Fm1Ch1 XMQs and MS

(Total: 61 marks)

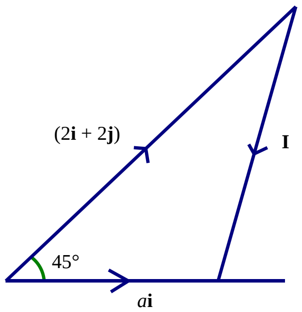
1. FM1_2019a Q3 . 9 marks - FM1ch1 Momentum and impulse
2. FM1_2020 Q1 . 7 marks - FM1ch1 Momentum and impulse
3. FM1_2021 Q4 . 8 marks - FM1ch1 Momentum and impulse
4. FM1_2022 Q1 . 8 marks - FM1ch1 Momentum and impulse
5. FM1_2022 Q3 . 5 marks - FM1ch1 Momentum and impulse
6. FM1_2019b Q3 . 9 marks - FM1ch1 Momentum and impulse
7. FM1_Sample Q1 . 6 marks - FM1ch1 Momentum and impulse
8. FM1_Specimen Q3 . 9 marks - FM1ch1 Momentum and impulse

3. A particle P, of mass 0.5 kg, is moving with velocity $(4\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{I}$ of magnitude 2.5 Ns.

As a result of the impulse, the direction of motion of P is deflected through an angle of 45°

Given that $\mathbf{I} = (\lambda\mathbf{i} + \mu\mathbf{j})$ Ns, find all the possible pairs of values of λ and μ.

(9)

Question	Scheme	Marks	AOs	Notes
3				
	Momentum of *P* after impulse = *a***i** (or *a***j**)	B1	2.2a	Correct interpretation of angle of deflection (velocity or momentum a multiple of **i** or **j**)
Either	Use of $\mathbf{I} = m(\mathbf{v} - \mathbf{u})$: $(\mathbf{I} =)0.5(2a\mathbf{i} - (4\mathbf{i} + 4\mathbf{j}))(= (a-2)\mathbf{i} - 2\mathbf{j})$	M1	3.3	Form vector triangle or equation for **v** or their *a***i**
	Use of Pythagoras to form equation in *a*	M1	3.4	Use trigonometry or Pythagoras' theorem to form equation in *a*
	$6.25 = 0.25((2a-4)^2 + 16)$ $(4a^2 - 16a + 7 = 0)$	A1ft A1	1.1b 1.1b	Unsimplified equation with at most one error. Follow their *a***i** Correct unsimplified equation
Or	$\lambda^2 + \mu^2 = \frac{25}{4}$	M1		
	$\mathbf{I} = \lambda\mathbf{i} + \mu\mathbf{j} = \frac{1}{2}((x-4)\mathbf{i} - 4\mathbf{j})$	M1		
	$\mu = -2$	A1		Dependent on 2nd M (for impulse)
	$\lambda^2 = \frac{9}{4}$	A1		

Or	Use of $\mathbf{I}=m(\mathbf{v}-\mathbf{u})$ to form vector triangle	M1	3.3	
	Form equation in their a	M1	3.4	
	$6.25=a^2+8-2a\sqrt{8}\times\frac{1}{\sqrt{2}}$ $\left(4\times 6.25=b^2+32-2b\sqrt{32}\times\frac{1}{\sqrt{2}}\text{ for velocity }b\mathbf{i}\right)$ $\left(4a^2-16a+7=0\right)$	A1ft A1	1.1b 1.1b	
	$a=\frac{7}{2},\ \frac{1}{2}\ \Rightarrow \mathbf{I}=\frac{3}{2}\mathbf{i}-2\mathbf{j}$ (Ns)	M1	1.1b	Complete correct method to solve to find a pair of values for λ and μ
	or $\mathbf{I}=-\frac{3}{2}\mathbf{i}-2\mathbf{j}$ (Ns)	A1	1.1b	Two correct pairs of values for λ and μ
	or $\mathbf{I}=-2\mathbf{i}-\frac{3}{2}\mathbf{j}$ (Ns)	M1	2.2a	Use symmetry in complete correct method to find one of the other pairs of values for λ and μ
	or $\mathbf{I}=-2\mathbf{i}+\frac{3}{2}\mathbf{j}$ (Ns)	A1	1.1b	All four correct pairs (They do not need to write out the impulse in full)
		(9)		
			(9 marks)	

1. A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{J}$ N s. Immediately after receiving the impulse, P is moving with velocity $(-\mathbf{i} + 6\mathbf{j})\,\text{m s}^{-1}$.

