified and isw after a correct equation.

(c)
M1: Solves their simultaneous equations to find both the *x* and *y* coordinate for the point *R*.
A1: Correct point of intersection, it does not need to be simplified.
dM1: Dependent on the first method mark. Multiplies *x* by *y* to reach a constant.
A1: Shows that $xy = 32$ and hence **rectangular hyperbola**

5. The rectangular hyperbola H has equation $xy = c^2$, where c is a non-zero constant.

The point $P\left(cp, \frac{c}{p}\right)$, where $p \neq 0$, lies on H.

(a) Use calculus to show that an equation of the normal to H at P is

$$p^3x - py + c(1 - p^4) = 0$$

(4)

The normal to H at the point P meets H again at the point Q.

(b) Find the coordinates of the midpoint of PQ in terms of c and p, simplifying your answer where possible.

(6)

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Question	Scheme	Marks	AOs
5	$H: xy = c^2,\ c \neq 0;\ P\left(cp, \frac{c}{p}\right),\ p \neq 0,$ lies on H		
(a)	**Either** $y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2}$ or $-\frac{c^2}{x^2}$ **or** $xy = c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$ **or** $x = ct,\ y = \frac{c}{t} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}.\frac{\mathrm{d}t}{\mathrm{d}x} = -\left(\frac{c}{t^2}\right)\left(\frac{1}{c}\right)$ **and** so, at $P\left(cp, \frac{c}{p}\right),\ m_T = -\frac{1}{p^2}$	M1	2.1
	So, $m_N = p^2$	A1	2.2a
	$y - \frac{c}{p} = "p^2"(x - cp)$ **or** $\frac{c}{p} = "p^2"(cp) + b \Rightarrow y = "p^2"x +$ their b	M1	1.1b
	correct algebra leading to $p^3x - py + c(1 - p^4) = 0$ *	A1*	2.1
		(4)	
(b)	$y = \frac{c^2}{x} \Rightarrow p^3x - p\frac{c^2}{x} + c(1 - p^4) = 0$ **or** $x = \frac{c^2}{y} \Rightarrow p^3\frac{c^2}{y} - py + c(1 - p^4) = 0$	M1	3.1a
	$p^3x^2 + c(1 - p^4)x - c^2p = 0$ **or** $py^2 - c(1 - p^4)y - c^2p^3 = 0$	A1	1.1b
	$(x - cp)(p^3x + c) = 0 \Rightarrow x = ...$ **or** $\left(y - \frac{c}{p}\right)\left(yp + cp^4\right) = 0 \Rightarrow y = ...$	M1	3.1a
	$x = -\frac{c}{p^3}$ **and** $y = -cp^3$ **or** $\{Q\}\left(-\frac{c}{p^3}, -cp^3\right)$	A1	1.1b
	Midpoint is $\left(\frac{1}{2}\left(cp - \frac{c}{p^3}\right), \frac{1}{2}\left(\frac{c}{p} - cp^3\right)\right)$	M1	1.1b
		A1	1.1b
		(6)	
(b) Alt 1	Let Q be $\left(cq, \frac{c}{q}\right)$, so $p^3cq - p\frac{c}{q} + c(1 - p^4) = 0$	M1	3.1a
	$p^3cq^2 - pc + c(1 - p^4)q = 0 \Rightarrow p^3q^2 + (1 - p^4)q - p = 0$	A1	1.1b
	$(q - p)(p^3q + 1) = 0 \Rightarrow q = ...$	M1	3.1a
	$\{Q\}\left(-\frac{c}{p^3}, -cp^3\right)$ **or** $x = -\frac{c}{p^3}$ **and** $y = -cp^3$	A1	1.1b
			(10 marks)

