Fm1Ch4 XMQs and MS

(Total: 211 marks)

1. FM1_2019a Q1 . 8 marks - FM1ch4 Elastic collisions in one dimension
2. FM1_2019a Q5 . 11 marks - FM1ch4 Elastic collisions in one dimension
3. FM1_2020 Q3 . 14 marks - FM1ch4 Elastic collisions in one dimension
4. FM1_2021 Q2 . 14 marks - FM1ch4 Elastic collisions in one dimension
5. FM1_2022 Q5 . 10 marks - FM1ch4 Elastic collisions in one dimension
6. FM1_2019b Q1 . 8 marks - FM1ch4 Elastic collisions in one dimension
7. FM1_2019b Q5 . 11 marks - FM1ch4 Elastic collisions in one dimension
8. FM1_Sample Q3 . 8 marks - FM1ch4 Elastic collisions in one dimension
9. FM1_Sample Q8 . 14 marks - FM1ch4 Elastic collisions in one dimension
10. FM1_Specimen Q6 . 14 marks - FM1ch4 Elastic collisions in one dimension
11. FM1(AS)_2018 Q4 . 14 marks - FM1ch4 Elastic collisions in one dimension
12. FM1(AS)_2019 Q2 . 13 marks - FM1ch4 Elastic collisions in one dimension
13. FM1(AS)_2019 Q4 . 10 marks - FM1ch4 Elastic collisions in one dimension
14. FM1(AS)_2020 Q1 . 5 marks - FM1ch4 Elastic collisions in one dimension
15. FM1(AS)_2020 Q3 . 12 marks - FM1ch4 Elastic collisions in one dimension
16. FM1(AS)_2021 Q2 . 9 marks - FM1ch4 Elastic collisions in one dimension
17. FM1(AS)_2021 Q4 . 13 marks - FM1ch4 Elastic collisions in one dimension
18. FM1(AS)_2022 Q2 . 8 marks - FM1ch4 Elastic collisions in one dimension
19. FM1(AS)_2022 Q4 . 15 marks - FM1ch4 Elastic collisions in one dimension

1.

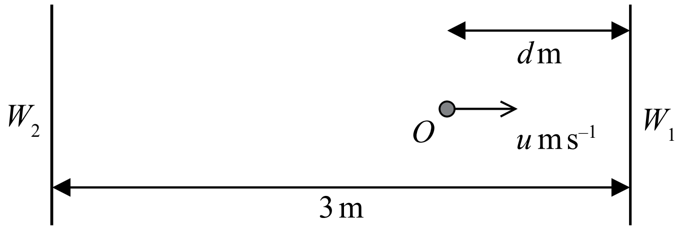


Figure 1

Figure 1 represents the plan of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls. The walls are 3 metres apart.

A particle lies at rest at a point O on the floor between the two walls, where the point O is d metres, $0 < d \leqslant 3$, from W_1

At time $t = 0$, the particle is projected from O towards W_1 with speed u m s^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$

The particle returns to O at time $t = T$ seconds, having bounced off each wall once.

(a) Show that $T = \dfrac{45 - 5d}{4u}$

(6)

The value of u is fixed, the particle still hits each wall once but the value of d can now vary.

(b) Find the least possible value of T, giving your answer in terms of u. You must give a reason for your answer.

(2)

P 6 2 6 7 4 A 0 2 2 8

Question	Scheme	Marks	AOs	Notes
1a	Speed after first impact $=\frac{2}{3}u$	B1	3.4	Correct use of impact law, seen or implied. Allow +/-
	Speed after second impact $=\frac{4}{9}u$	B1	3.4	Correct use of impact law a second time, seen or implied. Allow +/-
	Correct method for total time	M1	2.1	Use of $t=\frac{d}{v}$ or equivalent for at least 2 of the 3 parts added
	$T=\frac{d}{u}+\frac{3}{\frac{2}{3}u}+\frac{3-d}{\frac{4}{9}u}$	A1ft A1ft	1.1b 1.1b	Unsimplified expression for *T* with all 3 terms and at most one error. Follow their speeds. Correct unsimplified expression for *T*. Follow their speeds
	$=\frac{4d+18+27-9d}{4u}=\frac{45-5d}{4u}$ *	A1*	2.2a	Obtain **given answer** from correct working
		(6)		
1b	• Least *T* when *d* is maximum • Furthest distance at highest speed • Highest average speed • Sketch graph of function	B1	2.4	Correct reasoning
	i.e. $d=3$, least $T=\frac{30}{4u}=\frac{15}{2u}$	B1	2.2a	Correct answer only. Any equivalent form. $\left(\frac{7.5}{u}\right)$
		(2)		
(8 marks)				

5. A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$.

Immediately after the collision P and Q are moving in opposite directions.

The coefficient of restitution between P and Q is e.

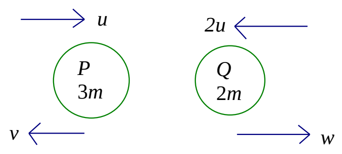
(a) Find the range of possible values of e, justifying your answer.

(8)

Given that Q loses 75% of its kinetic energy as a result of the collision,

(b) find the value of e.

(3)

Question	Scheme	Marks	AOs	Notes
5(a)				
	Use of CLM	M1	3.1a	Use of CLM. All terms required. Must be dimensionally correct. Condone sign errors
	$3mu-4mu=2mw-3mv \quad (-u=-3v+2w)$	A1	1.1b	Correct unsimplified equation
	Use of impact law	M1	3.4	Use of impact law. Must be dimensionally correct and used correctly. Condone sign errors
	$w+v=3ue$	A1	1.1b	Correct unsimplified equation Signs consistent with CLM equation
	Correct strategy to form equation in w and find critical value of $e\in(0,1)$ $(5w=u(9e-1))$	M1	3.1a	Correct overall strategy to find the critical value of e in (0,1) in e eg by using CLM and impact law to form equation or inequality in w and solve for e.
	$w>0: e>\frac{1}{9}$	A1	1.1b	One inequality for e correct Condone $e\geq\frac{1}{9}$
	Complete strategy to justify the range of values of e $(5v=u(1+6e)) \quad v>0$: true for all e	M1	3.1a	Correct strategy to find the range of possible value of e. i.e find second speed and form second inequality
	Therefore $\frac{1}{9}<e\leq 1$	A1	2.2a	Correct final conclusion
		(8)		

Question	Scheme	Marks	AOs	Notes
5(b)	Final KE = 25% of initial KE	M1	3.1a	Use KE to form equation in *e*. 25% should be used correctly Condone if mass cancelled throughout
	$\frac{1}{2}\times 2m\times\frac{u^2(9e-1)^2}{25}=\frac{1}{4}\times\frac{1}{2}\times 2m\times 4u^2$ $\left(\text{or } w=\frac{1}{2}\times 2u\right)$	A1ft	1.1b	Correct unsimplified equation – follow their *w*
	$\Rightarrow (9e-1)^2=25,\ \ e=\frac{2}{3}$ only	A1	1.1b	Or equivalent. Correct conclusion ISW after correct answer.
		(3)		
			(11marks)	

3. Two particles, A and B, have masses $3m$ and $4m$ respectively. The particles are moving in the same direction along the same straight line on a smooth horizontal surface when they collide directly. Immediately before the collision the speed of A is $2u$ and the speed of B is u.

The coefficient of restitution between A and B is e.

(a) Show that the direction of motion of each of the particles is unchanged by the collision.

