Fm1Ch5 XMQs and MS

(Total: 176 marks)

1. FM1_2019a Q2 . 11 marks - FM1ch5 Elastic collisions in two dimensions
2. FM1_2019a Q6 . 12 marks - FM1ch5 Elastic collisions in two dimensions
3. FM1_2020 Q4 . 9 marks - FM1ch5 Elastic collisions in two dimensions
4. FM1_2020 Q5 . 14 marks - FM1ch5 Elastic collisions in two dimensions
5. FM1_2020 Q7 . 11 marks - FM1ch5 Elastic collisions in two dimensions
6. FM1_2021 Q3 . 14 marks - FM1ch5 Elastic collisions in two dimensions
7. FM1_2021 Q5 . 10 marks - FM1ch5 Elastic collisions in two dimensions
8. FM1_2021 Q7 . 9 marks - FM1ch5 Elastic collisions in two dimensions
9. FM1_2022 Q4 . 9 marks - FM1ch5 Elastic collisions in two dimensions
10. FM1_2022 Q8 . 10 marks - FM1ch5 Elastic collisions in two dimensions
11. FM1_2019b Q2 . 11 marks - FM1ch5 Elastic collisions in two dimensions
12. FM1_2019b Q6 . 12 marks - FM1ch5 Elastic collisions in two dimensions
13. FM1_Sample Q4 . 9 marks - FM1ch5 Elastic collisions in two dimensions
14. FM1_Sample Q6 . 9 marks - FM1ch5 Elastic collisions in two dimensions
15. FM1_Specimen Q5 . 12 marks - FM1ch5 Elastic collisions in two dimensions
16. FM1_Specimen Q7 . 14 marks - FM1ch5 Elastic collisions in two dimensions

2.

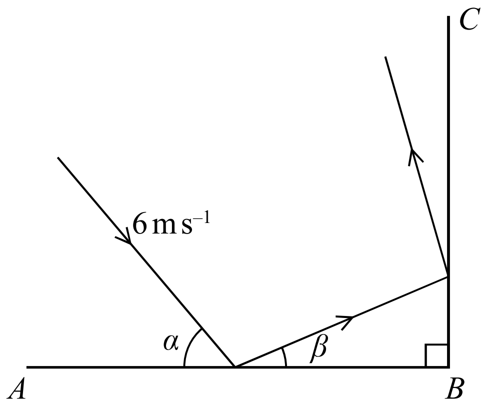


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where *AB* and *BC* are fixed vertical walls with *AB* perpendicular to *BC*.

A small ball is projected along the floor towards *AB* with speed $6\,\text{m s}^{-1}$ on a path that makes an angle α with *AB*, where $\tan\alpha = \frac{4}{3}$. The ball hits *AB* and then hits *BC*.

Immediately after hitting *AB*, the ball is moving at an angle β to *AB*, where $\tan\beta = \frac{1}{3}$

The coefficient of restitution between the ball and *AB* is *e*.

The coefficient of restitution between the ball and *BC* is $\frac{1}{2}$

By modelling the ball as a particle and the floor and walls as being smooth,

(a) show that the value of $e = \frac{1}{4}$

(5)

(b) find the speed of the ball immediately after it hits *BC*.

(4)

(c) Suggest two ways in which the model could be refined to make it more realistic.

(2)

Question	Scheme	Marks	AOs	Notes
2(a)	After hit *AB*: $\rightarrow 6\cos\alpha\,(=v\cos\beta)\quad(=3.6)$	B1	3.1b	Use model to find component parallel to the wall
	Use of impact law:	M1	3.4	Use model and impact law perpendicular to the wall
	$\uparrow 6e\sin\alpha\quad(=v\sin\beta)\left(=\frac{24e}{5}\right)(=4.8e)$	A1	1.1b	Correct perpendicular component
	$\tan\beta=\frac{1}{3}=\frac{6e\sin\alpha}{6\cos\alpha}\left(=\frac{24e}{5}\div\frac{18}{5}\right)$	M1	2.1	Use $\frac{1}{3}$ and their components to form equation in e $\left(v=\frac{6\sqrt{10}}{5}=3.79\right)$
	$e=\frac{18}{3\times 24}=\frac{1}{4}$ *	A1*	2.2a	Correct answer from correct exact working
				If only see $e\tan\alpha=\tan\beta$ with no explanation of where it comes from then score 0/5
		(5)		
(b)	After hit *BC*: $\uparrow\frac{6}{5}$	B1	1.1b	First component correct
	$\rightarrow\frac{1}{2}\times\frac{18}{5}\left(=\frac{9}{5}\right)$	B1	3.4	Second component correct
				Alternative: B1 for speed of impact with $BC=3.79.....$ B1 for path on leaving *BC* at $56.3...^\circ$ to *BC*
	Speed $=\frac{3}{5}\sqrt{2^2+3^2}$	M1	1.1b	Use Pythagoras' theorem or trigonometry to find the speed
	$=\frac{3\sqrt{13}}{5}$ (m s^{-1})	A1	1.1b	Any equivalent form. 2.2 or better $(2.1633....)$
		(4)		

(c)	An appropriate refinement	B1	3.5c	Two independent refinements relating to the modelling e.g. • Include friction between the floor and the ball • Include friction between the ball and the walls • Give the ball dimensions . • Consider air resistance • Spin / rotation Do not accept comments about mass / gravity / levels / perpendicularity
	A second independent appropriate refinement and no incorrect refinements	B1	3.5c	
		(2)		
(11 marks)				

6. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

A smooth uniform sphere A has mass 0.2 kg and another smooth uniform sphere B, with the same radius as A, has mass 0.4 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, the velocity of A is $(3\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-4\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

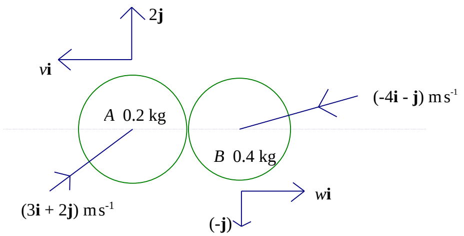
At the instant of collision, the line joining the centres of the spheres is parallel to **i**

The coefficient of restitution between the spheres is $\frac{3}{7}$

(a) Find the velocity of A immediately after the collision.
(7)

(b) Find the magnitude of the impulse received by A in the collision.
(2)

(c) Find, to the nearest degree, the size of the angle through which the direction of motion of A is deflected as a result of the collision.
(3)

Question	Scheme	Marks	AOs	Notes
6(a)				
	Perpendicular to line of centres: $2\mathbf{j}$	B1	3.4	Use the model to find the component perpendicular to the line of centres. Correct value seen or implied
	CLM parallel to the line of centres	M1	3.1b	Use of CLM parallel to line of centres. Need all terms and dimensionally correct. Condone sign errors
	$0.2\times3-0.4\times4=0.4w-0.2v \quad (-5=2w-v)$	A1	1.1b	Correct unsimplified equation.
	Impact law parallel to the line of centres	M1	3.4	Correct use of impact law parallel to the line of centres. Condone sign errors
	$7e=v+w \quad \Rightarrow 3=v+w$	A1	1.1b	Correct equation with $\frac{3}{7}$ used.
	Complete strategy to find $\mathbf{v}_A$	M1	3.1b	Complete strategy to find components parallel and perpendicular to line of centres, eg by using CLM and impact law
	$\mathbf{v}_A=-\frac{11}{3}\mathbf{i}+2\mathbf{j}\ \left(\mathrm{m\,s^{-1}}\right)$ follow their $2\mathbf{j}$	A1ft	1.1b	$\mathbf{v}_A$ correct, follow their $a\mathbf{j}$ for $2\mathbf{j}(a\neq0)$
		(7)		
6(b)	Magnitude of impulse on *A*: $0.2\left(\frac{11}{3}-(-3)\right)$	M1	3.1b	Evidence of use of $m(v-u)$ parallel to the line of centres
	$=0.2\left(\frac{11}{3}+3\right)=\frac{4}{3}$ (Ns)	A1	1.1b	1.3 (Ns) or better
		(2)		

Question	Scheme	Marks	AOs	Notes
6(c)	Use of scalar product to find the angle	M1	3.1a	Complete method for finding the required angle. Allow for $\tan^{-1}\frac{3}{2}$ or $\tan^{-1}\frac{2}{3}$ and $\tan^{-1}\frac{6}{11}$ or $\tan^{-1}\frac{11}{6}$
	$\cos\theta = \dfrac{(3\mathbf{i}+2\mathbf{j}).\left(-\frac{11}{3}\mathbf{i}+2\mathbf{j}\right)}{\sqrt{13}\times\sqrt{\frac{157}{9}}}$	A1ft	1.1b	A correct unsimplified expression Follow their $\mathbf{v}_A$. Do not ISW
	$\theta = 118°$	A1	1.1b	Correct answer only. (Q asks for the nearest degree) Do not ISW
	Alternative method: $180° - \tan^{-1}\frac{2}{3} - \tan^{-1}\frac{6}{11}$ Or $\tan^{-1}\frac{3}{2} + \tan^{-1}\frac{11}{6}$			$62°$ probably scores M1A0A0
		(3)		
		(Total 12 marks)		

4. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

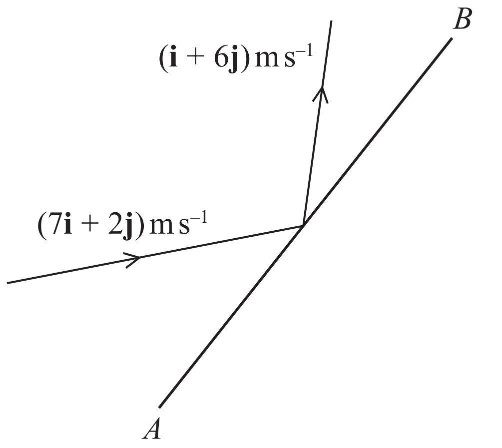


Figure 1

Figure 1 represents the plan view of part of a smooth horizontal floor, where *AB* represents a fixed smooth vertical wall.