Notes	
(a)	
M1:	Starts the process of establishing the gradient of the normal by differentiating $xy = c^2$ • to give $\frac{dy}{dx} = \pm k\, x^{-2}$; $k \neq 0$, or • by the product rule to give $\pm x\frac{dy}{dx} \pm y$, or • by parametric differentiation to give $\left(\text{their } \frac{dy}{dt}\right) \times \frac{1}{\left(\text{their } \frac{dx}{dt}\right)}$, condoning $t \equiv p$ **and** attempt to use $P\left(cp, \frac{c}{p}\right)$ to write down the gradient of the tangent to the curve in terms of p
A1:	Deduces the correct normal gradient p^2 from their tangent gradient which is found using calculus
M1:	Correct straight line method for an equation of a normal where $m_N\ (\neq m_T)$ is found by using calculus. **Note:** m_N must be a function of p for this mark
A1*:	Obtains $p^3x - py + c(1 - p^4) = 0$, by correct solution only
(b)	
M1:	Substitutes $y = \frac{c^2}{x}$ or $x = \frac{c^2}{y}$ into the printed equation to obtain an equation in x, c and p only **or** in y, c and p only
A1:	Obtains a 3TQ equation in x or a 3TQ equation in y
Note:	E.g. $p^3x^2 + cx - cp^4x = c^2p$ or $py^2 = cy - cp^4y + c^2p^3$ are acceptable for the 1st A mark
M1:	Recognises that one solution of the quadratic equation is already known and uses a correct factorisation method of solving a 3TQ to give either $x = ...$ or $y = ...$ Alternatively applies a correct quadratic formula method for solving a 3TQ
A1:	Correct coordinates for Q, which can be simplified or un-simplified Allow $x = -\frac{c}{p^3}$ and $y = -cp^3$
M1:	Uses $\left(cp, \frac{c}{p}\right)$ and their (x_Q, y_Q) and applies $\left(\frac{cp + \text{their } x_Q}{2}, \frac{\frac{c}{p} + \text{their } y_Q}{2}\right)$ to give (x_M, y_M), where x_M and y_M are both in terms of c and p only
A1:	Correct coordinates $\left(\frac{1}{2}\left(cp - \frac{c}{p^3}\right), \frac{1}{2}\left(\frac{c}{p} - cp^3\right)\right)$. Condone $\left(\frac{cp - \frac{c}{p^3}}{2}, \frac{\frac{c}{p} - cp^3}{2}\right)$
Note:	Condone $x = \frac{1}{2}\left(cp - \frac{c}{p^3}\right)$ **and** $y = \frac{1}{2}\left(\frac{c}{p} - cp^3\right)$ for the final A mark
Note:	You can apply isw after correctly stated coordinates for the midpoint of P and Q

Notes Continued	
(b)	
Alt 1	**(*for the first 4 marks*)**
M1:	Substitutes $x = cq$ and $y = \frac{c}{q}$ into the printed equation to obtain an equation in only *p*, *c* and *q*
A1:	Eliminates *c* and obtains a correct quadratic equation in *q*
Note:	E.g. $p^3q^2 + q - p^4q = p$ is acceptable for the 1st A mark
M1:	Recognises that one solution of the quadratic equation is already known and uses a correct factorisation method of solving a 3TQ to give $q = ...$ Alternatively applies a correct quadratic formula method for solving a 3TQ in *q*
A1:	Correct coordinates for *Q*, which can be simplified or un-simplified Allow $x = -\frac{c}{p^3}$ and $y = -cp^3$

5.

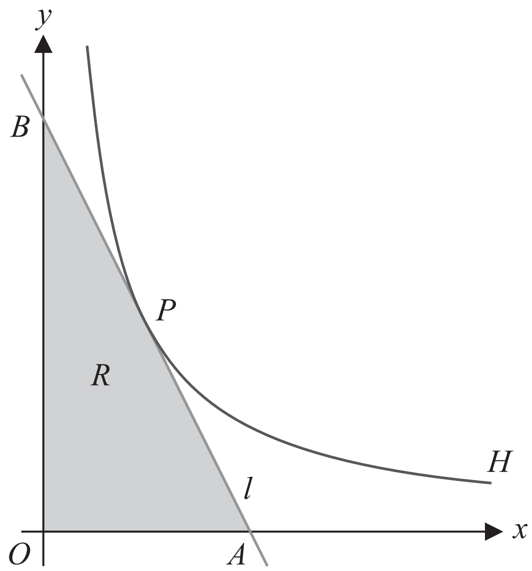

Figure 2

Figure 2 shows a sketch of part of the rectangular hyperbola H with equation

$$xy = c^2 \qquad x > 0$$

where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$ lies on H.