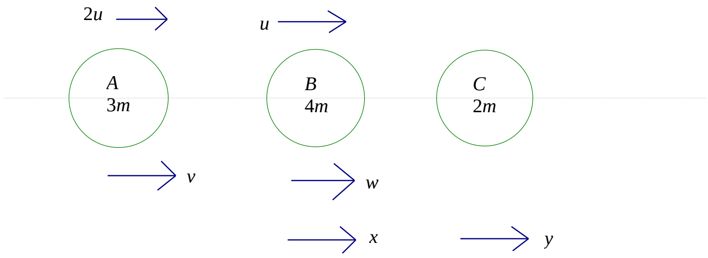
(8)

After the collision with A, particle B collides directly with a third particle, C, of mass $2m$, which is at rest on the surface.

The coefficient of restitution between B and C is also e.

(b) Show that there will be a second collision between A and B.

(6)

Question	Scheme	Marks	AOs
3(a)	Taking left to right as positive, 		
	CLM:	M1	3.1a
	$6mu+4mu\left(=10mu\right)=3mv+4mw \quad \left(10u=3v+4w\right)$	A1	1.1b
	Impact Law:	M1	3.4
	$w-v=e\left(2u-u\right)\left(=eu\right)$	A1	1.1b
	Solve for v or w	M1	2.1
	$w=\frac{u}{7}\left(10+3e\right)$	A1	1.1b
	$v=\frac{u}{7}\left(10-4e\right)$	A1	1.1b
	$0\le e\le 1 \quad \Rightarrow \quad 10+3e>0$ and $10-4e>0$ hence both particles still travelling in the original direction. *	A1*	2.2a
		(8)	
(b)	CLM: $4mw=4mx+2my \quad \left(2w=2x+y\right)$	M1	3.1a
	Impact: $y-x=ew$	M1	3.4
	$\Rightarrow w\left(2-e\right)=3x \ , \ x=\frac{u}{21}\left(10+3e\right)\left(2-e\right)$	M1	1.1b
	Consider $v-x$ i.e. $\frac{u}{7}\left(10-4e\right)-\frac{u}{21}\left(10+3e\right)\left(2-e\right)$ $\left(3e^2-8e+10\right)$	M1	2.1
	Show that $v-x>0 \ \forall e$	M1	1.1b
	Complete correct argument and conclusion *	A1*	2.2a
		(6)	
			(14 marks)

Notes:

(a)M1	All terms required. Condone sign errors.
A1	Correct unsimplified equation

M1	Law used correctly. Condone sign errors
A1	Correct unsimplified equation
M1	Use their correctly formed equations to solve for v or w
A1	Either velocity correct
A1	Both velocities correct
A1*	Use possible values of e to justify given result from correct working.
(b)M1	All terms required. Condone sign errors
M1	Correct use of impact law. Condone sign errors
M1	Use their correctly formed equtions to find velocity of B (x)
M1	Form relevant difference for a second collision
M1	Complete correct method (e.g. differentiation or completing the square or discriminant) to determine when inequality is true
A1*	Reach correct conclusion from correct work.

2. Two particles, A and B, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Particle A has mass $5m$ and particle B has mass $3m$.

The coefficient of restitution between A and B is e, where $e > 0$

Immediately **after** the collision the speed of A is v and the speed of B is $2v$.

Given that A and B are moving in the same direction after the collision,

(a) find the set of possible values of e.

(8)

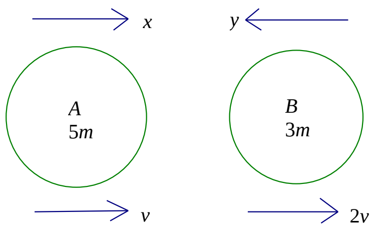
Given also that the kinetic energy of A immediately after the collision is 16% of the kinetic energy of A immediately before the collision,

(b) find

(i) the value of e,

(ii) the magnitude of the impulse received by A in the collision, giving your answer in terms of m and v.

(6)

Question	Scheme	Marks	AOs
2(a)			
	Use of CLM	M1	3.1a
	$5mv+6mv(=11mv)=5mx-3my \qquad (11v=5x-3y)$	A1	1.1b
	Use of impact law	M1	3.1a
	$v=e(x+y)$	A1	1.1b
	$\begin{cases}11ev=5ex-3ey\\ 3v=3ex-3ey\end{cases} \Rightarrow x=\frac{v}{8e}(11e+3)$	M1	3.1a
	$y=\frac{v}{8e}(5-11e)$	A1	1.1b
	$e>0 \quad (\Rightarrow \quad x>0) \quad \Rightarrow \quad 5-11e>0$	M1	3.4
	$\Rightarrow 0<e<\frac{5}{11}$	A1	2.2a
		(8)	
(b)	Form equation for KE	M1	2.1
	$\frac{1}{2}\times 5m\times v^2=\frac{16}{100}\times\frac{1}{2}\times 5m\times\frac{v^2}{64e^2}(11e+3)^2$	A1ft	1.1b
	$(4(11e+3)=(\pm)80e) \qquad e=\frac{1}{3}$	A1	1.1b
	Impulse $=-5m(v-x)$	M1	3.1a
	$=-5m\left(v-\frac{11v}{8}-\frac{3v}{8e}\right)$ Or: $3m\left(2v+\frac{5v}{8e}-\frac{11v}{8}\right)$	A1ft	1.1b
	Magnitude $=\frac{15}{2}mv$	A1	2.2a
		(6)	
Alt(b)	Form equation for KE	M1	2.1
	$\frac{1}{2}\times 5m\times v^2=\frac{16}{100}\times\frac{1}{2}\times 5m\times x^2$	A1	1.1b
	$\Rightarrow x=\frac{5v}{2},\ y=\frac{v}{2} \quad \Rightarrow e=\frac{1}{3}$	A1	1.1b

	Impulse $= -5m(v - x)$	M1	3.1a
	$= -5m\left(v - \frac{5v}{2}\right)$ Or: $3m\left(2v + \frac{v}{2}\right)$	**A1**	1,16
	Magnitude $= \frac{15}{2}mv$	A1	2.2a
		(6)	
(14 marks)			

Notes:	
(a)M1	All terms required. Dimensionally correct. Condone sign errors
A1	Correct unsimplified equation
M1	Used correctly. Condone sign errors
A1	Correct unsimplified equation
M1	Use their correctly formed equations to solve for *v* or *w* or a multiple of *v* or *w*
A1	Both velocities correct
M1	Use their velocities (in general form – not by considering one specific value) to form inequality for both moving in the same direction.
A1	Correct only.
(b)M1	Dimensionally correct. Condone 16% on wrong side Allow *M* or 5*m*
A1ft	Or equivalent. Correct unsimplified equation. Follow their *x* Allow *M* or 5*m*
A1	Correct answer only Allow *M* or 5*m*
M1	Correct use of $I = mv - mu$. Must be subtracting.
A1ft	Accept ± Follow their *x*, *y*, *e*
A1	Correct only. Must be positive.

5. Two particles, P and Q, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of P is $3m$ and the mass of Q is $4m$.
Immediately before the collision the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{u}{7}(9e + 2)$

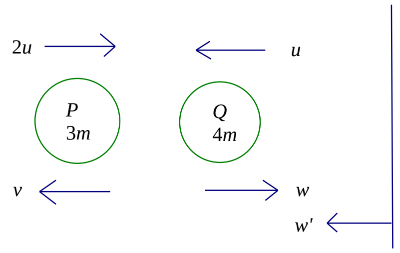
(6)

After the collision with P, particle Q collides directly with a fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of Q.

The coefficient of restitution between Q and the wall is $\frac{1}{2}$

(b) Find the complete range of possible values of e for which there is a second collision between P and Q.