(a) Find the magnitude of $\mathbf{J}$.

(4)

The angle between the direction of the impulse and the direction of motion of P immediately before receiving the impulse is $\alpha°$

(b) Find the value of α

(3)

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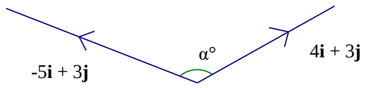
Question	Scheme	Marks	AOs
1a	Impulse-momentum equation	M1	3.1a
	$\mathbf{J} = 0.5(-\mathbf{i}+6\mathbf{j}-4\mathbf{i}-3\mathbf{j})$ $(\mathbf{J} = 0.5(-5\mathbf{i}+3\mathbf{j}))$	A1	1.1b
	Find magnitude of **J**:	M1	1.1b
	$\lvert\mathbf{J}\rvert^2 = \frac{1}{4}(25+9)$, $\lvert\mathbf{J}\rvert = \frac{\sqrt{34}}{2}$ (N s)	A1	1.1b
		(4)	
1b			
	Correct use of trig	M1	3.1a
	$\alpha° = 180° - \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{3}{5}$ or $\alpha° = \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{3}$	A1ft	1.1b
	$\alpha = 112$	A1	1.1b
		(3)	
1balt	Use scalar product of $\mu\mathbf{J}$ and $4\mathbf{i}+3\mathbf{j}$ to find the angle	M1	3.1a
	$\cos\alpha° = \frac{-20+9}{\sqrt{34}\times 5}$	A1ft	1.1b
	$\alpha = 112$	A1	1.1b
		(3)	
1balt	Use of cosine rule in triangle of momenta or equivalent	M1	3.1a
	$\alpha° = 180° - \cos^{-1}\left(\frac{34+25-37}{2\times 5\times\sqrt{34}}\right)$	A1ft	1.1b
	$\alpha = 112$	A1	1.1b
		(3)	
		(7 marks)	

Notes:

(a)M1	Dimensionally correct. Must be subtracting, but condone subtracting in the wrong order.
A1	Correct unsimplified equation
M1	Correct application of Pythagoras to find the magnitude. (from $\pm\mathbf{J}$)
A1	2.9 or better (2.9154….) (from $\pm\mathbf{J}$)

(b)M1	Correct use of trig to find a relevant angle using $4\mathbf{i}+3\mathbf{j}$ and their **J** i.e. $\alpha°$ or $180°-\alpha°$ Allow $\left\|\frac{\mathbf{a.b}}{\|\mathbf{a}\|\|\mathbf{b}\|}\right\|$
A1ft	Correct unsimplified expression for the required angle. Follow their **J** A0 for $\left\|\frac{\mathbf{a.b}}{\|\mathbf{a}\|\|\mathbf{b}\|}\right\|$ Do not ISW
A1	110 or better (112.166…..) or accept $247.8....°$

4. A particle P has mass 0.5 kg. It is moving in the xy plane with velocity $8\mathbf{i}\,\text{m}\,\text{s}^{-1}$ when it receives an impulse $\lambda(-\mathbf{i}+\mathbf{j})\,\text{N}\,\text{s}$, where λ is a positive constant.

The angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse is $\theta°$

Immediately after receiving the impulse, P is moving with speed $4\sqrt{10}\,\text{m}\,\text{s}^{-1}$

Find (i) the value of λ

(ii) the value of θ

(8)

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Question	Scheme	Marks	AOs
4	Use of Impulse = change in momentum	M1	3.1a
	$0.5(\mathbf{v}-8\mathbf{i})=\lambda(-\mathbf{i}+\mathbf{j})$ $\left(\mathbf{v}=(-2\lambda+8)\mathbf{i}+2\lambda\mathbf{j}\right)$	A1	1.1b
	Use of Pythagoras:	M1	3.1a
	e.g. $160=(-2\lambda+8)^2+(2\lambda)^2$ $\left(160=4\lambda^2-32\lambda+64+4\lambda^2\right)$	A1	1.1b
	Form and solve quadratic in λ : $8\lambda^2-32\lambda-96=0$ $\left(\lambda^2-4\lambda-12=(\lambda-6)(\lambda+2)=0\right)$	M1	2.1
	$\Rightarrow \lambda=6$	A1	1.1b
	Find the required angle: $180°-\tan^{-1}3$	M1	1.1b
	$\theta=108$	A1	2.2a
		(8)	
			(8 marks)

Notes:

M1	Must be subtracting two values for momentum, but condone subtraction in the wrong order
A1	Correct unsimplified equation
M1	Correct use of final speed with their **v**
A1	Correct unsimplified equation in one unknown or pair of simultaneous equations
M1	Simplify and solve for λ from correct working
A1	Correct positive solution only
M1	Complete method to solve for θ
A1	108 or better (108.4349….)