The line l is the tangent to H at the point P.

The line l crosses the x-axis at the point A and crosses the y-axis at the point B.

The region R, shown shaded in Figure 2, is bounded by the x-axis, the y-axis and the line l.

Given that the length OB is twice the length of OA, where O is the origin, and that the area of R is 32, find the exact coordinates of the point P.

(10)

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Question	Scheme	Marks	AOs
5	$H: xy = c^2, c > 0$; $P\left(ct, \frac{c}{t}\right)$ lies on H; $OB = 2OA$; Area$(OAB) = 32$		
Way 1	**Either** $y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{dy}{dx} = -c^2x^{-2}$ or $-\frac{c^2}{x^2}$ **or** $xy = c^2 \Rightarrow x\frac{dy}{dx} + y = 0$ **or** $x = cp,\ y = \frac{c}{p} \Rightarrow \frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = -\left(\frac{c}{p^2}\right)\left(\frac{1}{c}\right)$; condone $t \equiv p$ **and** so, at $P\left(ct, \frac{c}{t}\right)$, $m_T = -\frac{1}{t^2}$	M1	3.1a
	$y - \frac{c}{t} = "-\frac{1}{t^2}"(x - ct)$	M1	1.1b
	or $\frac{c}{t} = "-\frac{1}{t^2}"(ct) + b \Rightarrow y = "-\frac{1}{t^2}"x + \text{their } b \Rightarrow y = -\frac{1}{t^2}x + \frac{2c}{t}$	A1	1.1b
	$y = 0 \Rightarrow x = 2ct\ \{\Rightarrow x_A = 2ct\}$, $x = 0 \Rightarrow y = \frac{2c}{t}\ \left\{\Rightarrow y_B = \frac{2c}{t}\right\}$	M1	1.1b
		A1	1.1b
	$\{OB = 2OA \Rightarrow\}\ \frac{2c}{t} = 2(2ct) \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}\ t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707	A1	1.1b
	$\{\text{Area }(OAB) = 32 \Rightarrow\}\ \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 32 \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	2.1
	Deduces the ***numerical*** value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
Way 2	Same requirement as the 1st M mark in Way 1	M1	3.1a
	e.g. $\left\{t = \frac{1}{\sqrt{2}} \Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right) \Rightarrow\right\}\ y - \sqrt{2}c = -2\left(x - \frac{c}{\sqrt{2}}\right)$	M1	1.1b
	using $m_T = -2$ and their P which has been found by a correct method	A1	1.1b
	$y = 0 \Rightarrow x = \sqrt{2}c\ \{\Rightarrow x_A = \sqrt{2}c\}$, $x = 0 \Rightarrow y = 2\sqrt{2}c\ \{\Rightarrow y_B = 2\sqrt{2}c\}$	M1	1.1b
		A1	1.1b
	$\{OB = 2OA \Rightarrow\}\ m_T = -2$ and their $m_T = -\frac{1}{t^2} = -2 \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}\ t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707 $\left\{\Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right)\right\}$	A1	1.1b
	$\{\text{Area }(OAB) = 32 \Rightarrow\}\ \frac{1}{2}\sqrt{2}c\left(2\sqrt{2}c\right) = 32 \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	2.1
	Deduces the ***numerical*** value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
			(10 marks)