A small ball of mass 0.5 kg is moving on the floor when it strikes the wall.

Immediately before the impact the velocity of the ball is $(7\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$.

Immediately after the impact the velocity of the ball is $(\mathbf{i} + 6\mathbf{j})\,\text{m s}^{-1}$.

The coefficient of restitution between the ball and the wall is e.

(a) Show that *AB* is parallel to $(2\mathbf{i} + 3\mathbf{j})$.

(4)

(b) Find the value of e.

(5)

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Question	Scheme	Marks	AOs
4(a)	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ or $\mathbf{v} - \mathbf{u}$	M1	2.1
	$\mathbf{I} = 0.5\left((\mathbf{i}+6\mathbf{j})-(7\mathbf{i}+2\mathbf{j})\right) \quad \left(=(-3\mathbf{i}+2\mathrm{j})\right)$	A1	1.1b
	Use of scalar product $(-3\mathbf{i}+2\mathbf{j}).(2\mathbf{i}+3\mathbf{j}) = -6+6=0$	M1	1.1b
	Hence impulse perpendicular to $(2\mathbf{i}+3\mathbf{j})$, so *AB* must be parallel to $(2\mathbf{i}+3\mathbf{j})$. *	A1*	2.2a
		(4)	
4(a) alt	Components of velocities parallel to $(2\mathbf{i}+3\mathbf{j})$:	M1	2.1
	$\left(\frac{1}{\sqrt{13}}\right)(7\mathbf{i}+2\mathbf{j}).(2\mathbf{i}+3\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(14+6)$ $\left(\frac{1}{\sqrt{13}}\right)(\mathbf{i}+6\mathbf{j}).(2\mathbf{i}+3\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(2+18)$	A1	1.1b
	Simplify and compare values	M1	1.1b
	Hence component of velocity parallel to $(2\mathbf{i}+3\mathbf{j})$ is unchanged, so *AB* must be parallel to $(2\mathbf{i}+3\mathbf{j})$. *	A1*	2.2a
		(4)	
4(a) alt	Use conservation of velocity parallel to $a\mathbf{i}+b\mathbf{j}$	M1	2.1
	$(7\mathbf{i}+2\mathbf{j}).(a\mathbf{i}+b\mathbf{j}) = (\mathbf{i}+6\mathbf{j}).(a\mathbf{i}+b\mathbf{j})$ $(\Rightarrow 7a+2b = a+6b)$	A1	1.1b
	Find ratio of *a* and *b* to obtain direction: $\left(b = \frac{2}{3}a\right)$	M1	1.1b
	Hence *AB* must be parallel to $(2\mathbf{i}+3\mathbf{j})$. *	A1*	2.2a
		(4)	
4(b)	Use scalar product to find components of velocities perpendicular to the wall	M1	3.1b
	$\left(\frac{1}{\sqrt{13}}\right)(-3\mathbf{i}+2\mathbf{j})(7\mathbf{i}+2\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(-21+4) \quad \left(=\frac{-17}{\sqrt{13}}\right)$ $\left(\frac{1}{\sqrt{13}}\right)(-3\mathbf{i}+2\mathbf{j})(\mathbf{i}+6\mathbf{j}) = \left(\frac{1}{\sqrt{13}}\right)(-3+12) \quad \left(=\frac{9}{\sqrt{13}}\right)$	A1 A1	1.1b 1.1b
	Use of impact law	M1	3.4
	$e = \frac{9}{17}$	A1	1.1b

		(5)	
			(9 marks)

Notes:

(a)M1	Must be finding the difference between two momenta or two velocities
A1	Correct unsimplified equation for the impulse or for change in velocity
M1	Use of scalar product or equivalent. In the alt method allow full marks if $\sqrt{13}$ not used.
A1*	Reach given conclusion from correct working
	If working with angles, score M1 for correct method to find components parallel to the wall A1 for $\sqrt{53}\cos 40.36..°$ and $\sqrt{37}\cos 24.23..°$ M1 for comparing the two values A0 because the work has involved decimal approximations (since working towards an exact given answer). Alternative: Could use $e\tan 24.2..° = \tan 40.36..°$
(b)M1	Condone if not using a unit vector
A1 **A1**	One correct value Second correct values
	If working with angles, score M1A1A1 for $\sqrt{53}\sin 40.36..°$ and $\sqrt{37}\sin 24.23...°$
M1	Use their components the right way round in the impact law. Condone sign error.
A1	0.53 or better (0.52941……)

5. A smooth uniform sphere P has mass 0.3 kg. Another smooth uniform sphere Q, with the same radius as P, has mass 0.2 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of P is $(4\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the velocity of Q is $(-3\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to $\mathbf{i}$.

The kinetic energy of Q immediately after the collision is half the kinetic energy of Q immediately before the collision.

(a) Find

(i) the velocity of P immediately after the collision,

(ii) the velocity of Q immediately after the collision,

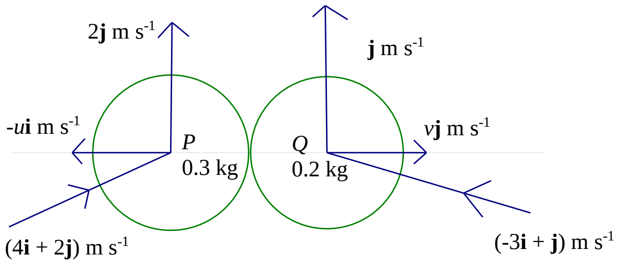
(iii) the coefficient of restitution between P and Q,

carefully justifying your answers.

(11)

(b) Find the size of the angle through which the direction of motion of P is deflected by the collision.

(3)

Question	Scheme	Marks	AOs
5(a)			
	Components perpendicular to the line of centres after the collision: $\mathbf{v}_{P\mathbf{j}} = 2\mathbf{j}\left(\text{m s}^{-1}\right)$, $\mathbf{v}_{Q\mathbf{j}} = \mathbf{j}\left(\text{m s}^{-1}\right)$	B1	3.4
	Kinetic energy:	M1	3.1a
	$\frac{1}{2}\times 0.2\times\left(v^2+1\right) = \frac{1}{2}\times\frac{1}{2}\times 0.2\times(9+1)$	A1	1.1b
	CLM parallel to line of centres:	M1	3.1a
	$0.3\times 4 - 0.2\times 3 = 0.2v - 0.3u \quad (6 = 2v - 3u)$	A1	1.1b
	Impact law parallel to line of centres	M1	3.1a
	$v + u = e(4+3)$	A1	1.1b
	Solve for $\mathbf{v}_P$, $\mathbf{v}_Q$ or e	M1	1.1b
	$\mathbf{v}_P = \frac{2}{3}\mathbf{i} + 2\mathbf{j}\left(\text{m s}^{-1}\right)$ and $\mathbf{v}_Q = 2\mathbf{i} + \mathbf{j}\left(\text{m s}^{-1}\right)$	A1	1.1b
	$e = \frac{4}{21}$	A1	1.1b
	$v = -2 \Rightarrow u = -\frac{10}{3} \Rightarrow$ *P* and *Q* have passed through each other: impossible, so solution is unique *	A1*	2.4
		(11)	
(b)	Use trig to find angle between velocities	M1	3.1a
	$\cos\theta = \left(\frac{\frac{8}{3}+4}{\sqrt{20}\sqrt{4\frac{4}{9}}}\right)$ or $\theta = \tan^{-1}\frac{2}{2/3} - \tan^{-1}\frac{1}{2}$	A1ft	1.1b
	$\theta = 45° \left(\frac{\pi}{4}\text{ rads}\right)$	A1	1.1b
		(3)	
		(14 marks)	

Notes:

(a)B1	Seen or implied. Correct only
M1	Equation for KE of Q. Dimensionally correct. Condone $\frac{1}{2}$ on the wrong side.
A1	Correct unsimplified equation in v^2
M1	Equation for CLM. Correct terms required. Condone sign errors. Dimensionally correct.
A1	Correct unsimplified equation
M1	Correct use of impact law. Condone sign errors
A1	Correct unsimplified equation.
M1	Complete method to solve for $\mathbf{v}_P, \mathbf{v}_Q$ or e
	(Working in e gives $v = \frac{1}{5}(6 + 21e)$ and $441e^2 + 252e - 64 = 0$)
A1	Both velocities correct. Need to see answers in the form $a\mathbf{i} + b\mathbf{j}$ or equivalent
A1	Correct only. 0.19 or better (0.19047….)
A1*	Or equivalent justification of given result. e.g. a negative value for e is not possible
(b) M1	Use of trig or equivalent to find a relevant angle between two velocities e.g by scalar product or difference between angles.
A1ft	Correct unsimplified equation in θ . Follow their $\mathbf{v}_P$
A1	Correct only. (0.785… radians) Do not ISW

7.