1. A particle A of mass $3m$ and a particle B of mass m are moving along the same straight line on a smooth horizontal surface. The particles are moving in opposite directions towards each other when they collide directly.

Immediately before the collision, the speed of A is ku and the speed of B is u.
Immediately after the collision, the speed of A is v and the speed of B is $2v$.

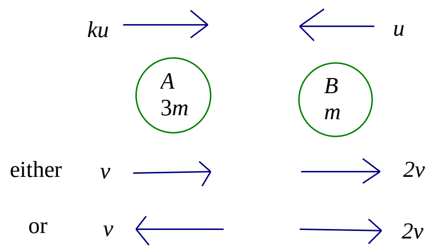
The magnitude of the impulse received by B in the collision is $\frac{3}{2}mu$.

(a) Find v in terms of u only.

(3)

(b) Find the two possible values of k.

(5)

Question	Scheme	Marks	AOs
1a			
	Note that if they start with their 2v to the left this creates an impossible situation (the particles need to pass through each other). The maximum score is M1M1M1.		
	Impulse received by *B*:	M1	3.4
	$\frac{3}{2}mu = m\left(2v-\left(-u\right)\right)$	A1	1.1b
	$v = \frac{u}{4}$	A1	1.1b
		(3)	

1b	Use of CLM or Impulse-momentum for one option for *A*:	M1	3.4
	$3kmu - mu = 2mv + 3mv\left(= \frac{5mu}{4}\right)$ or $3m(v - ku) = -\frac{3mu}{2} \quad \left(3mu\left(\frac{1}{4} + \frac{1}{2}\right) = 3mku\right)$	A1ft	1.1b
	$k = \frac{3}{4}$	A1	1.1b
	Form a second equation in *k* $\left(3mku - mu = 2mv - 3mv\left(= -\frac{mu}{4}\right) \text{ or } 3m(v + ku) = \frac{3mu}{2}\right)$	M1	3.1a
	$k = \frac{1}{4}$	A1	1.1b
		(5)	
			(T0tal 8 Marks)

Notes

(a)M1	Form impulse-momentum equation for *B* (or *A*). May be expressed as either $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ or $\mathbf{I} + m\mathbf{u} = m\mathbf{v}$. Dimensionally correct. Must be considering difference in velocities Must have a correct combination of mass and velocity: pairing velocity of one with the mass of the other scores M0 Allow for subtraction the wrong way round or impulse in the wrong direction. Assuming that you have not seen an incorrect formula stated, allow for $2v + u$ without overt evidence of subtraction. Allow if the common factor of *m* is not seen
A1	Correct unsimplified equation for *B* (or *A*). Allow without *m*
A1	Correct answer only
(b) M1	Correct method to form an equation in *k*. Must be dimensionally correct Condone sign errors in CLM. Allows marks for CLM equation here if seen in (a) and used correctly to find *k* here. Rules for impulse-momentum as above. M1 is available if they have not reversed the direction of the impulse. An equation which allows for the change in direction by using **u** – **v** can score full marks. Could be working with either option for the direction of motion of *A*
A1ft	Correct unsimplified equation in *u*, *v* or their *v*
A1	One correct solution Be aware that a sign error in the impulse-momentum equation for *A* can lead to a fortuitous answer. A fortuitous answer scores A0 (FYI the incorrect answers are $\frac{-7}{4}$ and $\frac{1}{4}$)

M1	Correct method to form a second equation in k (reversing the direction of motion of A)
A1	Second correct solution

3.

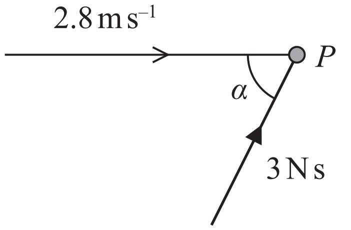


Figure 2

A particle P of mass 0.5 kg is moving in a straight line with speed 2.8 m s^{-1} when it receives an impulse of magnitude 3 N s.
The angle between the direction of motion of P immediately before receiving the impulse and the line of action of the impulse is α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

Find the speed of P immediately after receiving the impulse.