Question	Scheme	Marks	AOs
5	$H: xy = c^2,\ c > 0;\ P\left(ct, \frac{c}{t}\right)$ lies on H; $OB = 2OA$; Area$(OAB) = 32$		
Way 3	Same requirement as the 1st M mark in Way 1	M1	3.1a
	e.g. $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$	M1	1.1b
	using $m_T = -2$ and either their $A(4\sqrt{2}, 0)$ or their $B(0, 8\sqrt{2})$ which have been found by a correct method	A1	1.1b
	$\{$Area $(OAB) = 32,\ OB = 2OA \Rightarrow\}\ \frac{1}{2}(x)(2x) = 32 \Rightarrow x = ...$	M1	2.1
	$x = 4\sqrt{2}\ \{\Rightarrow x_A = 4\sqrt{2}\}$ or $y = 8\sqrt{2}\ \{\Rightarrow y_B = 8\sqrt{2}\}$	A1	1.1b
	$\{OB = 2OA \Rightarrow\}\ m_T = -2$ and their $m_T = -\frac{1}{t^2} = -2 \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}\ t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707 $\left\{\Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right)\right\}$	A1	1.1b
	$\sqrt{2}c - 8\sqrt{2} = -2\left(\frac{c}{\sqrt{2}} - 0\right) \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	1.1b
	Deduces the ***numerical*** value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
Way 4	Complete process substituting their $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$ **into** $xy = c^2$ and applying $b^2 - 4ac = 0$ to their resulting $2x^2 - 8\sqrt{2}x + c^2 = 0$	M1	3.1a
	e.g. $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$	M1	1.1b
	using $m_T = -2$ and either their $A(4\sqrt{2}, 0)$ or their $B(0, 8\sqrt{2})$ which have been found by a correct method	A1	1.1b
	$\{$Area $(OAB) = 32,\ OB = 2OA \Rightarrow\}\ \frac{1}{2}(x)(2x) = 32 \Rightarrow x = ...$	M1	2.1
	$x = 4\sqrt{2}\ \{\Rightarrow x_A = 4\sqrt{2}\}$ or $y = 8\sqrt{2}\ \{\Rightarrow y_B = 8\sqrt{2}\}$	A1	1.1b
	dependent on 2nd M mark $\{xy = c^2 \Rightarrow\}\ x(-2x + 8\sqrt{2}) = c^2\ \{\Rightarrow 2x^2 - 8\sqrt{2}x + c^2 = 0\}$	dM1	2.1
	or $\{xy = c^2 \Rightarrow\}\ \frac{1}{2}\left(8\sqrt{2} - y\right)y = c^2\ \{\Rightarrow y^2 - 8\sqrt{2}y + 2c^2 = 0\}$	A1	1.1b
	$\{b^2 - 4ac = 0 \Rightarrow\}\ (8\sqrt{2})^2 - 4(2)(c^2) = 0 \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	1.1b
	Deduces the ***numerical*** value x_P and y_P using their value of c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
Note:	**For the final M1 mark in Way 1, Way 2, Way 3 and Way 4** Allow final M1 for a correct method which gives any of $x_P = 2\sqrt{2}$ **or** $y_P = 4\sqrt{2}$ **or** $x_P =$ awrt 2.83 **or** $y_P =$ awrt 5.66 o.e.		

Notes for Question 5	
Way 1	
M1:	Establishes the gradient of the tangent by differentiating $xy = c^2$ • to give $\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k\,x^{-2}$; $k \neq 0$, or • by the product rule to give $\pm x\frac{\mathrm{d}y}{\mathrm{d}x} \pm y$, or • by parametric differentiation to give $\left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}t}\right) \times \frac{1}{\left(\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}\right)}$, condoning $p \equiv t$ **and** attempt to use $P\left(ct, \frac{c}{t}\right)$ to write down the gradient of the tangent to the curve in terms of t
M1:	Correct straight line method for an equation of a tangent where $m_T\ (\neq m_N)$ is found by using calculus. **Note:** m_T must be a function of t for this mark
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Attempts to find either the x-coordinate of A or the y-coordinate of B
A1:	**Both** {x-coordinate of A is} $2ct$ **and** the {y-coordinate of B is} $\frac{2c}{t}$
M1:	See scheme
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme
Way 2	
M1:	Same description as the 1st M mark in Way 1
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Attempts to find either the x-coordinate of A or the y-coordinate of B
A1:	**Both** {x-coordinate of A is} $\sqrt{2}c$ **and** the {y-coordinate of B is} $2\sqrt{2}c$
M1:	Recognising that the gradient of the tangent is -2 and puts this equal to their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and finds $t = ...$
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme
Way 3	
M1:	Same description as the 1st M mark in Way 1
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Uses $y = 2x$ and Area $(OAB) = 32$ to find either x_A or y_B
A1:	Either {x-coordinate of A is} $4\sqrt{2}$ or the {y-coordinate of B is} $8\sqrt{2}$
M1:	Recognising that the gradient of the tangent is -2 and puts this equal to their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and finds $t = ...$
A1:	See scheme
M1:	Substitutes their P (which is in terms of c, and has come from a correct method) into the equation of the tangent and finds $c = ...$
M1:	See scheme
A1:	See scheme

