Fd1Ch7 XMQs and MS

(Total: 121 marks)

1. FD1_Sample   Q5 . 15 marks - FD1ch7 The simplex algorithm
2. FD1_Sample   Q7 . 12 marks - FD1ch7 The simplex algorithm
3. FD1_Specimen Q5 . 16 marks - FD1ch7 The simplex algorithm
4. FD1_Specimen Q7 . 10 marks - FD1ch7 The simplex algorithm
5. FD1_2019   Q6 . 12 marks - FD1ch7 The simplex algorithm
6. FD1_2020   Q7 . 17 marks - FD1ch7 The simplex algorithm
7. FD1_2021   Q8 . 18 marks - FD1ch7 The simplex algorithm
8. FD1_2022   Q4 . 6 marks - FD1ch7 The simplex algorithm
9. FD1_2022   Q7 . 15 marks - FD1ch7 The simplex algorithm

5. A garden centre makes hanging baskets to sell to its customers. Three types of hanging basket are made, *Sunshine*, *Drama* and *Peaceful*. The plants used are categorised as *Impact*, *Flowering* or *Trailing*.

Each *Sunshine* basket contains 2 *Impact* plants, 4 *Flowering* plants and 3 *Trailing* plants.
Each *Drama* basket contains 3 *Impact* plants, 2 *Flowering* plants and 4 *Trailing* plants.
Each *Peaceful* basket contains 1 *Impact* plant, 3 *Flowering* plants and 2 *Trailing* plants.

The garden centre can use at most 80 *Impact* plants, at most 140 *Flowering* plants and at most 96 *Trailing* plants each day.

The profit on *Sunshine*, *Drama* and *Peaceful* baskets are £12, £20 and £16 respectively. The garden centre wishes to maximise its profit.

Let x, y and z be the number of *Sunshine*, *Drama* and *Peaceful* baskets respectively, produced each day.

(a) Formulate this situation as a linear programming problem, giving your constraints as inequalities.

(5)

(b) State the further restriction that applies to the values of x, y and z in this context.

(1)

The Simplex algorithm is used to solve this problem. After one iteration, the tableau is

b.v.	x	y	z	r	s	t	Value
r	$-\frac{1}{4}$	0	$-\frac{1}{2}$	1	0	$-\frac{3}{4}$	8
s	$\frac{5}{2}$	0	2	0	1	$-\frac{1}{2}$	92
y	$\frac{3}{4}$	1	$\frac{1}{2}$	0	0	$\frac{1}{4}$	24
P	3	0	–6	0	0	5	480

(c) State the variable that was increased in the first iteration. Justify your answer.

(2)

(d) Determine how many plants in total are being used after only one iteration of the Simplex algorithm.

(1)

(e) Explain why for a second iteration of the Simplex algorithm the 2 in the z column is the pivot value.

(2)

After a second iteration, the tableau is

b.v.	x	y	z	r	s	t	Value
r	$\frac{3}{8}$	0	0	1	$\frac{1}{4}$	$-\frac{7}{8}$	31
s	$\frac{5}{4}$	0	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	46
y	$\frac{1}{8}$	1	0	0	$-\frac{1}{4}$	$\frac{3}{8}$	1
P	$\frac{21}{2}$	0	0	0	3	$\frac{7}{2}$	756

(f) Use algebra to explain why this tableau is optimal.

(1)

(g) State the optimal number of each type of basket that should be made.

(1)

The manager of the garden centre is able to increase the number of *Impact* plants available each day from 80 to 100. She wants to know if this would increase her profit.

(h) Use your final tableau to determine the effect of this increase. (You should not carry out any further calculations.)