(5)

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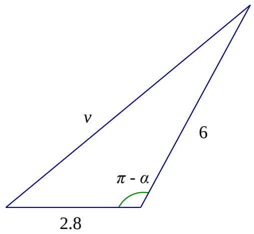
P 7 2 0 9 2 R A 0 1 0 3 2

Question	Scheme	Marks	AOs
3	Impulse momentum equation(s)	M1	3.1a
	$\begin{pmatrix} 3\times\cos\alpha \\ 3\times\sin\alpha \end{pmatrix} = \frac{1}{2}\begin{pmatrix} v_x - 2.8 \\ v_y \end{pmatrix}$ $\left(v_x = \frac{32}{5}, \quad v_y = \frac{24}{5} \right)$	A1 A1	1.1b 1.1b
	$v = \frac{1}{5}\sqrt{32^2 + 24^2}$	M1	1.1b
	$= 8\left(\text{m s}^{-1}\right)$	A1	1.1b
	Alternative working parallel and perpendicular to the impulse: $\begin{pmatrix} 3 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} v_1 - 2.8\times\cos\alpha \\ v_2 \pm 2.8\times\sin\alpha \end{pmatrix}$ $v_1 = 7.68, v_2 = \pm 2.24$ $v = \sqrt{7.68^2 + 2.24^2} = 8\left(\text{m s}^{-1}\right)$		
		(5)	
3alt			
	Using cosine rule:	M1	
	$v^2 = 2.8^2 + 6^2 - 2\times 2.8\times 6\cos(\pi - \alpha)$	A1 A1	
	Solve for v	M1	
	$v = 8\left(\text{m s}^{-1}\right)$	A1	
		(5)	
			(Total 5 marks)

Notes	
M1	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ in two dimensions. (i.e. resolving used) Dimensionally correct. Allow for a combined equation in vector format or for just one component. Condone sin/cos confusion. Allow if m seen but not substituted.
A1 A1	Equation for one component correct unsimplified Equations for both components correct unsimplified Allow A1A1 for a correct unsimplified vector equation Allow A marks if in terms of m and α
M1	Correct use of Pythagoras for their components to obtain the numerical value of the speed This may be seen or implied: an alert candidate might spot the 3, 4, 5 triangle.

A1	Correct only
Alt	
M1	Correct use of cosine rule in a dimensionally correct triangle. The lengths of the sides must be consistent, i.e. v, 2.8 and 6 or $\frac{1}{2}v$, 1.4 and 3 and it must be a correct vector triangle (vectors combined correctly)
A1 A1	Unsimpified equation with at most one error Correct unsimplified equation
M1	Substitute for trig. and solve for *v*
A1	Correct only

3. A particle P, of mass 0.5 kg, is moving with velocity $(4\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ when it receives an impulse $\mathbf{I}$ of magnitude 2.5 Ns.

As a result of the impulse, the direction of motion of P is deflected through an angle of 45°

Given that $\mathbf{I} = (\lambda\mathbf{i} + \mu\mathbf{j})$ Ns, find all the possible pairs of values of λ and μ.

(9)

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Question	Scheme	Marks	AOs
3	Use impulse-momentum principle	M1	3.1a
	$\lambda(2\mathbf{i}-\mathbf{j}) = 0.5(\mathbf{v}-(-\mathbf{i}+2\mathbf{j}))$	A1	1.1b
	$\mathbf{v} = (4\lambda-1)\mathbf{i}+(2-2\lambda)\mathbf{j}$	A1	1.1b
	Use of change in KE to set up quadratic equation in λ only.	M1	2.1
	$12 = \frac{1}{2}\times\frac{1}{2}\left\{(4\lambda-1)^2+(2-2\lambda)^2-((-1)^2+2^2)\right\}$	A1**ft**	1.1b
	Simplifying to $5\lambda^2-4\lambda-12=0$ and solving	M1	1.1b
	$\mathbf{I} = 4\mathbf{i}-2\mathbf{j}$	A1	2.2a
		(7)	
			(7 marks)

Notes: Allow column vectors throughout

M1: Allow **I =** … but must be a *difference* in momenta and dimensionally correct
A1: For LHS (This may be awarded later)
A1: For RHS
M1: All terms present but allow difference reversed
A1ft: Follow through their **v**
M1: Attempt to solve a 3 term quadratic
A1: $4\mathbf{i}-2\mathbf{j}$ only

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} + \mathbf{j})\,\text{m}\,\text{s}^{-1}$ when it receives an impulse $(2\mathbf{i} - \mathbf{j})\,\text{N}\,\text{s}$.