(4)

Question	Scheme	Marks	AOs
5a			
	Using CLM:	M1	3.4
	$6mu - 4mu = -3mv + 4mw \quad (2u = -3v + 4w)$	A1	1.1b
	Use of impact law	M1	3.1a
	$w + v = e \times 3u$	A1	1.1b
	Complete method to find w	M1	2.1
	$\begin{cases} 3w + 3v = 9eu \\ -3v + 4w = 2u \end{cases} \Rightarrow 7w = 9eu + 2u, \quad w = \frac{u}{7}(9e+2)$ *	A1*	2.2a
		(6)	
5b	$w' = \frac{1}{2} \times \frac{u}{7}(9e+2) \quad \left(= \frac{u}{14}(9e+2)\right)$	B1	1.1b
	$v = \frac{u}{7}(12e-2)$	B1	1.1b
	For a second collision: $w' > v$	M1	3.3
	$9e + 2 > 2(12e - 2), \quad 0 < e < \frac{2}{5}$	A1	1.1b
		(4)	
		(Total 10 marks)	

Notes

(a) M1	Use of CLM. Need all terms. Must be dimensionally correct. Condone sign errors. Accept consistent cancelling of m
A1	Correct unsimplified equation for CLM. They can have v in either direction
M1	Correct use of the impact law (used the right way round) Condone sign errors in finding speed of approach and speed of separation.
A1	Correct unsimplified equation. Signs consistent with equation for CLM.
M1	Complete method to find w e.g. by forming simultaneous equations using CLM and Impact Law and solving. This requires both of the preceding M marks
A1*	Obtain **given answer** from correct working. Accept with 2 + 9e in place of 9e + 2

	Check that the answer does follow from the working.
(b) B1	Speed of *Q* after impact with the wall. Any equivalent form. Correct speed can be implied by a correct negative velocity.
B1	Speed of *P* after impact with *Q*. Accept $\pm$. Any equivalent form in *u* and *e* (seen or implied)
M1	Form correct inequality using their *v* and *w*'. A correct inequality has *P* and *Q* both moving away from the wall
A1	Correct interval only. Accept unsimplified fraction. Need both ends of the interval. Must be strict inequality at both ends.

1.

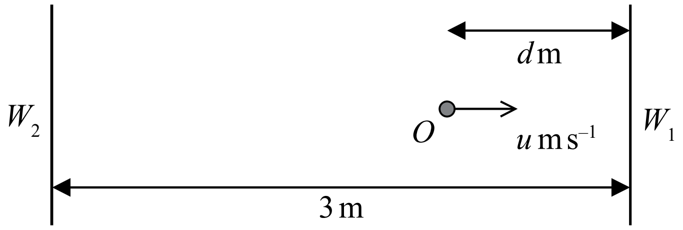


Figure 1

Figure 1 represents the plan of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls. The walls are 3 metres apart.

A particle lies at rest at a point O on the floor between the two walls, where the point O is d metres, $0 < d \leqslant 3$, from W_1

At time $t = 0$, the particle is projected from O towards W_1 with speed u m s^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$

The particle returns to O at time $t = T$ seconds, having bounced off each wall once.

(a) Show that $T = \dfrac{45 - 5d}{4u}$

(6)

The value of u is fixed, the particle still hits each wall once but the value of d can now vary.

(b) Find the least possible value of T, giving your answer in terms of u. You must give a reason for your answer.

(2)

9FM0/3C: Further Mechanics 1 (replaced paper) mark scheme – Summer 2019

Question	Scheme	Marks	AOs
1(a)	Use of $P = Fv$: $F = \dfrac{13000}{20}$	B1	3.3
	Using the model to set up an equation of motion	M1	3.4
	$\dfrac{13000}{20} - 20\lambda - 750g \times \dfrac{1}{21} = 0$	A1	1.1b
	$\lambda = 15$ *	A1*	1.1b
		(4)	
(b)	Using the model to set up equation of motion	M1	3.3
	$\dfrac{11250}{U} - 15U = 750 \times 0.1$	A1	1.1b
	3 term quadratic and solve: $15U^2 + 75U - 11250 = 0$	M1	1.1b
	$U = 25$	A1	2.2a
		(4)	
			(8 marks)

Notes:

(a)
B1: Use of $P = Fv$
M1: Correct number of terms with weight resolved.
A1: Correct equation
A1*: Given answer

(b)
M1: Correct number of terms
A1: Correct equation
M1: This mark can be implied by a correct value of U
A1: $U = 25$

5. A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$.

Immediately after the collision P and Q are moving in opposite directions.

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e, justifying your answer.

(8)

Given that Q loses 75% of its kinetic energy as a result of the collision,

(b) find the value of e.

(3)

P 6 2 6 7 4 A 0 1 8 2 8

Question	Scheme	Marks	AOs
5(a)	CLM parallel to line of centres (loc)	M1	3.1a
	$-4mv_P + 5mv_Q = 4mu\cos\alpha - 5mu\cos\alpha$ $-\ 4v_P + 5v_Q = -u\cos\alpha$	A1	1.1b
	Correct use of NIL	M1	3.4
	$v_P + v_Q = 2ue\cos\alpha$	A1	1.1b
	Solve for v_Q	M1	1.1b
	$v_Q = \dfrac{(8e-1)u\cos\alpha}{9}$	A1	1.1b
	Velocity component of Q perp to loc $= u\sin\alpha$	B1	3.4
	$\tan\theta = \dfrac{u\sin\alpha}{v_Q}$	M1	3.1a
	$\tan\theta = \dfrac{u\sin\alpha}{\dfrac{(8e-1)u\cos\alpha}{9}}$	M1	1.1b
	$\tan\theta = \dfrac{9\tan\alpha}{8e-1}$ *	A1*	2.1
		(10)	
(b) (i)	Perp to loc $\Rightarrow v_Q = 0 \Rightarrow 8e-1=0 \Rightarrow e = \dfrac{1}{8}$	B1	2.2a
(ii)	$v_P = \dfrac{1}{4}u\cos\alpha$	B1	1.1b
	$\tan\phi = \dfrac{u\sin\alpha}{v_P} = \dfrac{u\sin\alpha}{\frac{1}{4}u\cos\alpha} = 4\tan\alpha = 4$	M1	3.1a
	$\phi = \tan^{-1}4 = 76^{\circ}$ or better (1.3^{c}) to the line of centres oe	A1	1.1b
		(4)	
(c)	Impulse between spheres acts horizontally i.e. parallel to the plane $\Rightarrow$momentum conserved horizontally	B1	2.4
		(1)	
			(15 marks)
Notes:			
(a) **M1:** Need all four terms			

A1: Correct unsimplified equation **M1:** *e* must be on the correct side of the equation **A1:** Correct unsimplified equation **M1:** Solve for v_Q **A1:** Correct unsimplified equation **B1:** Use the model to find the velocity component perpendicular to loc **M1:** Overall strategy to find $\tan\theta$ **M1:** Sub for v_Q and simplify **A1*:** Given answer
(b)(i) **B1:** Clear explanation. May use $\theta = 90 \Rightarrow 8e - 1 = 0 \Rightarrow e = \frac{1}{8}$ **(b)(ii)** **B1:** Use $v_Q = 0$ to find v_P **M1:** Complete method to solve the problem and find the angle **A1:** Answers in degrees (76°) or rads (1.3) or better, are acceptable.
(c) **B1:** Clear explanation

3. A particle of mass m kg lies on a smooth horizontal surface.

Initially the particle is at rest at a point O between two fixed parallel vertical walls.

The point O is equidistant from the two walls and the walls are 4 m apart.

At time $t = 0$ the particle is projected from O with speed u m s^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{3}{4}$

The magnitude of the impulse on the particle due to the first impact with a wall is λmu N s.

(a) Find the value of λ.