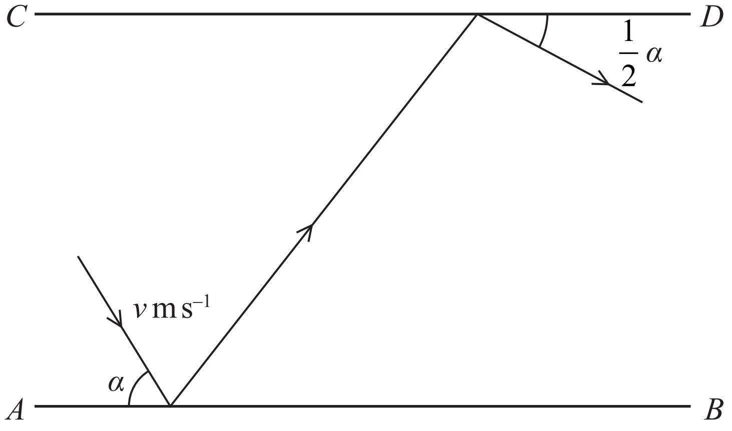


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and CD represent fixed vertical walls, with AB parallel to CD.

A small ball is projected along the floor towards wall AB. Immediately before hitting wall AB, the ball is moving with speed v m s^{-1} at an angle α to AB, where $0 < \alpha < \frac{\pi}{2}$

The ball hits wall AB and then hits wall CD.

After the impact with wall CD, the ball is moving at angle $\frac{1}{2}\alpha$ to CD.

The coefficient of restitution between the ball and wall AB is $\frac{2}{3}$

The coefficient of restitution between the ball and wall CD is also $\frac{2}{3}$

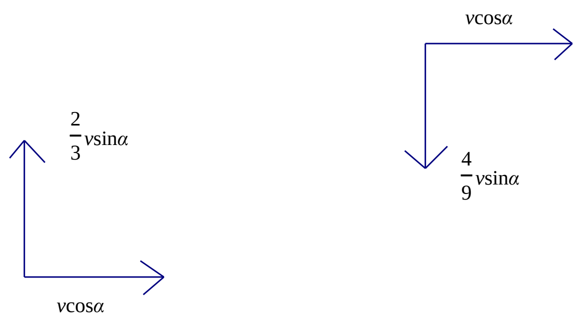
The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

(a) Show that $\tan\left(\frac{1}{2}\alpha\right) = \frac{1}{3}$

(7)

(b) Find the percentage of the initial kinetic energy of the ball that is lost as a result of the two impacts.

(4)

Question	Scheme	Marks	AOs
7(a)	Use model to find components of velocity after the impacts: 	B1 B1 B1 B1	3.1b 3.4 3.1b 3.4
	$\tan\frac{\alpha}{2}=\frac{\frac{4}{9}v\sin\alpha}{v\cos\alpha}\left(=\frac{4}{9}\tan\alpha\right)$	M1	3.1b
	$t=\tan\frac{\alpha}{2}\quad\Rightarrow t=\frac{4\times 2t}{9\left(1-t^2\right)}$	M1	1.1b
	$1-t^2=\frac{8}{9},\quad t=\frac{1}{3}\ *$	A1*	2.2a
		(7)	
(b)	$\tan\alpha=\frac{\frac{2}{3}}{1-\frac{1}{9}}=\frac{3}{4}$	B1	1.1b
	change in KE $\quad\frac{1}{2}mv^2-\frac{1}{2}m\left(v^2\cos^2\alpha+\left(\frac{4}{9}v\right)^2\sin^2\alpha\right)$	M1	3.1b
	% of KE lost $=100\left(1-\frac{\frac{1}{2}mv^2\left(\frac{16}{25}+\frac{16}{81}\times\frac{9}{25}\right)}{\frac{1}{2}mv^2}\right)$	M1	1.1b
	$=28.888...(\%)$	A1	1.1b
		(4)	
			(11 marks)

Notes:

(a)B1 **B1** **B1** **B1**	One mark for each component correct.
M1	Form expression for $\tan\frac{\alpha}{2}$ in terms of $\tan\alpha$

M1	Form and solve equation in $\tan\frac{\alpha}{2}$
A1*	Obtain given answer from correct working
	NB: This is a "Show that .." question. A candidate who assumes, without proof, that $\tan\frac{\alpha}{2} = e^2 \tan\alpha$ can only score the last two marks.
(b)B1	Correct use of $t = \frac{1}{3}$ Must be seen / used in part (b)
M1	Dimensionally correct expression for change in KE NB note that they may not show component parallel to the wall
M1	Dimensionally correct expression for the percentage of KE lost.
A1	Accept $29(\%)$ or better Accept $\frac{260}{9}$

3. [*In this question*, **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

A smooth uniform sphere P has mass 0.3 kg. Another smooth uniform sphere Q, with the same radius as P, has mass 0.5 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of P is $(u\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$, where u is a positive constant, and the velocity of Q is $(-4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$

At the instant when the spheres collide, the line joining their centres is parallel to **i**.

The coefficient of restitution between P and Q is $\frac{3}{5}$

As a result of the collision, the direction of motion of P is deflected through an angle of 90° and the direction of motion of Q is deflected through an angle of $\alpha°$

(a) Find the value of u

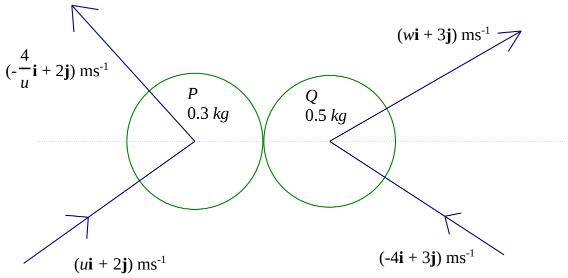
(8)

(b) Find the value of α

(5)

(c) State how you have used the fact that P and Q have equal radii.

(1)

Question	Scheme	Marks	AOs
3(a)			
	For P after: Component in $\mathbf{j}$ direction $=2$	B1	3.4
	Deflected through $90°$ so velocity after $=\left(-\frac{4}{u}\mathbf{i}+2\mathbf{j}\right)\left(\text{m s}^{-1}\right)$	B1	3.4
	CLM parallel to line of centres:	M1	3.1a
	$0.3\left(u+\frac{4}{u}\right)=0.5(4+w)$	A1ft	1.1b
	Impact law parallel to line of centres:	M1	3.1a
	$w+\frac{4}{u}=\frac{3}{5}(u+4)$	A1ft	1.1b
	$\begin{cases} 3u+\frac{12}{u}=20+5w \\ \frac{20}{u}+5w=3u+12 \end{cases}$	M1	1.1b
	$\left(\Rightarrow\frac{32}{u}=32,\right)\quad u=1$	A1	2.2a
		(8)	
(b)	For Q after: $w=-1$	B1	1.1b
	$\mathbf{v}=-\mathbf{i}+3\mathbf{j}$	B1ft	1.1b
	Find relevant angle between directions	M1	3.1a
	$\alpha°=\tan^{-1}3-\tan^{-1}\frac{3}{4}$ or $\alpha°=\cos^{-1}\left(\frac{4+9}{5\times\sqrt{10}}\right)$	A1ft	1.1b
	$\alpha=34.7\quad(35)$	A1	1.1b
		(5)	
(c)	The line of centres is parallel to the surface the spheres are moving on, so the impulse acts parallel to the surface.	B1	3.5a
		(1)	
			(14 marks)

Notes:	
(a) B1	Correct only Check the diagram
B1	Correct only. Seen or implied.
M1	Correct use of CLM. Need all terms. Condone sign errors
A1ft	Follow their components of velocity of *P*, with or without a value for the **i** component.
M1	Correct use of the impact law. Condone sign errors
A1ft	Follow their components of velocity of *P*, with or without a value for the **i** component.
M1	Solve their correctly formed simultaneous equations to obtain value of *u*.
A1	Correct only
(b) B1	Correct only
B1ft	Follow their *w*
M1	Correct method to find a relevant angle between the directions
A1ft	Correct unsimplified expression. Follow their **v**
A1	35 or better (34.695…) 0.61 radians
(c)B1	Or equivalent that explains that the line of centres is parallel to the surface

5.

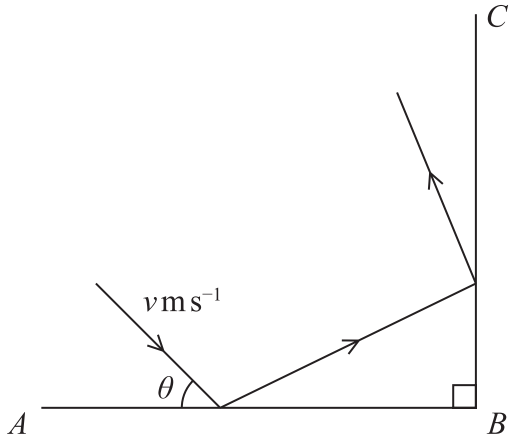


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls, with AB perpendicular to BC.

A small ball is projected along the floor towards the wall AB. Immediately before hitting the wall AB the ball is moving with speed $v\,\text{m s}^{-1}$ at an angle θ to AB.

The ball hits the wall AB and then hits the wall BC.

The coefficient of restitution between the ball and the wall AB is $\frac{1}{3}$

The coefficient of restitution between the ball and the wall BC is e.

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall AB.

(a) Find the exact value of $\cos\theta$.

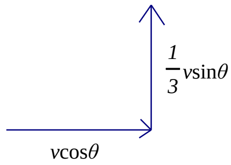
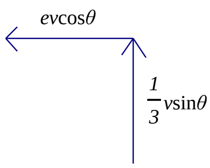
(5)

The ball loses half of its remaining kinetic energy in the impact with the wall BC.

(b) Find the exact value of e.