Notes for Question 5	
Way 4	
M1:	See scheme
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Uses $y = 2x$ and Area $(OAB) = 32$ to find either x_A or y_B
A1:	Either {x-coordinate of A is} $4\sqrt{2}$ or the {y-coordinate of B is} $8\sqrt{2}$
M1:	See scheme
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme

4.

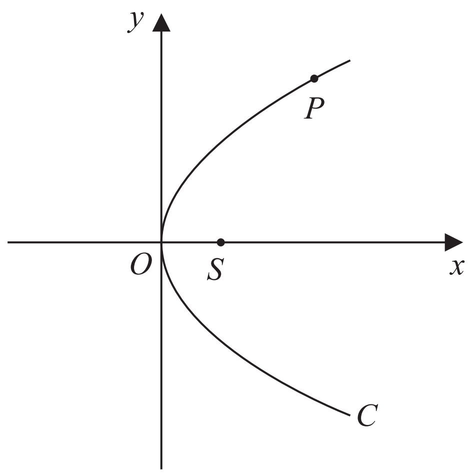

Figure 2

Figure 2 shows a sketch of the parabola C with equation $y^2 = 4ax$, where a is a positive constant. The point S is the focus of C and the point $P(ap^2, 2ap)$ lies on C where $p > 0$

(a) Write down the coordinates of S.

(1)

(b) Write down the length of SP in terms of a and p.

(1)

The point $Q(aq^2, 2aq)$, where $p \neq q$, also lies on C.
The point M is the midpoint of PQ.

Given that $pq = -1$

(c) prove that, as P varies, the locus of M has equation

$$y^2 = 2a(x - a)$$

(5)

Question	Scheme	Marks	AOs
4(a)	$(a, 0)$	B1	1.1b
		(1)	
(b)	$SP = ap^2 + a$ Note that if focus-directrix property not used may use Pythagoras: E.g. $SP = \sqrt{4a^2p^2 + (ap^2 - a)^2} = ... = ap^2 + a$	B1	1.1b
		(1)	
(c)	M has coordinates $\left(\frac{ap^2 + aq^2}{2}, \frac{2ap + 2aq}{2}\right)$	B1	1.1b
	$y^2 = a^2\left(p^2 + 2pq + q^2\right)$	M1	1.1b
	$y^2 = a^2\left(p^2 - 2 + q^2\right)$	A1	2.1
	$2a(x-a) = 2a\left(\frac{1}{2}ap^2 + \frac{1}{2}aq^2 - a\right) = a^2\left(p^2 + q^2 - 2\right)$	M1	1.1b
	$\Rightarrow y^2 = 2a(x-a)$*	A1*	2.1
		(5)	
	Alternative for (c)		
	M has coordinates $\left(\frac{ap^2 + aq^2}{2}, \frac{2ap + 2aq}{2}\right)$	B1	1.1b
	$\frac{y}{a} = p + q$	M1	1.1b
	$\frac{y^2}{a^2} = p^2 + q^2 + 2pq = p^2 + q^2 - 2$	A1	2.1
	$\frac{2x}{a} = p^2 + q^2$	M1	1.1b
	$\frac{y^2}{a^2} = \frac{2x}{a} - 2 \Rightarrow y^2 = 2a\left(x - a\right)$*	A1*	2.1
		(5)	
			(7 marks)