(3)

The particle returns to O, having bounced off each wall once, at time $t = 7$ seconds.

(b) Find the value of u.

(5)

Question	Scheme	Marks	AOs
3(a)	Use NEL to find the speed of particle after the first impact $= eu = \frac{3}{4}u\frac{\pi}{2}$	B1	3.4
	Impulse = $\lambda mu = mv - mu = \pm\left[\frac{3}{4}mu - (-mu)\right]$	M1	3.1b
	$\lambda = \frac{7}{4}$	A1	1.1b
		(3)	
(b)	Use NEL to find the speed of the particle after the second impact $= \frac{3}{4} \times \frac{3}{4}u = \frac{9}{16}u$	B1	3.4
	Use of $s = vt$ to find total time	M1	3.1b
	$7 = \frac{2}{u} + \frac{4}{\frac{3}{4}u} + \frac{2}{\frac{9}{16}u}\left(= \frac{2}{u} + \frac{16}{3u} + \frac{32}{9u}\right)$	A1	1.1b
	Solve for *u*: $63u = 18 + 48 + 32$	M1	1.1b
	$u = \frac{98}{63} = \frac{14}{9}\left(= 1.\dot{5}\right)$	A1	1.1b
		(5)	
			(8 marks)

Notes:

(a)

B1: Using Newton's experimental law as a model to find the speed after the first impact

M1: Must be a difference of two terms, taking account of the change in direction of motion

A1: cao

(b)

B1: Using NEL as a model to find the speed after the second impact

M1: Needs to be used for at least one stage of the journey

A1: Ur equivalent

M1: Solve their linear equation for *u*

A1: Accept 1.56 or better

8. A particle P of mass $2m$ and a particle Q of mass $5m$ are moving along the same straight line on a smooth horizontal plane.

They are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is $2u$ and the speed of Q is u.

The direction of motion of Q is reversed by the collision.

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e.

(8)

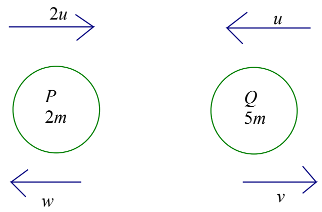
Given that $e = \frac{1}{3}$

(b) show that the kinetic energy lost in the collision is $\frac{40mu^2}{7}$.

(5)

(c) Without doing any further calculation, state how the amount of kinetic energy lost in the collision would change if $e > \frac{1}{3}$

(1)

Question	Scheme	Marks	AOs
8(a)			
	Complete overall strategy to find v	M1	3.1a
	Use of CLM	M1	3.1a
	$2m \times 2u - 5m \times u = 5m \times v - 2m \times w$, $(-u = 5v - 2w)$	A1	1.1b
	Use of Impact law:	M1	3.1a
	$v + w = e(2u + u)$	A1	1.1b
	Solve for v: $-u = 5v - 2w$ $6eu = 2v + 2w$		
	$7v = u(6e - 1)$ $\left(v = \frac{u}{7}(6e - 1)\right)$	A1	1.1b
	Direction of Q reversed: $v > 0$	M1	3.4
	$\Rightarrow 1 \geq e > \frac{1}{6}$	A1	1.1b
		(8)	
(b)	$e = \frac{1}{3} \Rightarrow v = \frac{u}{7}$, $w = \frac{6u}{7}$	B1	2.1
	Equation for KE lost	M1	2.1
	$\frac{1}{2} \times 2m\left(4u^2 - \frac{36u^2}{49}\right) + \frac{1}{2} \times 5m\left(u^2 - \frac{u^2}{49}\right)$	A1 A1	1.1b 1.1b
	$\frac{1}{2}mu^2\left(8 - \frac{72}{49} + 5 - \frac{5}{49}\right) = \frac{40mu^2}{7}$ *	A1*	2.2a
		(5)	
(c)	Increase $e \Rightarrow$ more elastic $\Rightarrow$ less energy lost	B1	2.2a
		(1)	
			(14 marks)

Question 8 notes:	
(a)	
M1:	Complete strategy to form sufficient equations in v and w and solve for v
M1:	Use CLM to form equation in v and w
	Needs all 4 terms & dimensionally correct
A1:	Correct unsimplified equation
M1:	Use NEL as a model to form a second equation in v and w. Must be used the right way round
A1:	Correct unsimplified equation
A1:	for v or $7v$ correct
M1:	Use the model to form a correct inequality for their v
A1:	Both limits required
(b)	
B1:	Or equivalent statements
M1:	Terms of correct structure combined correctly
A1:	Fully correct unsimplified A1A1
	One error on unsimplified expression A1A0
A1*:	cso. plus a 'statement' that the required result has been achieved
(c)	
B1:	"less energy lost" or equivalent

6. A particle, P, of mass $4m$ is moving along a straight line on a smooth horizontal plane.

A particle, Q, of mass $3m$ is at rest on the plane on the same straight line.

Particle P collides directly with particle Q.

Immediately before the collision the speed of P is ku, where k is a constant.

Immediately after the collision the speed of P is u and the speed of Q is $\frac{3u}{2}$

The coefficient of restitution between P and Q is e.

(a) (i) Show that there is only one possible value of k.

(ii) State the value of k and the value of e.

(11)

(b) Find the total kinetic energy lost in the collision between P and Q.

(3)

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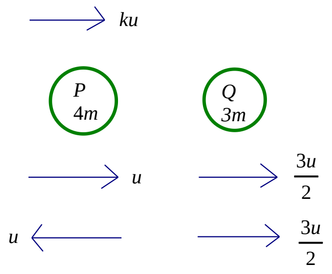
Question	Scheme	Marks	AOs
6(a)			
	Two correct possibilities identified	B1	2.1
	Form and solve a pair of simultaneous equations in *k* and *e*	M1	3.1a
	Use of CLM:	M1	3.1a
	$4mu + 3m \times \frac{3u}{2} = 4mku$ or $-4mu + 3m \times \frac{3u}{2} = 4mku$	A1	1.1b
	Use of impact law:	M1	3.1a
	$\frac{3}{2}u - u = e \times ku$ or $\frac{3}{2}u + u = e \times ku$	A1	1.1b
	$\frac{17}{2} = 4k$ and $\frac{1}{2} = ek \Rightarrow k = \frac{17}{8}$, $e = \frac{4}{17}$	A1	1.1b
	Second pair of simultaneous equations	M1	3.4
	Both equations correct	A1	1.1b
	$\frac{1}{2} = 4k$ and $\frac{5}{2} = ek \Rightarrow k = \frac{1}{8}$		
	$e = 20$ impossible since max $e = 1$	M1	1.1b
	Convincing argument to support just one possible value for *k**.	A1*	2.2a
	Alternative for last 4 marks:		
	Second CLM equation	M1	3.4
	$\frac{1}{2} = 4k \Rightarrow k = \frac{1}{8}$	A1	1.1b
	$k = \frac{1}{8} \Rightarrow$ both particles gain KE, which is impossible	M1	1.1b
	Convincing argument to support just one possible value for *k**.	A1*	2.2a
		(11)	

6(b)	KE lost = difference of two KEs	M1	3.1a
	$=\frac{1}{2}\times 4m\times(ku)^2-\frac{1}{2}\times 4m\times u^2-\frac{1}{2}\times 3m\times\left(\frac{3}{2}u\right)^2$ $=mu^2\left(2k^2-2-\frac{27}{8}\right)$	A1ft	1.1b
	$=\frac{117}{32}mu^2$ or equivalent	A1	1.1b
		(3)	
			(14 marks)

Notes:

6a	B1	Identify all possible options from given information
	M1	Complete strategy to find a pair of values for *k* and *e*
	M1	Correct use of CLM. All terms needed. Condone sign errors. Dimensionally correct
	A1	Correct unsimplified equation (for either option)
	M1	Correct use of impact law.
	A1	Correct unsimplified equation (for the same option)
	A1	Correct solution for one pair of *k* and *e*
	M1	Form second pair of simultaneous equations to fit the model.
	A1	Both equations correct unsimplified
	M1	Correct reasoning for elimination of one pair of values
	A1*	CSO. Deduce the **given result** having considered all the options.
6b	M1	Complete strategy to find an expression in *m*, (*k*) and *u* for the KE lost.
	A1ft	Correct unsimplified expression in *k* or their *k*
	A1	$3.7mu^2$ or better

4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal floor. A particle Q of mass $5m$ is moving in the opposite direction to P along the same straight line.