Show that the kinetic energy gained by P as a result of the impulse is 12 J.

(6)

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Paper 3C/4C: Further Mechanics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	Use Impulse-momentum principle	M1	2.1
	$2\mathbf{i}-\mathbf{j}=0.5\mathbf{v}-0.5(4\mathbf{i}+\mathbf{j})$	A1	1.1b
	$\frac{1}{2}\mathbf{v}=4\mathbf{i}-\frac{1}{2}\mathbf{j}$, $\mathbf{v}=8\mathbf{i}-\mathbf{j}$ (m s^{-1})	A1	1.1b
	Use of $\text{KE}=\frac{1}{2}m\lvert\mathbf{v}\rvert^2-\frac{1}{2}m\lvert\mathbf{u}\rvert^2$	M1	2.1
	$=\frac{1}{2}\times0.5\times\{(64+1)-(16+1)\}$	A1	1.1b
	$=\frac{1}{4}\times48=12$ (J) *	A1*	1.1b
		(6)	
			(6 marks)

Notes:

M1: Difference of terms & dimensionally correct
A1: Correct unsimplified equation
A1: cao
M1: Must be a difference of two terms
Must be dimensionally correct
A1: Correct unsimplified equation
A1*: Complete justification of given answer

3. A particle, P, of mass 3 kg is moving with velocity $(2\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{I}$ of magnitude $\sqrt{130}$ N s. Immediately after receiving the impulse, P is moving with velocity $(-\mathbf{i} + \lambda\mathbf{j})\,\text{m s}^{-1}$, where λ is a positive constant.

(a) Find $\mathbf{I}$, giving your answer in terms of $\mathbf{i}$ and $\mathbf{j}$.

(6)

The angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse is $\theta°$

(b) Find the value of θ

(3)

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Question	Scheme	Marks	AOs
3(a)	Impulse momentum equation	M1	2.1
	$\mathbf{I} = 3(-\mathbf{i} + \lambda\mathbf{j}) - 3(2\mathbf{i} + \mathbf{j})(= -9\mathbf{i} + (3\lambda - 3)\mathbf{j})$	A1	1.1b
	Magnitude of the impulse	M1	1.1b
	$130 = 81 + (3\lambda - 3)^2$ Follow their **I**	A1ft	1.1b
	$3\lambda - 3 = (\pm)7$, $\lambda = \frac{10}{3}$	M1	2.2a
	$\mathbf{I} = -9\mathbf{i} + 7\mathbf{j}$ (Ns)	A1	1.1b
		(6)	
3(b)	Use of scalar product: $(2\mathbf{i} + \mathbf{j}) \cdot \left(-\mathbf{i} + \frac{10}{3}\mathbf{j}\right) = \frac{4}{3}$	M1	3.1a
	$\cos\theta = \dfrac{\frac{4}{3}}{\sqrt{\frac{109}{9}}\sqrt{5}} \left(= \dfrac{4}{\sqrt{545}}\right)$ follow their λ	A1ft	1.1b
	$\theta = 80.1$	A1	1.1b
		(3)	
3(b) alt	Use trig to find 2 relevant angles: $\tan^{-1} \pm \frac{10}{3}$, $\tan^{-1} \frac{1}{2}$	(M1)	3.1a
	$73.30°$ or $106.70°$, $26.57°$	(A1)	1.1b
	$\theta = 80.1$	(A1)	1.1b
		(3)	
			(9 marks)

Notes:

3a	M1	Use impulse momentum to find the impulse in terms of λ
	A1	Correct unsimplified equation
	M1	Use Pythagoras and the given modulus
	A1ft	Correct unsimplified expression using their **I**
	M1	Solve for λ (or $3\lambda - 3$) and find **I**
	A1	Correct answer
3b	M1	Complete strategy, using vectors or equivalent, to find relevant angle Could be working with velocity or momentum.
	A1ft	Single trig ratio or all relevant angles. Follow their λ
	A1	80 or better