(2)

(Total for Question 5 is 15 marks)

Question	Scheme	Marks	AOs
5(a)	Maximise $P = 12x + 20y + 16z$	B1	3.3
	Subject to $2x+3y+z \le 80$ $4x+2y+3z \le 140$ $3x+4y+2z \le 96$ $x, y, z \ge 0$	M1 A1 A1 B1	3.3 1.1b 1.1b 3.3
		(5)	
(b)	The values must all be integers	B1	3.3
		(1)	
(c)	Variable y entered the basic variable column…	M1	2.4
	…so y was increased first	A1	2.2a
		(2)	
(d)	$(80+140+96)-(8+92)=216$ plants	B1	3.2a
		(1)	
(e)	The next pivot must come from a column which has a negative value in the objective row so therefore the pivot must come from column z	M1	2.4
	The pivot must be positive and the least of 92/2 = 46 and 24/0.5 = 48 so the pivot must be the 2 (from column z)	A1	2.2a
		(2)	
(f)	$P + 10.5x + 3s + 3.5t = 756$ so increasing x, s or t will decrease profit	B1	2.4
		(1)	
(g)	Make 1 *Drama* basket and 46 *Peaceful* baskets	B1	2.2a
		(1)	
(h)	The slack variable, r, associated with this type of plant, is currently at 31. Increasing the number of *Impact* plants by a further 20 would have no effect	M1 A1	3.1b 3.2a
		(2)	
		(15 marks)	

Question 5 notes:

(a)

B1: Correct objective function/expression (accept in pence rather than pounds e.g. $1200x + 2000y + 1600z$)

M1: Correct coefficients and correct right-hand side for at least one inequality – accept any inequality or equals

A1: Two correct (non-trivial) inequalities

A1: All three non-trivial inequalities correct

B1: $x, y, z \geq 0$

(b)

B1: cao

(c)

M1: Correct reasoning that y has become a basic variable

A1: Correct deduction that y was therefore increased first

(d)

B1: cao

(e)

M1: Correct reasoning given that the pivot value must come from column z

A1: Correctly deduce (from correctly stated calculations) that the pivot value is the 2 in column z

(f)

B1: States correct objective function and mention of increasing x, s or t will decrease profit

(g)

B1: cao – in context so not in terms of y and z

(h)

M1: Identifies the slack variable r and its current value of 31

A1: Correct interpretation that increasing the number of Impact plants would have no effect

7. A linear programming problem in x, y and z is described as follows.

Maximise $P = 3x + 2y + 2z$

subject to $2x + 2y + z \leqslant 25$

$x + 4y \leqslant 15$

$x \geqslant 3$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

The big-M method is to be used to solve this linear programming problem.

(b) Define, in this context, what M represents. You must use correct mathematical language in your answer.

(1)

The initial tableau for a big-M solution to the problem is shown below.

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	2	2	1	1	0	0	0	25
s_2	1	4	0	0	1	0	0	15
t_1	1	0	0	0	0	–1	1	3
P	$-(3 + M)$	–2	–2	0	0	M	0	$-3M$

(c) Explain clearly how the equation represented in the b.v. t_1 row was derived.

(1)

(d) Show how the equation represented in the b.v. P row was derived.

(2)

The tableau obtained from the first iteration of the big-M method is shown below.

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	0	2	1	1	0	2	–2	19
s_2	0	4	0	0	1	1	–1	12
x	1	0	0	0	0	–1	1	3
P	0	–2	–2	0	0	–3	$3 + M$	9

(e) Solve the linear programming problem, starting from this second tableau. You must

- give a detailed explanation of your method by clearly stating the row operations you use and
- state the solution by deducing the final values of P, x, y and z.

(7)

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

Question 7 continued

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops
P									

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops
P									

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

Question	Scheme	Marks	AOs
7(a)	Simplex can only work with $\leq$ constraints	B1	3.5b
		(1)	
(b)	M is an arbitrary large real number	B1	2.5
		(1)	
(c)	$x \geqslant 3 \Rightarrow x - s_3 + t_1 = 3$ where s_3 is a surplus variable and t_1 is an artificial variable	B1	2.4
		(1)	
(d)	Let $P = 3x + 2y + 2z - Mt_1$ (where M is an arbitrary large number) $\therefore P = 3x + 2y + 2z - M(3 - x + s_3)$ $= (3 + M)x + 2y + 2z - Ms_3 - 3M$ $\Rightarrow P - (3 + M)x - 2y - 2z + Ms_3 = -3M$	M1 A1	2.1 1.1b
		(2)	
(e)	(see first table below)	M1 A1 A1	1.1b 1.1b 1.1b
	(see second table below)	M1 A1 B1	1.1b 1.1b 2.4
	$P = 47, x = 3, y = 0, z = 19$	B1ft	1.1b
		(7)	
		(12 marks)	