The particles collide directly.

Immediately before the collision, the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{u}{8}(9e + 1)$

(6)

(b) Find the range of values of e for which the direction of motion of P is not changed as a result of the collision.

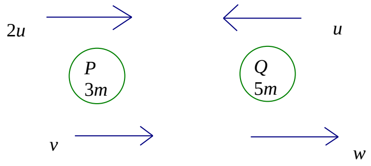
(2)

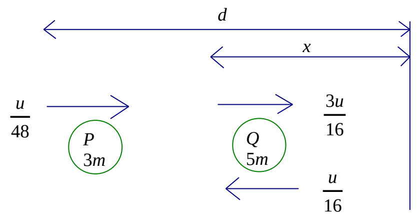
When P and Q collide they are at a distance d from a smooth fixed vertical wall, which is perpendicular to their direction of motion. After the collision with P, particle Q collides directly with the wall and rebounds so that there is a second collision between P and Q. This second collision takes place at a distance x from the wall.

Given that $e = \frac{1}{18}$ and the coefficient of restitution between Q and the wall is $\frac{1}{3}$

(c) find x in terms of d.

(6)

Qu	Scheme	Marks	AOs	Notes
4(a)	Complete strategy to find speed of Q	M1	3.1b	Complete strategy e.g. use of CLM, impact law and solution of simultaneous equations.
				
	Use of CLM	M1	3.1a	CLM equation. Requires all terms and dimensionally correct. Condone sign errors.
	$6mu-5mu\left(=mu\right)=3mv+5mw$	A1	1.1b	Correct unsimplified equation
	Use of impact law	M1	3.1a	Impact law. Condone sign error. Must be used the right way round.
	$w-v=3ue$	A1	1.1b	Correct unsimplified equation Signs consistent with CLM equation.
	$\left.\begin{array}{r}3v+5w=u\\3w-3v=9ue\end{array}\right\} \Rightarrow 8w=u+9ue\ ,\quad w=\frac{u}{8}\left(9e+1\right)$ *	A1*	2.1	Obtain **given answer** from correct working
		(6)		
4(b)	$v=w-3ue=\frac{u}{8}\left(1-15e\right)$ and $v>0$	M1	3.1b	Find speed of P and form correct inequality consistent with their directions.
	$\Rightarrow \left(0\le\right)e<\frac{1}{15}$	A1	1.1b	Correct solution. Need not mention the lower limit.
		(2)		

4(c)	Complete strategy to find time for Q to get to second collision	M1	3.1a	Complete strategy e.g. find time to wall and back again
	Speed of Q after impact with wall $=\frac{u}{16}$	B1	1.1b	Correct use of impact law
				
	Time for Q: $\frac{16d}{3u}+\frac{16x}{u}$ follow their $\frac{u}{16}$ and $\frac{16d}{3u}$	A1ft	1.1b	Correct unsimplified equation using time $=\frac{\text{distance}}{\text{speed}}$ and following their $\frac{u}{16}$ and $\frac{16d}{3u}$
	Complete strategy to find time for P to get to second collision $=\frac{48(d-x)}{u}$	B1ft	1.1b	Correct use of time $=\frac{\text{distance}}{\text{speed}}$ Follow their $\frac{u}{48}$
	Use both at the same place at the same	M1	2.1	find x by putting both particles in the same place at the same time. Must be valid expressions for the times.
	$x=\frac{128d}{192}=\frac{2d}{3}$	A1	1.1b	Correct answer or exact equivalent
		(6)		

4(c) alt	Complete strategy to find position of second collision	M1	3.1a	e.g. by considering distances and relative velocities
	Speed of Q after impact with wall $=\frac{u}{16}$	B1	1.1b	Correct use of impact law
	Distance apart when Q strikes the wall $=\frac{8d}{9}$	B1ft	1.1b	Follow their $\frac{u}{48}$ and $\frac{3u}{16}$
	Gap closing at $\frac{u}{16}+\frac{u}{48}$	A1ft	1.1b	Follow their $\frac{u}{16}$ and $\frac{u}{48}$
	$t=\frac{\frac{8d}{9}}{\frac{u}{16}+\frac{u}{48}}\quad\left(=\frac{32d}{3u}\right)$	M1	2.1	Correct use of $\text{time}=\frac{\text{distance}}{\text{speed}}$
	$x=\frac{u}{16}\times\frac{32d}{3u}=\frac{2d}{3}$	A1	1.1b	Correct answer
		(6)		
4(c) alt	Complete strategy to find position of second collision	M1	3.1a	e.g. by considering distances and relative velocities
	Speed of Q after impact with wall $=\frac{u}{16}$	B1	1.1b	Correct use of impact law
	Distance apart when Q strikes the wall $=\frac{8d}{9}$	B1ft	1.1b	Follow their $\frac{u}{48}$ and $\frac{3u}{16}$
	Ratio of speeds: $v_Q:v_P=3:1$	A1ft	1.1b	Follow their $\frac{u}{16}$ and $\frac{u}{48}$
	Distance travelled by Q $=\frac{3}{4}\times\frac{8d}{9}$	M1	2.1	Correct use of ratio to find x
	$x=\frac{2d}{3}$	A1	1.1b	Correct answer
		(6)		
(14 marks)				

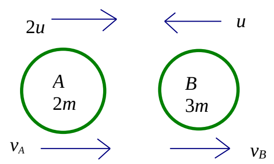
2. Two particles, A and B, of masses $2m$ and $3m$ respectively, are moving on a smooth horizontal plane. The particles are moving in opposite directions towards each other along the same straight line when they collide directly. Immediately before the collision the speed of A is $2u$ and the speed of B is u. In the collision the impulse of A on B has magnitude $5mu$.

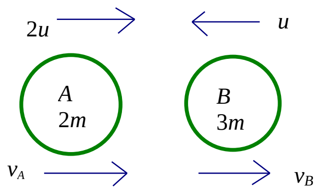
(a) Find the coefficient of restitution between A and B.

(9)

(b) Find the total loss in kinetic energy due to the collision.