(5)

P 6 6 8 0 0 A 0 1 6 2 8

Question	Scheme	Marks	AOs
5(a)	Use the model to find components of velocity after first impact: 	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{2}\times\frac{1}{2}mv^2=\frac{1}{2}m\left(v^2\cos^2\theta+\frac{1}{9}v^2\sin^2\theta\right)$	M1	3.1b
	$\frac{1}{2}=\frac{1}{9}+\frac{8}{9}\cos^2\theta$	M1	1.1b
	$\frac{7}{16}=\cos^2\theta,\quad \cos\theta=\frac{\sqrt{7}}{4}$	A1	1.1b
		(5)	
(a) alt	Working with initial velocity $\mathbf{v}=x\mathbf{i}-y\mathbf{j}$, after impact $\mathbf{v}=x\mathbf{i}+\frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4
	KE: $\frac{1}{2}\times\frac{1}{2}m\left(x^2+y^2\right)=\frac{1}{2}m\left(x^2+\frac{1}{9}y^2\right)$	M1	3.1b
	$y^2=\frac{9}{7}x^2,\quad \frac{y}{x}=\tan\theta=\frac{3}{\sqrt{7}}$	M1	1.1b
	$\cos\theta=\frac{\sqrt{7}}{4}$	A1	1.1b
		(5)	
(b)	Use the model to find components of velocity after second impact: 	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{4}\times\frac{1}{2}mv^2=\frac{1}{2}m\left(e^2v^2\cos^2\theta+\frac{1}{9}v^2\sin^2\theta\right)$ or $\frac{1}{2}\times\frac{1}{2}m\left(v^2\cos^2\theta+\frac{1}{9}v^2\sin^2\theta\right)=\frac{1}{2}m\left(e^2v^2\cos^2\theta+\frac{1}{9}v^2\sin^2\theta\right)$	M1	3.1b
	$\frac{1}{4}=\frac{7}{16}e^2+\frac{1}{9}\times\frac{9}{16}$	M1	1.1b
	$\Rightarrow e^2=\frac{3}{7},\quad e=\sqrt{\frac{3}{7}}$	A1	1.1b
		(5)	
(b) alt	After second impact $\mathbf{v}=-ex\mathbf{i}+\frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4

	KE: $\frac{1}{4}\times\frac{1}{2}m\left(x^2+y^2\right)=\frac{1}{2}m\left(e^2x^2+\frac{1}{9}y^2\right)$	M1	3.1b
	$4e^2x^2+\frac{4}{9}y^2=x^2+y^2,\quad 4e^2=1+\frac{5}{9}\left(\frac{y}{x}\right)^2$	M1	1.1b
	$\Rightarrow e^2=\frac{3}{7},\quad e=\sqrt{\frac{3}{7}}$	A1	1.1b
		(5)	
			(10 marks)

Notes:

(a) B1 **B1**	Parallel component correct Perpendicular component correct Check the diagram
M1	Equation for KE in v,θ . Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side
M1	Form and solve equation in $\cos\theta$
A1	Or exact equivalent
(b) B1 **B1**	Parallel component correct Perpendicular component correct
M1	Equation for KE in x,y. Dimensionally correct. Includes all components. Condone $\frac{1}{2}$ used on wrong side
M1	Use their $\cos\theta$ to form and solve equation in e
A1	Or exact equivalent

7. [*In this question*, **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

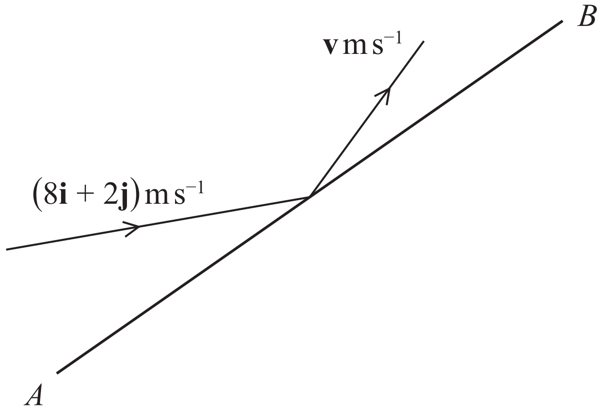


Figure 3

Figure 3 represents the plan view of part of a smooth horizontal floor, where *AB* is a fixed smooth vertical wall.

The direction of $\overrightarrow{AB}$ is in the direction of the vector $(\mathbf{i} + \mathbf{j})$

A small ball of mass 0.25 kg is moving on the floor when it strikes the wall *AB*.

Immediately before its impact with the wall *AB*, the velocity of the ball is $(8\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$

Immediately after its impact with the wall *AB*, the velocity of the ball is $\mathbf{v}\,\mathrm{m\,s^{-1}}$

The coefficient of restitution between the ball and the wall is $\frac{1}{3}$

By modelling the ball as a particle,

(a) show that $\mathbf{v} = 4\mathbf{i} + 6\mathbf{j}$

(6)

(b) Find the magnitude of the impulse received by the ball in the impact.

(3)

Question	Scheme	Marks	AOs
7(a)	Component parallel to the wall: $\left[\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j}).(8\mathbf{i}+2\mathbf{j})\right]$	M1	2.1
	$=5\sqrt{2}$	A1	1.1b
	Use of impact law perpendicular to wall:	M1	3.4
	Component perpendicular to wall after impact $\frac{1}{3}\left[\frac{1}{\sqrt{2}}(-\mathbf{i}+\mathbf{j}).(8\mathbf{i}+2\mathbf{j})\right]=-\sqrt{2}$	A1	1.1b
	For a complete method to find **v**	M1	1.1b
	$\Rightarrow \mathbf{v}=(5\mathbf{i}+5\mathbf{j})+(-\mathbf{i}+\mathbf{j})=(4\mathbf{i}+6\mathbf{j})$ *	A1*	2.2a
		(6)	
(a) alt	If $\mathbf{v}=a\mathbf{i}+b\mathbf{j}$ component parallel to the wall:	M1	2.1
	$(8\mathbf{i}+2\mathbf{j}).(\mathbf{i}+\mathbf{j})=(a\mathbf{i}+b\mathbf{j}).(\mathbf{i}+\mathbf{j})$ $(a+b=10)$	A1	1.1b
	Use of impact law:	M1	3.4
	$-\frac{1}{3}(8\mathbf{i}+2\mathbf{j}).(-\mathbf{i}+\mathbf{j})=(a\mathbf{i}+b\mathbf{j}).(-\mathbf{i}+\mathbf{j})$ $(2=-a+b)$	A1	1.1b
	For a complete method to find **v**	M1	1.1b
	$\Rightarrow \mathbf{v}=(4\mathbf{i}+6\mathbf{j})$ *	A1*	2.2a
		(6)	
(a) alt 2	Angle to wall $=31°$, component parallel to the wall:	M1	2.1
	$=\sqrt{68}\cos 31°=7.07$	A1	1.1b
	Component perpendicular to the wall	M1	3.4
	$=\frac{1}{3}\sqrt{68}\sin 31°=1.42$	A1	1.1b
	For a complete method to find **v**	M1	1.1b
	$\Rightarrow \mathbf{v}=\left(\sqrt{52}\cos 56.3°\mathbf{i}+\sqrt{52}\sin 56.3°\mathbf{j}\right)=(4\mathbf{i}+6\mathbf{j})$	A1*	2.2a
		(6)	
(b)	$\mathbf{I}=0.25(4\mathbf{i}+6\mathbf{j})-0.25(8\mathbf{i}+2\mathbf{j})$ $(\mathbf{I}=0.25(-\mathbf{i}+\mathbf{j})-0.25(3\mathbf{i}-3\mathbf{j}))$ $(\mathbf{I}=(-\mathbf{i}+\mathbf{j}))$	M1	3.1b

	Use of Pythagoras	M1	1.1b
	$\lvert \mathbf{I} \rvert = \sqrt{2}$ (N s)	A1	1.1b
		(3)	
			(9 marks)

Notes:	
(a)M1	Use of scalar product or equivalent. Allow M1 if not using unit vector
A1	Correct unsimplified expression for component parallel to wall
M1	Correct use of impact law perpendicular to the wall. Condone sign error
A1	Correct unsimplified expression for component perpendicular to wall
M1	Complete method to solve for $\mathbf{v}$
A1*	Obtain given result from correct working
(b) M1	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ with velocities or perpendicular components of velocities. Must be subtracting but allow subtraction in either order.
M1	Correct use of Pythagoras to find modulus
A1	Accept 1.4 Ns or better

4.

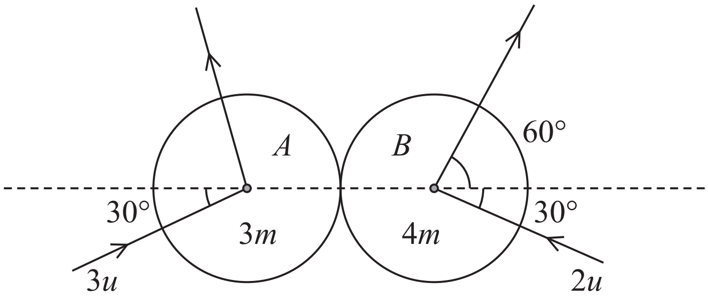


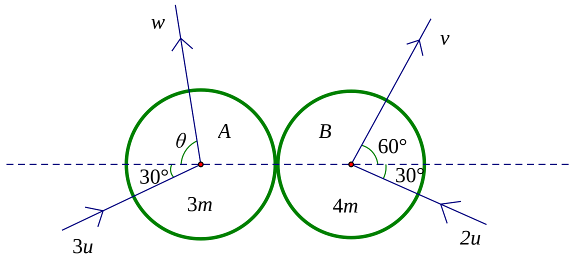
Figure 3

Two smooth uniform spheres, A and B, have equal radii. The mass of A is $3m$ and the mass of B is $4m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before they collide, A is moving with speed $3u$ at 30° to the line of centres of the spheres and B is moving with speed $2u$ at 30° to the line of centres of the spheres. The direction of motion of B is turned through an angle of 90° by the collision, as shown in Figure 3.