Notes
(a) B1: Correct coordinates (b) B1: Correct expression (c) B1: Correct coordinates for the midpoint M1: Squares their y coordinate of the midpoint A1: Uses $pq = -1$ to obtain a correct expression for y^2 M1: Attempts $2a(x - a)$ using the x coordinate of their midpoint and attempts to simplify A1*: Fully correct completion to show $y^2 = 2a(x - a)$ Alternative B1: Correct coordinates for the midpoint M1: Uses their y coordinate of the midpoint to find $p + q$ A1: Square and uses $pq = -1$ to obtain a correct expression for y^2/a^2 M1: Uses the x coordinate of their midpoint to find $p^2 + q^2$ A1*: Fully correct completion to show $y^2 = 2a(x - a)$

5. The point $P\ (ap^2, 2ap)$, where a is a positive constant, lies on the parabola with equation

$$y^2 = 4ax$$

The normal to the parabola at P meets the parabola again at the point $Q\ (aq^2, 2aq)$

(a) Show that

$$q = \frac{-p^2 - 2}{p}$$

(5)

(b) Hence show that

$$PQ^2 = \frac{ka^2}{p^4}\left(p^2 + 1\right)^n$$

where k and n are integers to be determined.

(5)

Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx}=\frac{2a}{2ap}=\frac{1}{p}$ or $y=2\sqrt{a}\sqrt{x}\Rightarrow\frac{dy}{dx}=\frac{\sqrt{a}}{\sqrt{x}}=\frac{1}{p}$ or $2y\frac{dy}{dx}=4a\Rightarrow\frac{dy}{dx}=\frac{2a}{y}=\frac{1}{p}$	B1	1.1b
	$y-2ap=-p\left(x-ap^2\right)$	M1	2.1
	$2aq-2ap=-p\left(aq^2-ap^2\right)$ $pq^2+2q-2p-p^3=0$	A1	1.1b
	$\left(q-p\right)\left(pq+p^2+2\right)=0\Rightarrow q=...$	M1	3.1a
	$q=\frac{-p^2-2}{p}$ *	A1*	1.1b
		(5)	
(b)	$PQ^2=\left(ap^2-aq^2\right)^2+\left(2ap-2aq\right)^2$	M1	1.1b
	$=a^2\left(p-q\right)^2\left(p+q\right)^2+4a^2\left(p-q\right)^2$ $=a^2\left(p-q\right)^2\left[\left(p+q\right)^2+4\right]$ $=a^2\left(2p+\frac{2}{p}\right)^2\left[\left(-\frac{2}{p}\right)^2+4\right]$	M1 A1	2.1 1.1b
	$=\frac{4a^2}{p^2}\left(p^2+1\right)^2\frac{4}{p^2}\left(p^2+1\right)=\frac{16a^2}{p^4}\left(p^2+1\right)^3$	A1 A1	1.1b 1.1b
		(5)	
			(10 marks)

Notes

(a)

B1: Deduces the correct tangent gradient

M1: Correct strategy for the equation of the normal

A1: Correct equation in terms of *p* and *q*

M1: Applies a correct strategy for finding *q* in terms of *p*. E.g. uses the fact that *q* = *p* is known and uses inspection or long division to find the other root

A1*: Correct proof with no errors

Alternative:

B1: As above

M1A1: $\frac{2aq-2ap}{aq^2-ap^2}\times\frac{1}{p}=-1$

M1: Finds gradient of *PQ* and uses product of gradients = −1

A1: Correct equation

M1A1: As above

(b)
M1: Applies Pythagoras correctly to find PQ^2
M1: Uses their q in terms of p to obtain an expression in terms of p only
A1: Correct expression in any form in terms of p only
A1: $k = 16$ or $n = 3$
A1: $k = 16$ and $n = 3$

4. The parabola C has equation $y^2 = 10x$

The point F is the focus of C.

(a) Write down the coordinates of F.

(1)

The point P on C has y coordinate q, where $q > 0$

(b) Show that an equation for the tangent to C at P is given by

$$10x - 2qy + q^2 = 0$$

(3)

The tangent to C at P intersects the directrix of C at the point A.

The point B lies on the directrix such that PB is parallel to the x-axis.

(c) Show that the point of intersection of the diagonals of quadrilateral $PBAF$ always lies on the y-axis.