(4)

Question	Scheme	Marks	AOs
2 (a)	Using the Impulse-momentum principle for *B*	M1	3.1a
	$5mu == 3m(v_B - -u)$	A1	1.1b
	$v_B = \frac{2u}{3}$	A1	1.1b
	Use of conservation of momentum	M1	3.1a
	$4mu - 3mu = 2mv_A + 3mv_B \left(= 2mv_A + 3m.\frac{2u}{3}\right)$	A1ft	1.1b
	$v_A = -\frac{u}{2}$	A1	1.1b
	Use of NLR	M1	3.4
	$e = \frac{v_B - v_A}{2u + u} \left(= \frac{\frac{u}{2} + \frac{2u}{3}}{2u + u} \right)$	A1ft	1.1b
	$e = \frac{7}{18} = 0.39$ or better	A1	1.1b
			
		(9)	
(b)	KE Loss = Initial KE – Final KE	M1	2.1
	$= \frac{1}{2}.2m(2u)^2 + \frac{1}{2}.3mu^2 - \left(\frac{1}{2}.2m\left(-\frac{u}{2}\right)^2 + \frac{1}{2}.3m\left(\frac{2u}{3}\right)^2\right)$	A1ft A1ft	1.1b 1.1b
	$= \frac{55mu^2}{12}$	A1	1.1b
		(4)	
			(13 marks)

Notes		
(a)	M1	Correct no. of terms and dimensionally correct but condone sign errors but must be a difference of momenta
	A1	Correct unsimplified equation
	A1	Correct appropriate velocity
	M1	Use of CLM with correct no. of terms and dimensionally correct but condone sign errors **Alternative**: Use Impulse - momentum for *A*
	A1ft	Correct unsimplified CLM equation **Or:** $-5mu = 2m(v_A - 2u)$
	A1	Correct speed
	M1	Use of NLR with *e* on the correct side
	A1ft	Correct unsimplified equation
	A1	Correct answer
ALT		 Could find v_A before v_B : M1A1A1 for first velocity, M1A1A1 for second M1A1A1 for *e* found correctly
		Candidates are approaching this in many different ways. They need - two of momentum impulse equation for each particle and CLM - impact law M1A1 for each correct equation (in the order seen) Of the remaining 3 A marks, A1 for a correct expression for v_A or v_B A1 for a correct expression in *e* A1 for the correct answer

Notes Continued		
e.g	M1A1	CLM: $4mu-3mu=2mv_A+3mv_B$
	M1A1	Impact: $v_B-v_A=3ue$
	A1	$v_B=\frac{u}{5}(1+6e)$ *or* $v_A=\frac{u}{5}(1-9e)$
	M1A1	$5mu=3m(v_B-(-u))$ $\left(=3m\left(\frac{u}{5}(1+6e)+u\right)\right)$ Or $-5mu=2m(v_A-2u)$ $\left(=2m\left(\frac{u}{5}(1-9e)-2u\right)\right)$
	A1	$5=3\left(\frac{1}{5}(1+6e)+1\right)$ or $-5=2\left(\frac{1}{5}(1-9e)-2\right)$
	A1	$e=\frac{7}{18}=0.39$ or better
(b)	M1	Correct no. of terms and must be a difference. Must be dimensionally correct at the point when they state their expression for the loss (change) in KE
	A1ft	Unsimplified expression in u with at most 1 error, ft on their speeds from (a)
	A1ft	Correct unsimplified expression in u. (These first 3 marks can be scored for a correct loss or gain in KE), ft on their speeds from (a)
	A1	cso Accept $4.58mu^2$ or $4.6mu^2$

4. Three particles, P, Q and R, are at rest on a smooth horizontal plane. The particles lie along a straight line with Q between P and R. The particles Q and R have masses m and km respectively, where k is a constant.

Particle Q is projected towards R with speed u and the particles collide directly.

The coefficient of restitution between each pair of particles is e.

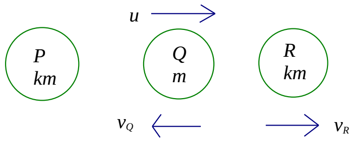
(a) Find, in terms of e, the range of values of k for which there is a second collision.

(9)

Given that the mass of P is km and that there is a second collision,

(b) write down, in terms of u, k and e, the speed of Q after this second collision.

(1)

Question	Scheme	Marks	AOs
4(a)			
	Use of conservation of momentum	M1	3.1a
	$mu = -mv_Q + kmv_R$	A1	1.1b
	Use of NLR	M1	3.4
	$eu = v_Q + v_R$	A1	1.1b
	Using correct strategy to solve problem by finding v_Q	M1	3.1a
	$v_Q = \dfrac{u(ke-1)}{k+1}$ or $v_Q = \dfrac{v_R(ke-1)}{1+e}$	A1	1.1b
	For second collision, $v_Q > 0$	M1	3.1a
	$\dfrac{u(ke-1)}{k+1} > 0$	M1	1.1b
	$k > \dfrac{1}{e}$	A1	1.1b
		(9)	
(b)	$\dfrac{u(ke-1)^2}{(k+1)^2}$	B1	2.2a
		(1)	
			(10 marks)

Notes		
(a)	M1	Correct no. of terms and dimensionally correct but condone sign errors
	A1	Correct equation
	M1	Use of NLR with *e* on the correct side
	A1	Correct equation (any equivalent form) Signs consistent with CLM equation
	M1	Solving for v_Q - complete correct strategy (i.e. correct use of CLM and of NLR)
	A1	Correct expression for their v_Q Can be implied by a correct multiple of v_Q
	M1	Use of appropriate condition for their v_Q
	M1	Complete correct strategy to find values for *k* (i.e. set up and solve inequality)
	A1	cso
(b)	B1	Or equivalent cao

1. Two particles P and Q have masses m and $4m$ respectively. The particles are at rest on a smooth horizontal plane. Particle P is given a horizontal impulse, of magnitude I, in the direction PQ. Particle P then collides directly with Q. Immediately after this collision, P is at rest and Q has speed w. The coefficient of restitution between the particles is e.

(a) Find I in terms of m and w.

(2)

(b) Show that $e = \dfrac{1}{4}$

(1)

(c) Find, in terms of m and w, the total kinetic energy lost in the collision between P and Q.

(2)

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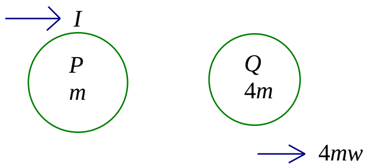
Question	Scheme	Marks	AOs
1a			
	Use of CLM: $m \times \frac{I}{m} = 4mw$	M1	3.1a
	$I = 4mw$	A1	1.1b
		(2)	
1b	$e = \frac{w}{4w} = \frac{1}{4}$ *	B1*	3.4
		(1)	
1c	KE Loss = $\frac{1}{2}m(4w)^2 - \frac{1}{2}4mw^2$	M1	3.4
	$= 6mw^2$	A1	1.1b
		(2)	
			(5 marks)

Notes

1a	M1	Correct no. of terms, condone extra g s, sign errors (must be equation in I, m and w only)
	A1	Correct equation
		Answer not given, so a correct answer with no clear error seen will score M1A1 An answer that relies on an impulse-momentum equation using $4m$ will score M0
1b	B1*	Use of NLR to obtain **given answer**
1c	M1	Allow negative loss
	A1	cao

3. Three particles A, B and C are at rest on a smooth horizontal plane. The particles lie along a straight line with B between A and C.

Particle B has mass $4m$ and particle C has mass km, where k is a positive constant. Particle B is projected with speed u along the plane towards C and they collide directly.

The coefficient of restitution between B and C is $\frac{1}{4}$

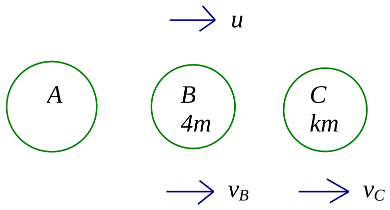
(a) Find the range of values of k for which there would be no further collisions.

(8)

The magnitude of the impulse on B in the collision between B and C is $3mu$

(b) Find the value of k.