7(e) first tableau:

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops
s_3	0	1	½	½	0	1	−1	19/2	$r_1 = (1/2)R_1$
s_2	0	3	−1/2	−1/2	1	0	0	5/2	$R_2 - r_1$
x	1	1	½	½	0	0	0	25/2	$R_3 + r_1$
P	0	1	−1/2	3/2	0	0	M	75/2	$R_4 + 3r_1$

7(e) second tableau:

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value	Row Ops
z	0	2	1	1	0	2	−2	19	$r_1 = 2R_1$
s_2	0	4	0	0	1	1	−1	12	$R_2 + (1/2)r_1$
x	1	0	0	0	0	−1	1	3	$R_3 - (1/2)r_1$
P	0	2	0	2	0	1	$M - 1$	47	$R_4 + (1/2)r_1$

Question 7 notes:

(a)

B1: Correctly states the limitation of the Simplex model – Simplex involves iterations which allow movement from one vertex in the feasible region to another vertex (in the feasible region). If all constraints are of the form ≤ this means that the origin is always a feasible solution and therefore can act as the initial starting point for the problem. However, the constraint $x \geqslant 3$ means that the origin is not feasible and so the algorithm is unable to begin.

(b)

B1: cao including the correct mathematical language (must include 'arbitrary', 'large' and 'real')

(c)

(B1: Correctly states both the inequality $x \geq 3$ and the equation $x - s_3 + t_1 = 3$ together with an explanation of the meaning behind the variables s_3 and t_1

(d)

M1: $P = 3x + 2y + 2z - Mt_1$ and substitutes their expression for t_1

A1: Correct mathematical argument including sufficient detail to allow the line of reasoning to be followed to the correct conclusion – dependent on previous B mark in (c)

(e)

M1: Correct pivot located, attempt to divide row. If negative value used then no marks

A1: Pivot row correct (including change of b.v.) and row operations used at least once, one of columns y, z, s_1, t_1 or Value correct

A1: cao for values (ignore b.v. column and Row Ops)

M1: Pivot row consistent (following their previous table) including change of b.v. and row operations used at least once, one of columns y, s_1, s_3, t_1 or Value correct

A1: cao on final table (ignore Row Ops)

B1: The correct Row Operations explained either in terms of the 'old' or 'new' pivot rows

B1ft: Correctly states the final values of P, x, y and z from their correct corresponding rows of the final table

5. Dale is planning a production run of three types of desk. The three types are lectern desk, roll top desk and writing desk.

In total, Dale has 400 m^2 of wood available; each lectern desk requires 3 m^2, each roll top desk requires 5 m^2, and each writing desk requires 8 m^2

In total, Dale has 350 hours available; each lectern desk requires 3 hours, each roll top desk requires 6 hours, and each writing desk requires 10 hours.

Once complete, the desks need to be stored in a warehouse. The warehouse has 75 m^3 of storage space available; each lectern desk requires 1 m^3, each roll top desk requires 1.5 m^3 and each writing desk requires 1.25 m^3

The profit on each lectern desk sold is £40, the profit on each roll top desk sold is £50 and the profit on each writing desk sold is £65

Dale wants to maximise his profit.

Let x, y and z be the number of lectern desks, roll top desks and writing desks made respectively during the production run.

(a) Formulate this situation as a linear programming problem, giving your constraints as inequalities.

(4)

(b) Complete the initial tableau in the answer book for this linear programming problem.