(i) Find the size of the angle through which the direction of motion of A is turned as a result of the collision.

(ii) Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(9)

Question	Scheme	Marks	AOs
4			
	Parallel to line of centres:	M1	3.1b
	$9mu\cos 30° - 8mu\cos 30° = 4mv\cos 60° - 3mw\cos\theta$ $(u\cos 30° = 2v - 3w\cos\theta)\quad (u\cos 30° = 2v - 3w_x)$	A1	1.1b
	$\updownarrow A:\ (w_y =)\, w\sin\theta = 3u\sin 30°\left(=\frac{3u}{2}\right)$ $\updownarrow B:\ v\sin 60° = 2u\sin 30° (= u)\quad \left(v = \frac{2u}{\sqrt{3}}\right)$	B1 B1	3.4 3.4
	$(w_x =)\, w\cos\theta = \frac{1}{3}(2v - u\cos 30°) = \frac{5u\sqrt{3}}{18}$ $(w_y =)\, w\sin\theta = \frac{3u}{2}$ $\left(\Rightarrow \tan\theta = \frac{9\sqrt{3}}{5},\quad \theta = 72.2°\right)$	M1	1.1b
	Direction deflected by $77.8°$ ($78°$ or better)	A1	2.2a
	Magnitude of impulse	M1	3.1b
	$= 4m(v\cos 60° - (-2u\cos 30°))$	A1	1.1b
	$= 4m\left(\frac{1}{2}\times\frac{u}{\sin 60°} - \left(-2u\frac{\sqrt{3}}{2}\right)\right) = \frac{16\sqrt{3}}{3}mu$	A1	2.2a
	OR: magnitude $= 3m(3u\cos 30° + w\cos\theta)$ $= 3m\left(+\frac{5u\sqrt{3}}{18} - \left(-\frac{3u\sqrt{3}}{2}\right)\right) = \frac{16\sqrt{3}}{3}mu$		
		(9)	
		(Total 9 Marks)	

M1	Use of CLM parallel to the line of centres. Need all 4 terms. Dimensionally correct. Condone sign errors and sin/cos confusion.

A1	Correct unsimplified equation. Allow e.g. w_x in place of $w\cos\theta$ and v_x in place of $v\cos 60°$. Allow if they have divided through by a common factor e.g. m
	NB there is no mark for the correct use of the impact law because the candidates are not required to find the coefficient of restitution. They might however find it as part of an alternative method. In this case, the M marks below are for a complete correct method to achieve the required result. Ignore work to find e if it is not used.
B1 B1	No change perpendicular to line of centres for one sphere. Allow e.g. w_y in place of $w\sin\theta$. Check the diagrams – the vertical components are often shown there. No change perpendicular to line of centres for both spheres
M1	Use scalar product or solve simultaneous equations to find θ for a relevant angle using their w_x They need to get as far as θ = a numerical value for a relevant angle
A1	$78°$ or better
M1	Use of $I = mv - mu$ in direction of line of centres. Condone subtraction in either order Allow M1 if they think that they have subtracted but they have not actually taken account of the change of direction. Allow M1 if they go direct to the correct expression with a + without telling you that they have taken account of the change in direction Allow M1 if they go straight to an unsimplified expression in surds using values already found earlier.
A1	Correct unsimplified expression. Allow the negative of this
A1	Any equivalent simplified form. Must be positive. Condone if they change sign at the very end without explaining why. Accept $9.2(376...)mu$ (2 sf or better)
	NB You might see candidates using the right angle and matrix multiplication to rotate the initial velocity of *B* to find the correct components of the velocity of *B* after impact.

8.

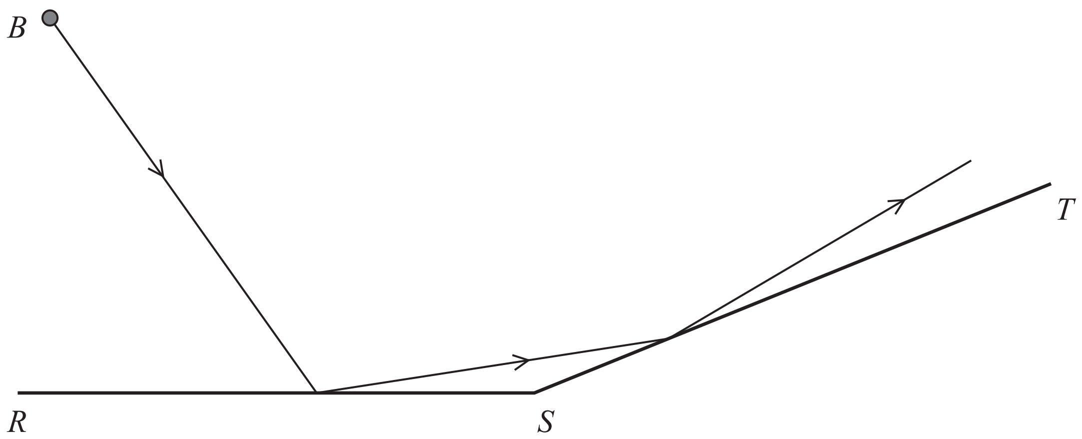


Figure 5

Figure 5 represents the plan view of part of a smooth horizontal floor, where *RS* and *ST* are smooth fixed vertical walls. The vector $\overrightarrow{RS}$ is in the direction of **i** and the vector $\overrightarrow{ST}$ is in the direction of $(2\mathbf{i} + \mathbf{j})$.

A small ball *B* is projected across the floor towards *RS*. Immediately before the impact with *RS,* the velocity of *B* is $(6\mathbf{i} - 8\mathbf{j})\,\text{m s}^{-1}$. The ball bounces off *RS* and then hits *ST*.

The ball is modelled as a particle.

Given that the coefficient of restitution between *B* and *RS* is *e*,

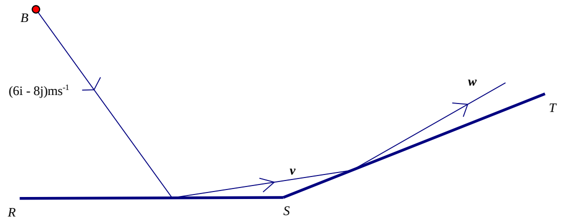
(a) find the full range of possible values of *e*.

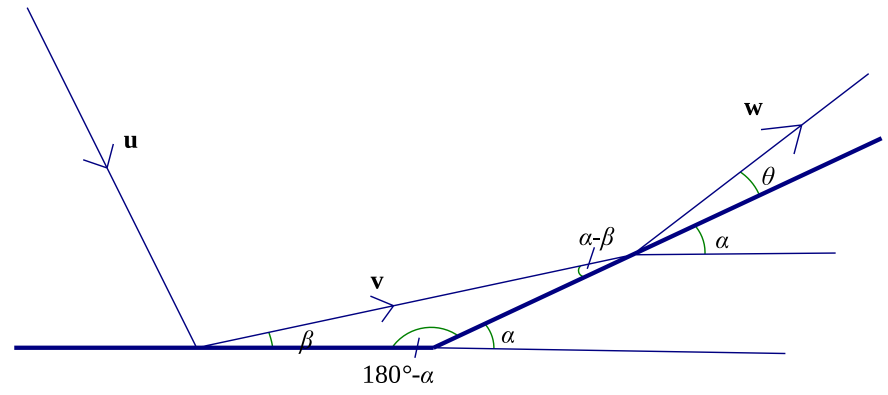
(3)

It is now given that $e = \dfrac{1}{4}$ and that the coefficient of restitution between *B* and *ST* is $\dfrac{1}{2}$

(b) Find, in terms of **i** and **j**, the velocity of *B* immediately after its impact with *ST*.

(7)

Question	Scheme	Marks	AOs
8a			
	$\mathbf{v} = 6\mathbf{i} + ...$	B1	3.4
	$.....8e\mathbf{j}$	B1	3.4
	impact with $ST \Rightarrow \frac{8e}{6} < \frac{1}{2}$, $0 < e < \frac{3}{8}$	B1	3.1b
		(3)	
8b	Perpendicular to ST: direction $\pm\mu(-\mathbf{i}+2\mathbf{j})$	B1	1.2
	Component parallel to ST: $(6\mathbf{i}+2\mathbf{j})\cdot\lambda(2\mathbf{i}+\mathbf{j})$	M1	3.1b
	$= \left((6\mathbf{i}+2\mathbf{j})\cdot\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{j}) = \right)\frac{1}{\sqrt{5}}(12+2)$	A1	1.1b
	Component perpendicular to ST: $\pm\left(\frac{1}{2}(6\mathbf{i}+2\mathbf{j})\cdot\gamma(-\mathbf{i}+2\mathbf{j})\right)$	M1	3.4
	$= \frac{1}{2\sqrt{5}}(-6+4)$	A1	1.1b
	$\mathbf{w} = \frac{14}{\sqrt{5}}\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{j}) + \frac{1}{\sqrt{5}}\frac{1}{\sqrt{5}}(-\mathbf{i}+2\mathbf{j})$	M1	3.1b
	$\mathbf{w} = \left(\frac{28}{5}-\frac{1}{5}\right)\mathbf{i} + \left(\frac{14}{5}+\frac{2}{5}\right)\mathbf{j} = \left(\frac{27}{5}\mathbf{i}+\frac{16}{5}\mathbf{j}\right)$ or $(5.4\mathbf{i}+3.2\mathbf{j})$ (m s^{-1})	A1	2.2a
		(7)	
8b alt 1	Perpendicular to ST: direction $\pm\mu(-\mathbf{i}+2\mathbf{j})$	B1	1.2
	$\mathbf{w} = a\mathbf{i}+b\mathbf{j} \Rightarrow (6\mathbf{i}+2\mathbf{j}).(2\mathbf{i}+\mathbf{j}) = (a\mathbf{i}+b\mathbf{j}).(2\mathbf{i}+\mathbf{j})$	M1	
	$14 = 2a+b$	A1	
	$\pm\frac{1}{2}(6\mathbf{i}+2\mathbf{j}).(-\mathbf{i}+2\mathbf{j}) = (a\mathbf{i}+b\mathbf{j}).(-\mathbf{i}+2\mathbf{j})$	M1	
	$2b - a = \pm 1$	A1	
	Solve simultaneous equations for a and b	M1	
	$\mathbf{w} = \left(\frac{27}{5}\mathbf{i}+\frac{16}{5}\mathbf{j}\right)$ or $(5.4\mathbf{i}+3.2\mathbf{j})$ (m s^{-1})	A1	
		(7)	
8balt 2	Perpendicular to ST: direction $\pm\mu(-\mathbf{i}+2\mathbf{j})$	B1	1.2
	$\mathbf{v} = 6\mathbf{i}+2\mathbf{j} = p(2\mathbf{i}+\mathbf{j}) + q(-\mathbf{i}+2\mathbf{j})$	M1	3.1b