(4)

Question	Scheme	Marks	AOs
3a			
	Use of CLM	M1	3.1a
	$4mu = 4mv_B + kmv_C$	A1	1.1b
	Use of NLR	M1	3.1a
	$\frac{1}{4}u = -v_B + v_C$	A1	1.1b
	Solve for v_B	M1	1.1b
	$v_B = \frac{u(16-k)}{4(k+4)}$ $\left(v_C = \frac{5u}{k+4}\right)$	A1	1.1b
	Use of $v_B \geq 0$ and solve for k	M1	3.4
	$(0<)\,k \leq 16$	A1	1.1b
	Alternative for last 4 marks		
	Solve for v_B in terms of v_C only	M1	
	$v_B = \frac{(16-k)v_C}{20}$	A1	
	Use of $v_B \geq 0$ and $v_C > 0$ to solve for k	M1	
	$(0<)\,k \leq 16$	A1	
		(8)	
3b	Impulse-momentum equation	M1	3.1a
	$-3mu = 4m(v_B - u)$ $\left(v_B = \frac{u}{4}\right)$ or $3mu = kmv_C$	A1	1.1b
	Complete method to solve for k	M1	1.1b
	$k = 6$	A1	2.2a
		(4)	
		(12 marks)	

Notes

3a	M1	Correct no. of terms, condone extra *g* s, sign errors
	A1	Correct equation
	M1	*e* must be on correct side
	A1	Correct equation
	M1	Complete method to solve for v_B (or a multiple of v_B)
	A1	Correct expression for *their* v_B or a multiple of *their* v_B
	M1	Use of appropriate inequality, allow strict inequality for method mark
	A1	Cao LHS not needed, but if there it must be correct.
3b	M1	Correct no. of terms, condone sign errors, but must be subtracting momentum terms
	A1	Correct equation
	M1	Eliminate and solve for *k*
	A1	$k = 6$

2.

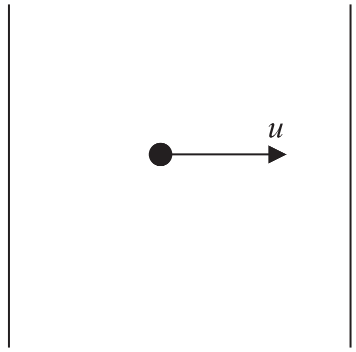


Figure 2

A particle of mass em is at rest on a smooth horizontal plane between two smooth fixed parallel vertical walls, as shown in the plan view in Figure 2. The particle is projected along the plane with speed u towards one of the walls and strikes the wall at right angles. The coefficient of restitution between the particle and each wall is e and air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m, u and e, an expression for the total loss in the kinetic energy of the particle as a result of the first two impacts.

(3)

Given that e can vary such that $0 < e < 1$ and using the model,

(b) find the value of e for which the total loss in the kinetic energy of the particle as a result of the first two impacts is a maximum,

(4)

(c) describe the subsequent motion of the particle.

(2)

Question	Scheme	Marks	AOs
2(a)	Speeds after 1st and 2nd impacts: eu and e^2u	B1	3.4
	KE Loss, $K = \frac{1}{2}emu^2 - \frac{1}{2}em(e^2u)^2$ (difference in KE's)	M1	3.3
	$\frac{1}{2}mu^2(e - e^5)$	A1	1.1b
		(3)	
(b)	Differentiate wrt e	M1	2.1
	$\frac{dK}{de} = \frac{1}{2}mu^2(1 - 5e^4)$	A1	1.1b
	Equate to zero and solve for e	M1	3.1a
	$e^4 = \frac{1}{5} \Rightarrow e = 0.67$ or better	A1	1.1b
		(4)	
(c)	Particle continues to bounce off each wall (indefinitely).	B1	2.4
	Speed of particle decreases oe	B1	2.4
		(2)	
			(9 marks)

Notes:

2a	B1	Need both for the mark
	M1	Allow terms reversed
	A1	cao
2b	M1	Clear attempt to differentiate their KE loss, in terms of e, wrt e, with powers decreasing by 1

	A1	Correct derivative
		If working from $\frac{1}{2}mu^2\left(1-e^4\right)$ allow M1A0 for a correct argument leading to $e=0$
	M1	Clear attempt to equate to zero
	A1	cao
2c	B1	Any clear equivalent statement
	B1	Any clear equivalent statement. Allow speed tends to 0.

Question	**Scheme**	**Marks**	**AOs**
3(a)	Freewheeling down: Equation of motion down the plane and using the model	M1	3.1b
	$100g\sin\alpha - kV^2 = 0$ $\left(kV^2 = \frac{100g}{35}\right)$	A1	1.1b
	Cycling up: Equation of motion up the plane and using the model	M1	3.1b
	$F - 100g\sin\beta - kV^2 = 0$	A1	1.1b
	Use of $F = \frac{P}{V}$ $\left(\frac{P}{V} = \frac{100g}{70} + \frac{100g}{35}\right)$	M1	3.3
	Solve the problem by solving for *P* in terms of *V* and substituting for $\sin\alpha$ and $\sin\beta$	M1	1.1b
	$\left(P = \frac{300gV}{70}\right)$ $P = 42V$	A1	1.1b
		(7)	
(b)	Equation of motion horizontally and using the model	M1	3.4
	$\frac{35V}{U} - kU^2 = 0$	A1	1.1b
	Solve for *U* in terms of *V* $\left(\frac{35V}{U} - \frac{100g}{35V^2}U^2 = 0\right)$	M1	3.1b
	$U = 1.1V$ or $U = 1.08V$	A1	1.1b
		(4)	
			(11 marks)
Notes:			

4. Two particles, P and Q, have masses m and em respectively. The particles are moving on a smooth horizontal plane in the same direction along the same straight line when they collide directly. The coefficient of restitution between P and Q is e, where $0 < e < 1$

Immediately before the collision the speed of P is u and the speed of Q is eu.

(a) Show that the speed of Q immediately after the collision is u.

(6)

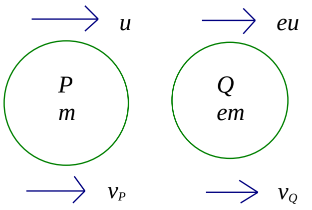
(b) Show that the direction of motion of P is unchanged by the collision.

(3)

The magnitude of the impulse on Q in the collision is $\frac{2}{9}mu$

(c) Find the possible values of e.

(4)

Question	Scheme		Marks	AOs
4(a)				
	Conservation of momentum		M1	3.4
	$mu + e^2mu = mv_P + emv_Q$		A1	1.1b
	Newton's Impact Law		M1	3.4
	$e(u - eu) \quad = -v_P + \ v_Q$		A1	1.1b
	Solve these equations for v_Q		M1	3.1a
	$v_Q = u$ *		A1*	1.1b
			(6)	
(b)	$v_P = \ u(e^2 - e + 1) \ \left(= \dfrac{(e^3+1)u}{e+1} \right)$		M1	1.1b
	$= u\left((e - \frac{1}{2})^2 + \frac{3}{4} \right)$		A1	1.1b
	> 0 so P continues to move in the same direction *		A1*	1.1b
			(3)	
			(9)	
(c)	Use impulse-momentum principle		M1	3.4
	$I = em(u - eu)$ or $m(-u(e^2 - e + 1) - (-u))$ $\quad (= (e - e^2)mu)$		A1	1.1b
	$(e - e^2) = \dfrac{2}{9}$ and solve		M1	1.1b
	$e = \dfrac{1}{3}$ or $\dfrac{2}{3}$		A1	1.1b
			(4)	
			(13 marks)	
Notes:				
4a	M1	Correct no. of terms, allow consistent cancelled m's $\left(u + e^2u = v_P + ev_Q\right)$		
	A1	Correct unsimplified equation		
	M1	Correct no. of terms, with e on correct side		

	A1	Correct unsimplified equation
	M1	Solve for v_Q
	A1*	cao
4b	M1	Solve for v_P
	M1	Completing the square or any other appropriate method
	A1*	Correct conclusion correctly reached
4c	M1	Correct no. of terms, dimensionally correct. Must be subtracting. Needs to be in terms of e and u.
	A1	Correct unsimplified expressiom (allow -ve answer at this stage)
	M1	Solving an appropriate quadratic equation
	A1	Two correct answers

2. Two particles, A and B, have masses m and $3m$ respectively. The particles are moving in opposite directions along the same straight line on a smooth horizontal plane when they collide directly.