(5)

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Question	Scheme	Marks	AOs
4(a)	$\left(\frac{5}{2}, 0\right)$ o.e.	B1	2.2a
		(1)	
(b)	$\frac{dy}{dx} = \frac{5}{q}$	B1	1.1b
	At P, $x = \frac{q^2}{10}$ so tangent has equation $y - q = \text{their}\frac{5}{q}\left(x - \frac{q^2}{10}\right)$ or $q = \left(\text{their}\frac{5}{q}\right)\left(\frac{q^2}{10}\right) + c \Rightarrow c = \ldots$ to reach an equation for y	M1	1.1b
	$\Rightarrow qy - q^2 = 5x - \frac{q^2}{2} \Rightarrow 10x - 2qy + q^2 = 0$ * cso or $\Rightarrow y = \frac{5}{q}x + \frac{q}{2} \Rightarrow 10x - 2qy + q^2 = 0$ * cso	A1*	2.1
		(3)	
(c)	B is $\left(-\frac{5}{2}, q\right)$ o.e.	B1	2.2a
	So diagonal BF has equation $\frac{y-0}{q-0} = \frac{x - \frac{5}{2}}{-\frac{5}{2} - \frac{5}{2}}$ or $y = -\frac{q}{5}\left(x - \frac{5}{2}\right)$	M1	1.1b
	(AP is a tangent so) diagonals meet when $10x - 2q\left(-\frac{q}{5}\left(x - \frac{5}{2}\right)\right) + q^2 = 0$ or $x = \frac{2qy - q^2}{10}$ therefore $y = -\frac{q}{5}\left(\frac{2qy - q^2}{10} - \frac{5}{2}\right)$ leading to $y = \ldots$ $\left\{y = \frac{25q + q^3}{50 + 2q^2}\right\}$	dM1	3.1a
	$\Rightarrow 10x + \frac{2q^2}{5}x - q^2 + q^2 = 0 \Rightarrow x\left(10 + \frac{2q^2}{5}\right) = 0$ or $x = \frac{1}{10}\left(2q\left(\frac{25q + q^3}{50 + 2q^2}\right) - q^2\right)$	M1	1.1b
	But $10 + \frac{2q^2}{5} > 0$ so not zero, hence $x = 0$, so the intersection lies on the y-axis.	A1	2.4

	Or achieves $x = 0$ (with no errors), so the intersection lies on the y axis.		
		(5)	
	Alternative for the last three marks		
	When $x = 0$ for BF $y = -\frac{q}{5}\left(-\frac{5}{2}\right) = \ldots$ **or** for AP $2qy = q^2 \Rightarrow y = \ldots$	**M1**	1.1b
	For BF y intercept is $\frac{q}{2}$ **and** for AP y intercept is $\frac{q}{2}$	**M1**	3.1a
	Since both diagonals always cross the y-axis at the same place, their intersection must always be on the y axis.	**A1**	2.4
			(9 marks)

Notes:

(a)
B1: Deduces correct coordinates.

(b)
B1: Using or deriving $\frac{dy}{dx} = \frac{5}{q}$

M1: Finds the equation of the tangent using the equation of a line formula with $y_1 = q$, $x = \frac{q^2}{10}$ (or clear attempt at it) and $m = \frac{2 \times \text{their}'a'}{q}$.
If uses $y = mx + c$ must find a value for c and substitute back to find an equation for the tangent
A1*: Completes correctly to the given equation, no errors seen.

(c)
B1: B is $\left(-\frac{5}{2}, q\right)$ seen or used.
M1: A correct method to find the equation of the diagonal BF using their coordinates of F and B
dM1: Uses the printed answer in (b) and their equation of the diagonal BF to form an equation just involving x or solves the two diagonals simultaneously to find an expression for y
M1: Correctly factors out the x to achieve $x(\ldots) = 0$ or uses their expression for y to find an expression for x
A1: Conclusion given including reference to $10 + \frac{2q^2}{5} \neq 0$
Alternative for last three marks
M1: Attempts to find the y intercept for at least one of the two diagonals.
M1: Finds y intercept for both diagonals in order to compare
A1: Both intercepts correct and suitable conclusion giving reference to both diagonals always crossing y-axis at same point.