	$6 = 2p - q\ ,\ \ 2 = p + 2q\ \ \left(p = \frac{14}{5},\ \ q = \frac{-2}{5}\right)$	A1	1.1b
	Component perpendicular to ST $\pm\frac{1}{2}\times q(-\mathbf{i}+2\mathbf{j})$	M1	3.4
	$\pm\frac{1}{2}\times q(-\mathbf{i}+2\mathbf{j})$	A1	1.1b
	Solve for p and q to obtain velocity $\mathbf{w} = \frac{14}{5}(2\mathbf{i}+\mathbf{j})+\frac{1}{2}\times\frac{2}{5}(-\mathbf{i}+2\mathbf{j})$	M1	3.1b
	$\mathbf{w} = \left(\frac{27}{5}\mathbf{i}+\frac{16}{5}\mathbf{j}\right)$ or $(5.4\mathbf{i}+3.2\mathbf{j})$ (m s^{-1})	A1	2.2a
		(7)	
8balt 3			
	$\alpha - \beta = 8.1...°$	B1	
	Component of **w** parallel to ST is $\lvert\mathbf{v}\rvert\cos(\alpha-\beta)$	M1	
	$= \sqrt{40}\cos(\alpha-\beta)\left(= \sqrt{40}\times\frac{7}{\sqrt{50}} = 6.26..\right)$	A1	
	Component of **w** perpendicular to ST is $\frac{1}{2}\lvert\mathbf{v}\rvert\sin(\alpha-\beta)$	M1	
	$\pm\frac{1}{2}\times\sqrt{40}\sin(\alpha-\beta)\left(= \frac{\sqrt{40}}{2}\times\frac{1}{\sqrt{50}} = 0.447...\right)$	A1	
	$\mathbf{w} = \lvert\mathbf{w}\rvert\cos(\alpha+\theta)\mathbf{i} + \lvert\mathbf{w}\rvert\sin(\alpha+\theta)\mathbf{j}$	M1	
	$\mathbf{w} = \left(\frac{27}{5}\mathbf{i}+\frac{16}{5}\mathbf{j}\right)$ or $(5.4\mathbf{i}+3.2\mathbf{j})$ (m s^{-1})	A1	
		(7)	
			(Total 10 marks)

8a	
B1	Component parallel to the wall unchanged. Could be on a diagram or implied if they use $e\tan\alpha = \tan\beta$
B1	Use of impact law perpendicular to the wall. Could be on a diagram or implied if they use $e\tan\alpha = \tan\beta$

B1	Use the direction to determine the range for *e*. (could come via $e\tan\alpha = \tan\beta < 1/2$)
8b	
B1	Correct vector perpendicular to *ST* seen or implied μ can have any scalar value
M1	Use scalar product to find component of **v** parallel to *ST*. λ can have any scalar value
A1	Correct unsimplified expression for the magnitude
M1	Use scalar product and impact law perpendicular to *ST* to find magnitude of component perpendicular to the wall. For their perpendicular vector Must clearly be using $e = \frac{1}{2}$. γ can have any scalar value.
A1	Correct unsimplified expression for the perpendicular component. Allow ±
M1	Combine the magnitudes and directions to obtain the velocity. The perpendicular should now be in the correct direction.
A1	Correct simplified velocity.
8b alt	
B1	Correct vector perpendicular to *ST* seen or implied. μ can have any scalar value
M1	Correct method for component parallel to *ST*
A1	Correct equation in *a* and *b*
M1	Correct method for component perpendicular to *ST* Allow ± For their perpendicular vector
A1	Correct equation in *a* and *b*
M1	Solve for *a* and *b* to obtain velocity. Using the correct direction for the perpendicular component
A1	Correct simplified answer.
8b alt2	
B1	Correct vector perpendicular to *ST* seen or implied. μ can have any scalar value
M1	Split **v** into components parallel and perpendicular to *ST*
A1	Two equations in *p* and *q*
M1	Use the impact law perpendicular to *ST* For their perpendicular vector
A1	Correct unsimpified perpendicular component. With *q* or their *q*

M1	Solve for p and q to obtain velocity Using the correct direction for the perpendicular component
A1	Correct simplified total.
8balt3	
B1	Seen or implied. $\sin(\alpha-\beta)=\frac{1}{\sqrt{50}}, \cos(\alpha-\beta)=\frac{7}{\sqrt{50}},$ $\tan(\alpha-\beta)=\frac{1}{7}$
M1	Correct use of their $\lvert\mathbf{v}\rvert$ and their $\alpha-\beta$
A1	Correct unsimplified
M1	Correct use of $\frac{1}{2}$, their $\lvert\mathbf{v}\rvert$ and their $\alpha-\beta$
A1	Correct unsimplified
M1	Use of Pythagoras and correct method for $\theta+\alpha$. $\cos(\alpha+\theta)=\frac{27}{\sqrt{5}\sqrt{197}}, \sin(\alpha+\theta)=\frac{16}{\sqrt{5}\sqrt{197}}$ $\alpha+\theta=30.65°$
A1	Correct simplified total.

2.

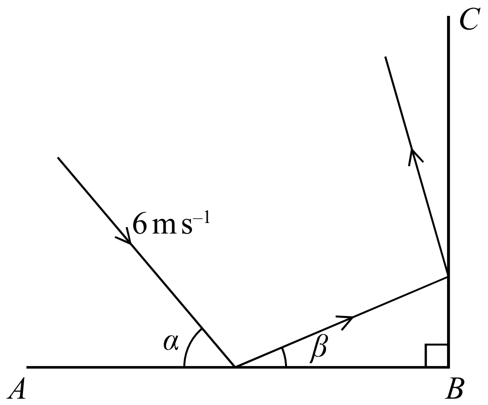


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where *AB* and *BC* are fixed vertical walls with *AB* perpendicular to *BC*.

A small ball is projected along the floor towards *AB* with speed $6\,\mathrm{m\,s^{-1}}$ on a path that makes an angle α with *AB*, where $\tan\alpha = \frac{4}{3}$. The ball hits *AB* and then hits *BC*.

Immediately after hitting *AB*, the ball is moving at an angle β to *AB*, where $\tan\beta = \frac{1}{3}$

The coefficient of restitution between the ball and *AB* is e.

The coefficient of restitution between the ball and *BC* is $\frac{1}{2}$

By modelling the ball as a particle and the floor and walls as being smooth,

(a) show that the value of $e = \frac{1}{4}$

(5)

(b) find the speed of the ball immediately after it hits *BC*.

(4)

(c) Suggest two ways in which the model could be refined to make it more realistic.

(2)

P 6 2 6 7 4 A 0 6 2 8

Question	Scheme	Marks	AOs
2(a)	$F_{\max} = \frac{1}{4}mg\cos\alpha = \frac{1}{5}mg$	B1	1.2
	$mg\sin\alpha = \frac{3}{5}mg > \frac{1}{5}mg \Rightarrow$ slides down	B1	2.2a
(b)		**(2)**	
	Using work-energy principle to solve the problem	M1	3.4
	$\frac{1}{2}m\times(7^2 - V^2) = \frac{1}{5}mg\times 2\times\frac{25}{8}$	A1	1.1b
	OR: $mg\times\frac{25}{8}\times\frac{3}{5} - \frac{1}{2}mV^2 = \frac{1}{5}mg\times\frac{25}{8}$	A1	1.1b
	$V = 4.9$ or 4.95	A1	1.1b
		(4)	
(c)	e.g. Include air resistance in the model.	B1	3.5c
		(1)	
			(7 marks)

Notes:

(a)
B1: Correct expression for max friction
B1: Correct deduction from comparing weight component with Fmax

(b)
M1: Using the work-energy principle with correct no. of terms (either start to finish or descent only)
A1: Correct equation, condone 1 error
A1: Correct equation
A1: 4.9 or 4.95 (m)

(c)
B1: Other refinements e.g. allow for spin of box, dimensions of box, more accurate value of g

6. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

A smooth uniform sphere A has mass 0.2 kg and another smooth uniform sphere B, with the same radius as A, has mass 0.4 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, the velocity of A is $(3\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-4\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

At the instant of collision, the line joining the centres of the spheres is parallel to **i**

The coefficient of restitution between the spheres is $\frac{3}{7}$

(a) Find the velocity of A immediately after the collision.