Immediately before they collide, A is moving with speed $2u$ and B is moving with speed u.

The direction of motion of each particle is reversed by the collision.

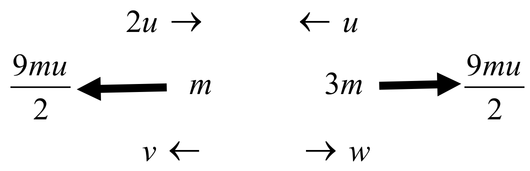
In the collision, the magnitude of the impulse exerted on A by B is $\dfrac{9mu}{2}$

(a) Find the value of the coefficient of restitution between A and B.

(7)

(b) Hence, write down the total loss in kinetic energy due to the collision, giving a reason for your answer.

(1)

Question	Scheme	Marks	AOs
2(a)			
	Use of Impulse-momentum principle for A or B	M1	3.4
	A: $\frac{9mu}{2} = m(v - -2u)$ **or** B: $\frac{9mu}{2} = 3m(w - -u)$	A1	1.1b
	Use of Impulse-momentum principle for B or A or CLM	M1	3.4
	$\frac{9mu}{2} = 3m(w - -u)$ **or** $\frac{9mu}{2} = m(v - -2u)$ **or** $2mu - 3mu = -mv + 3mw$	A1	1.1b
	$v = \frac{5u}{2}$ and $w = \frac{u}{2}$	A1	1.1b
	$e = \dfrac{\frac{5u}{2} + \frac{u}{2}}{2u + u}$	M1	3.1a
	$e = 1$	A1cso	1.1b
	ALTERNATIVE:		
	NEL is written down before v and w are found: $v + w = 3ue$	3rd M1	
	Use of Impulse-momentum principle for A or B	1st M1	
	A: $\frac{9mu}{2} = m(v - -2u)$ **or** B: $\frac{9mu}{2} = 3m(w - -u)$	1st A1	
	Use of Impulse-momentum principle for B or A or CLM	2nd M1	
	$\frac{9mu}{2} = 3m(w - -u)$ **or** $\frac{9mu}{2} = m(v - -2u)$ **or** $2mu - 3mu = -mv + 3mw$	2nd A1	
	An equation (not an identity) in u and e only is produced	3rd A1	
	$e = 1$	A1cso	
		(7)	
2(b)	Perfectly elastic (or the coefficient of restitution is 1) so no loss in kinetic energy. Allow a direct evaluation of the KE loss i.e. $\frac{1}{2}m(2u)^2 + \frac{1}{2} \times 3mu^2 - \left(\frac{1}{2}m\left(\frac{5u}{2}\right)^2 + \frac{1}{2} \times 3m\left(\frac{u}{2}\right)^2\right) = 0$ B0 if incorrect extras	DB1	2.4
		(1)	
			(8 marks)

Notes:		
N.B. Ignore diagrams if it helps the candidate. Equations need to be consistent, where appropriate, to earn A marks.		
2a	M1	Use of Impulse-momentum principle for *A* or *B*, condone sign errors but M0 if dimensionally incorrect e.g. if *m* missing
	A1	Correct unsimplified equation
	M1	Use of Impulse-momentum principle for other particle or CLM, condone sign errors but M0 if dimensionally incorrect e.g. if *m* missing from impulse For CLM, allow consistent missing *m*'s or extra *g*'s.
	A1	Correct unsimplified equation
	A1	Cao for both. Allow one or both negative if correct for their symbols.
	M1	Use of NEL to obtain $e = \ldots$, condone sign errors in numerator but must be terms in u only AND must be $(2u + u)$ in denominator. M0 if inverted
	A1	cso
2b	DB1	Dependent on $e = 1$ **correctly** obtained in (a) A correct statement e.g. zero, 0 etc and a correct reason

4. A particle P of mass $2m$ kg is moving with speed $2u\,\text{m}\,\text{s}^{-1}$ on a smooth horizontal plane. Particle P collides with a particle Q of mass $3m$ kg which is at rest on the plane. The coefficient of restitution between P and Q is e. Immediately after the collision the speed of Q is $v\,\text{m}\,\text{s}^{-1}$

(a) Show that $v = \dfrac{4u(1+e)}{5}$

(6)

(b) Show that $\dfrac{4u}{5} \leqslant v \leqslant \dfrac{8u}{5}$

(2)

Given that the direction of motion of P is reversed by the collision,

(c) find, in terms of u and e, the speed of P immediately after the collision.

(2)

After the collision, Q hits a wall, that is fixed at right angles to the direction of motion of Q, and rebounds.

The coefficient of restitution between Q and the wall is $\dfrac{1}{6}$

Given that P and Q collide again,

(d) find the full range of possible values of e.

(5)

Question	Scheme	Marks	AOs
4(a)	$2u \rightarrow$ 0 $P\ (2m)$ $Q(3m)$ $w \leftarrow$ $\rightarrow v$		
	Use of CLM	M1	3.4
	$2m \times 2u = -2mw + 3mv$	A1	1.1b
	Use of NEL	M1	3.4
	$2ue = w + v$	A1	1.1b
	Solve for v	D M1	1.1b
	$v = \frac{4u(1+e)}{5}$ *	A1*	2.2a
		(6)	
4(b)	Since $0 \le e \le 1$, $\frac{4u(1+0)}{5} \le v \le \frac{4u(1+1)}{5}$	M1	3.1a
	i.e. $\frac{4u}{5} \le v \le \frac{8u}{5}$ *	A1*	2.2a
		(2)	
4(c)	Solve for w	M1	1.1b
	$w = \frac{2u(3e-2)}{5}$ oe (m s^{-1}) or $\left\lvert\frac{2u(2-3e)}{5}\right\rvert$ oe	A1	1.1b
		(2)	
4(d)	Speed of Q after hitting the wall $= \frac{1}{6}v$ (m s^{-1})	M1	3.4
	For a further collision between P and Q, $\frac{1}{6}v > w$	M1	3.1a
	Substitute for v and w and solve for e	M1	1.1b
	$e < \frac{7}{8}$	A1	1.1b
	$\frac{2}{3} < e < \frac{7}{8}$	A1	1.1b
		(5)	
		(15 marks)	

Notes:

4a	M1	Correct no. of terms, condone sign errors, allow consistently cancelled m's or extra g's or common factors throughout
	A1	Correct equation; they may have w instead of $-w$
	M1	Correct no. of terms, condone sign errors. M0 if e on the wrong side of the equation

	A1	Correct equation; they may have w instead of $-w$
	DM 1	Solve for v, dependent on previous two marks
	A1*	Correct answer correctly obtained
4b	M1	Use of $0 \le e \le 1$ in the given answer; allow use of $e = 0$ and $e = 1$ to obtain the min and max expressions M1A0 for 'verification'.
	A1*	Correct answer correctly obtained (including use of max and min)
4c	M1	Solve for their w
	A1	cao
4d	M1	Speed so must see a positive quantity M0 if $\frac{1}{6}$ is on the wrong side of the equation
	M1	Correct inequality for their w (allow even if their w is dimensionally incorrect)
	M1	Independent M mark but must have an inequality in v and w: Substitute for v, using given answer, and w and solve for e
	A1	Correct upper bound for e
	A1	cao