(7)

(b) Find the magnitude of the impulse received by A in the collision.

(2)

(c) Find, to the nearest degree, the size of the angle through which the direction of motion of A is deflected as a result of the collision.

(3)

Question	Scheme	Marks	AOs
6(a)	Overall strategy to set up an equation in one unknown using equilibrium condition and resolving vertically: $2T\times\frac{4}{5}=4mg$	M1	3.1a
	$T=\frac{5mg}{2}$	A1	1.1b
	Use of Hooke's Law	M1	3.1a
	$\frac{5mg}{2}=\frac{5mg}{3}\frac{\left(5a-\frac{1}{2}l\right)}{\frac{1}{2}l}$ **OR** $\frac{5mg}{3}\frac{(10a-l)}{l}$	A1	1.1b
	$l=4a$ *	A1*	1.1b
		(5)	
(b)	Max speed is at equilibrium position	B1	3.1a
	Use of EPE = $\frac{\lambda x^2}{2l}$	M1	3.1a
	Use of conservation of energy principle	M1	3.1a
	$\frac{5mg}{3\times 8a}\left\{(6a)^2-(2a)^2\right\}=4mg\times 4a-\frac{1}{2}4mv^2$	A1	1.1b
		A1	1.1b
	$v=\sqrt{\frac{14ag}{3}}$	A1	1.1b
		(6)	
		(11 marks)	

Notes:

(a)
M1: Correct no. of terms with T resolved and correct equation in T only
A1: Correct tension
M1: Use of Hooke's Law
A1: Correct unsimplified equation
A1*: Given answer

(b)
B1: Use of max speed at equilm to solve the problem
M1: Use of EPE formula
M1: Use of Conservation of energy to solve the problem
A1: Correct unsimplified equation with one error
A1: Correct unsimplified equation
A1: cao oe

4.

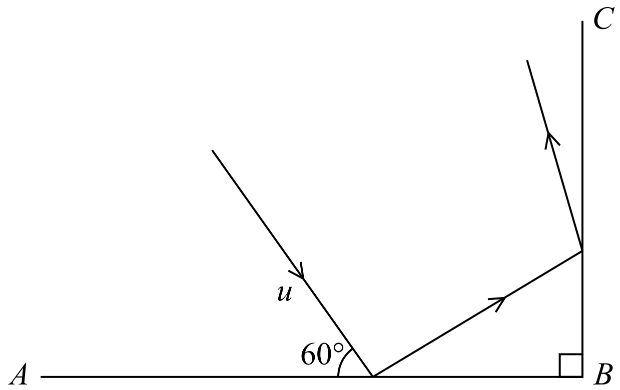


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where *AB* and *BC* are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards *AB* with speed $u\,\mathrm{m\,s^{-1}}$ on a path at an angle of 60° to *AB*. The ball hits *AB* and then hits *BC*.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall *AB* is $\dfrac{1}{\sqrt{3}}$

The coefficient of restitution between the ball and wall *BC* is $\sqrt{\dfrac{2}{5}}$

(a) Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

(8)

(b) In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

(1)

Question	Scheme	Marks	AOs
4(a)	Complete strategy to find the kinetic energy after the second impact	M1	3.1b
	Parallel to AB after collision: $u\cos 60°$	M1	3.1b
	Perpendicular to AB after collision: $\frac{1}{\sqrt{3}}u\sin 60°$	M1	3.4
	Components of velocity after first impact: $\frac{u}{2}, \frac{u}{2}$	A1	1.1b
	Parallel to BC after collision: $\frac{u}{2}$ $\left(u\times\frac{1}{\sqrt{3}}\sin 60°\right)$	M1	3.1b
	Perpendicular to BC after collision: $\sqrt{\frac{2}{5}}\times\frac{u}{2}\left(=\frac{1}{\sqrt{10}}u\right)$ $\left(\sqrt{\frac{2}{5}}\times u\cos 60°\right)$	M1	3.4
	Components of velocity after second impact: $\frac{u}{2}, \frac{u}{\sqrt{10}}$	A1	1.1b
	Final KE = $\frac{1}{2}m\left(\frac{u^2}{4}+\frac{u^2}{10}\right)$ $\left(=\frac{mu^2}{2}\times\frac{7}{20}\right)$		
	Fraction of initial KE $=\dfrac{\frac{mu^2}{2}\times\frac{7}{20}}{\frac{mu^2}{2}}=\frac{7}{20}=35\%$ *	A1*	2.2a
		(8)	
(b)	The answer is too large - rough surface means resistance so final speed will be lower	B1	3.5a
		(1)	
		(9 marks)	

Notes:

(a)

M1: Use of CLM parallel to the wall. Condone sin/cos confusion

M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion

A1: Both components correct with trig substituted (seen or implied)

M1: Use of CLM parallel to the wall. Condone sin/cos confusion

M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion

A1: Both components correct with trig substituted (seen or implied)

M1: Correct expression for total KE using their components after 2nd collision

A1*: Obtain **given answer** with sufficient working to justify it

(b)

B1: Clear explanation of how the modelling assumption has affected the outcome

6. [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass $3m$ kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of A is $(3\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\frac{1}{4}$

(a) Find the velocity of B immediately after the collision.

(7)

(b) Find, to the nearest degree, the size of the angle through which the direction of motion of B is deflected as a result of the collision.

(2)

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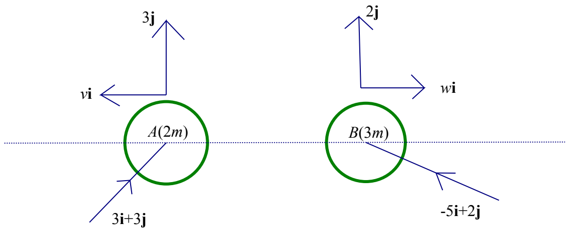
Question	Scheme	Marks	AOs
6(a)			
	Overall strategy to find $\mathbf{v}_A$	M1	3.1a
	Velocity of A perpendicular to loc after collision $= 3\mathbf{j}$ (m s^{-1})	B1	3.4
	CLM parallel to loc	M1	3.1a
	$2m \times 3 - 3m \times 5 = 3mw - 2mv \quad (-9 = 3w - 2v)$	A1	1.1b
	Correct use of impact law	M1	3.1a
	$v + w = \frac{1}{4}(3+5) \; (= 2)$	A1	1.1b
	Solve for w $\quad 3w - 2v = -9$, $\; 2v + 2w = 4$		
	$\mathbf{v}_B = -\mathbf{i} + 2\mathbf{j}$ (m s^{-1}),	A1ft	1.1b
		(7)	
(b)	$\cos\theta = \frac{(-5\mathbf{i}+2\mathbf{j}).(-\mathbf{i}+2\mathbf{j})}{\sqrt{29}\sqrt{5}}$	M1	3.1a
	$\theta = 41.63...° = 42°$ (nearest degree)	A1	1.1b
	Alternative method: $\tan^{-1} 2 - \tan^{-1}\frac{2}{5} = 41.63...° = 42°$ (nearest degree)		
		(2)	
			(9 marks)

Notes:

(a)

M1: Correct overall strategy to form sufficient equations and solve for $\mathbf{v}_A$

B1: Use the model to find the component of $\mathbf{v}_A$ perpendicular to the line of centres

M1: Use CLM to form equation in v and w. Need all 4 terms, dimensionally correct

A1: Correct unsimplified

M1: Must be used the right way round

A1: Correct unsimplified

A1ft: $\mathbf{v}_B$ correct. Follow their $2\mathbf{j}$

(b)

M1: Complete method for finding the required angle. Follow their $\mathbf{v}_B$

A1: cao

5. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

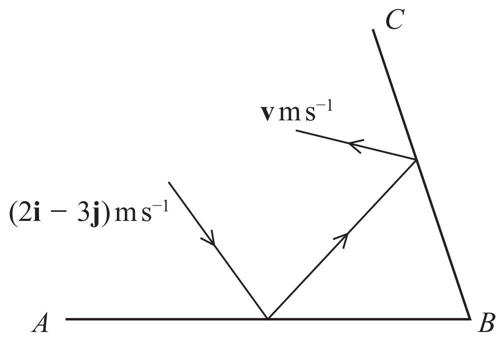


Figure 3

Figure 3 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls. The direction of $\overrightarrow{AB}$ is in the direction of the vector **i** and the direction of $\overrightarrow{BC}$ is in the direction of the vector (−**i** + 3**j**).

A small ball is projected along the floor towards wall AB so that, immediately before hitting wall AB, the velocity of the ball is $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$.

The ball hits wall AB and then hits wall BC.

The coefficient of restitution between the ball and wall AB is $\frac{1}{2}$

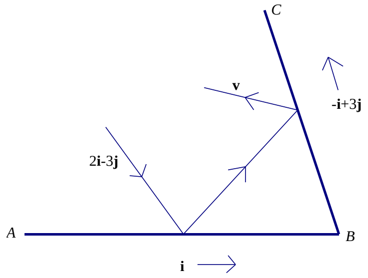
The coefficient of restitution between the ball and wall BC is $\frac{1}{3}$

The velocity of the ball immediately after hitting wall BC is $\mathbf{v}\,\text{m s}^{-1}$.

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

Show that $\mathbf{v} = \left(-\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$.

(12)

Question	Scheme	Marks	AOs
5			
	After first impact: parallel to *AB* $2\mathbf{i}$	B1	2.1
	Use of impact law perpendicular to *AB*	M1	3.4
	$-\frac{1}{2}(-3\mathbf{j}) \;= \frac{3}{2}\mathbf{j}$	A1	1.1b
	Strategy to find final velocity	M1	3.1b
	Second impact: parallel to *BC* $\mathbf{v}\cdot\left(\frac{-\mathbf{i}+3\mathbf{j}}{\left(\sqrt{10}\right)}\right)$	M1	3.1b
	$\left(\left(2\mathbf{i}+\frac{3}{2}\mathbf{j}\right)\cdot\left(\frac{-\mathbf{i}+3\mathbf{j}}{\left(\sqrt{10}\right)}\right)=\frac{5}{2\sqrt{10}}\right)$ follow their **v**	A1ft	1.1b
	Component of velocity $=\frac{5}{2\sqrt{10}}\times\left(\frac{-\mathbf{i}+3\mathbf{j}}{\sqrt{10}}\right)=\frac{1}{4}(-\mathbf{i}+3\mathbf{j})$	A1	1.1b
	Vector perpendicular to the wall $(3\mathbf{i}+\mathbf{j})$	B1	3.1b
	Use of impact law:	M1	3.4
	$-\frac{1}{3}\left(2\mathbf{i}+\frac{3}{2}\mathbf{j}\right)\cdot\frac{(3\mathbf{i}+\mathbf{j})}{\left(\sqrt{10}\right)}$ Follow their velocity and their perpendicular vector	A1ft	1.1b
	Component of velocity $=-\frac{5}{2\sqrt{10}}\times\left(\frac{3\mathbf{i}+\mathbf{j}}{\sqrt{10}}\right)=-\frac{1}{4}(3\mathbf{i}+\mathbf{j})$	A1	1.1b
	$\Rightarrow \mathbf{v}=\frac{1}{4}(-\mathbf{i}+3\mathbf{j})-\frac{1}{4}(3\mathbf{i}+\mathbf{j})$ (sum of their components)		
	$=\left(-\mathbf{i}+\frac{1}{2}\mathbf{j}\right)$ (m s^{-1}) *	A1*	2.2a
		(12)	
5 alt	***For the last 9 marks***		
	Strategy to find final velocity	M1	
	Perpendicular to $-\mathbf{i}+3\mathbf{j}$ is $-3\mathbf{i}-\mathbf{j}$	B1	
	Find components of the initial velocity parallel and perpendicular to $-\mathbf{i}+3\mathbf{j}$: $\mathbf{v}=p(-\mathbf{i}+3\mathbf{j})+q(-3\mathbf{i}-\mathbf{j})$	M1	

	$\begin{cases} 2 = -p - 3q \\ \frac{3}{2} = 3p - q \end{cases} \Rightarrow p = \frac{1}{4}$	A1	
	$q = -\frac{3}{4}$, $\left(\mathbf{v} = \frac{1}{4}(-\mathbf{i} + 3\mathbf{j}) - \frac{3}{4}(-3\mathbf{i} - \mathbf{j})\right)$	A1	
	Impact law perpendicular to plane: $\pm\frac{1}{3} \times -\frac{3}{4}(-3\mathbf{i} - \mathbf{j})$	M1	
	Follow their perpendicular component	A1ft	
	Parallel component: $\frac{1}{4}(-\mathbf{i} + 3\mathbf{j})$ Follow their parallel component	A1ft	
	Final velocity $= \frac{1}{4}(-\mathbf{i} + 3\mathbf{j}) + \frac{1}{4}(-3\mathbf{i} - \mathbf{j}) = -\mathbf{i} + \frac{1}{2}\mathbf{j}$ *	A1*	
			(12 marks)

Notes:

5	B1	Conservation of component parallel to the first wall
	M1	Use the impact law on the model to find the component of the velocity perpendicular to *AB* after the impact
	A1	Correct value
	M1	Complete strategy to find final velocity: find components parallel and perpendicular to *BC* and add.
	M1	Scalar product of their velocity with a vector parallel to *BC* . Condone missing modulus.
	A1	Correct unsimplified (follow their $2\mathbf{i} + \frac{3}{2}\mathbf{j}$)
	A1	Correct parallel component
	B1	Any parallel vector
	M1	Correct use of the model and the impact law to find the magnitude of the perpendicular component. Condone missing modulus.
	A1ft	Correct unsimplified. Follow their $2\mathbf{i} + \frac{3}{2}\mathbf{j}$ and their perpendicular vector
	A1	Correct perpendicular component
	A1*	Combine the components to deduce the **given answer**

7.

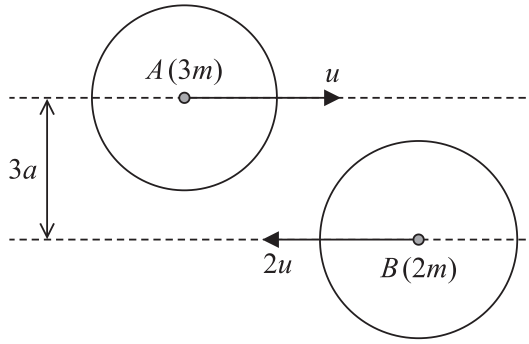


Figure 4

Two smooth uniform spheres, A and B, are moving with speeds u and $2u$ respectively on a smooth horizontal surface.

Sphere A has mass $3m$ and radius $2a$. Sphere B has mass $2m$ and radius $2a$.

The centres of the spheres are moving towards each other on parallel paths. The paths are at a distance $3a$ apart, as shown in Figure 4.

The spheres collide. The coefficient of restitution between A and B is $\frac{1}{3}$

(a) Show that the magnitude of the impulse received by A in the collision is $\frac{6\sqrt{7}}{5}mu$.

(10)

(b) Find the speed of A immediately after the collision.

(3)

(c) State how you have used the fact that the spheres are smooth when considering their collision.

(1)

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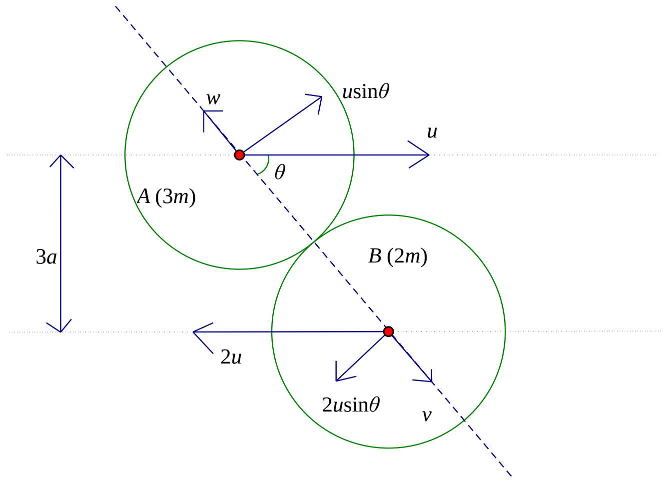
Question	Scheme	Marks	AOs
7(a)			
	Complete strategy to find impulse	M1	3.1a
	CLM parallel to line of centres	M1	3.1a
	$3mw-2mv=2m.2u\cos\theta-3m.u\cos\theta$ $(=mu\cos\theta)$	A1	1.1b
	Use of impact law parallel to line of centres	M1	3.1a
	$w+v=\frac{1}{3}(u\cos\theta+2u\cos\theta)(=u\cos\theta)$	A1	1.1b
	Solve for v or w: $\begin{cases}3w-2v=u\cos\theta\\2w+2v=2u\cos\theta\end{cases}\Rightarrow w=\frac{3}{5}u\cos\theta\quad\left(v=\frac{2}{5}u\cos\theta\right)$	A1	1.1b
	Correct trig ratio used $\left(\cos\theta=\frac{\sqrt{7}}{4},\quad\sin\theta=\frac{3}{4}\right)$	B1	1.1b
	Magnitude of impulse $=\lvert 3m(w\pm u\cos\theta)\rvert$	M1	3.1a
	$=\left\lvert 3m\left(\frac{3}{5}u\cos\theta+u\cos\theta\right)\right\rvert$	A1	1.1b
	$=\left\lvert 3m\left(\frac{8}{5}u\times\frac{\sqrt{7}}{4}\right)\right\rvert=\frac{6\sqrt{7}}{5}mu$ *	A1*	2.2a
		(10)	
7(b)	Component of velocity perpendicular to line of centres $=u\sin\theta$	B1	3.4
	Speed $=\sqrt{(u\sin\theta)^2+\left(\frac{3}{5}u\cos\theta\right)^2}$ for their w	M1	2.1
	$=u\sqrt{\frac{9}{16}+\frac{9\times7}{25\times16}}=\frac{3\sqrt{2}}{5}u$	A1	1.1b
		(3)	

7(c)	Impulse only acts along the line of centres	B1	3.5b
		(1)	
			(14 marks)

Notes:

7a	M1	Over all strategy: form and solve simultaneous equations and use impulse/momentum.
	M1	Use of CLM parallel to l of c. All terms needed. Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation
	M1	Must be used the right way round. Follow their components of u and $2u$.
	A1	Correct unsimplified equation
	A1	v or w correct in terms of u and θ
	B1	Correct trig ratio seen or implied
	M1	Magnitude of impulse on either particle. Must be using change in component of velocity.
	A1	Correct unsimplified in terms of m, u and θ
	A1*	Substitute trig values and deduce the **given result**
7b	B1	Use conservation of component of velocity perpendicular to line of centres
	M1	Use of Pythagoras to combine the components parallel and perpendicular to the line of centres. Follow their w.
	A1	Any equivalent simplified form
7c	B1	Any valid modelling assumption – no spin, no friction, no change perpendicular to the line of centres