(2)

(c) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the Simplex algorithm. Give an explanation of the method by clearly stating the row operations you use.

(4)

After a second iteration, the exact values in the tableau are

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	0	−0.52	0	1	−0.68	−0.96	90
z	0	0.24	1	0	0.16	−0.48	20
x	1	1.2	0	0	−0.2	1.6	50
P	0	13.6	0	0	2.4	32.8	3300

(d) Use algebra to explain how you know that this tableau is optimal.

(1)

(e) (i) State the optimal number of each type of desk that should be made.

(ii) State the maximum total profit.

(2)

(f) Explain, in context, the meaning of the 90 in the value column.

(2)

(g) Give a reason why the profit may be less than the value stated in (e)(ii).

(1)

(Total for Question 5 is 16 marks)

5.

(b)

b.v.	x	y	z	s_1	s_2	s_3	Value
							400
							350
							75
P							

(c)

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
P								

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
P								

Question	Scheme	Marks	AOs
5(a)	Maximise $P = 40x + 50y + 65z$	B1	2.5
	Subject to $3x + 5y + 8z \le 400$, $3x + 6y + 10z \le 350$, $x + 1.5y + 1.25z \le 75$, $x, y, z \ge 0$	M1 A1 B1	3.3 1.1b 3.3
		(4)	
(b)	*(see table below)*	M1 A1	3.4 1.1b
		(2)	
(c)	*(see table below)*	M1 A1ft A1 B1ft	2.1 1.1b 1.1b 2.4
		(4)	
(d)	$P + 13.6y + 2.4s_2 + 32.8s_3 = 3300$ so increasing y, s_2 or s_3 will decrease profit	B1	2.4
		(1)	
(e)	(i) Make 50 lectern desks, 20 writing desks and no roll top desks	B1	3.2a
	(ii) £3300	B1	1.1b
		(2)	
(f)	The 90 is the value of the slack variable s_1 which comes from the constraint $3x + 5y + 8z \le 400$	B1	2.4
	Indicating that there is 90 m^2 of wood still available	B1	3.2a
		(2)	

(b)

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	3	5	8	1	0	0	400
s_2	3	6	10	0	1	0	350
s_3	1	1.5	1.25	0	0	1	75
P	–40	–50	–65	0	0	0	0

(c)

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
s_1	0.6	0.2	0	1	–0.8	0	120	$R_1 = r_1 - 8R_2$
z	0.3	0.6	1	0	0.1	0	35	$R_2 = 0.1r_2$
s_3	0.625	0.75	0	0	–0.125	1	31.25	$R_3 = r_3 - 1.25R_2$
P	–20.5	–11	0	0	6.5	0	2275	$R_4 = r_4 + 65R_2$

(g)	e.g. there is no guarantee that all the desks will be sold	B1	3.5b
		(1)	
			(16 marks)

Notes:

(a)
B1: Correct objective function/expression (accept in pence rather than pounds e.g. 4000x + 5000y + 6500z) together with 'maximise'
M1: Correct coefficients and correct right-hand side for at least one inequality – accept any inequality or equals
A1: All three correct (non-trivial) inequalities
B1: $x, y, z \geq 0$
(b)
M1: Constructing all four rows including slack variables with at least one negative in P row (allow sign/numerical slips)
A1: All four rows correct
(c)
M1: Correct pivot located, attempt to divide row
A1ft: Pivot row correct (including change of b.v.) and row operations used at least once, one of columns x, y, s_2 or Value correct
A1: Cao for values (ignore b.v. column and Row Ops)
B1ft: The correct Row Operations (on the ft) explained either in terms of the 'old' or 'new' pivot rows
(d)
B1: States correct objective function and mention of increasing y, s_2 or s_3 will decrease profit
(e)(i)
B1: Cao – in context so not in terms of x, y and z
(ii) B1: Cao
(f)
B1: Recognises that $s_1 = 90$ and is linked to the wood constraint
B1: Evaluates this value in context (so must see both units and mention of 'wood')
(g)
B1: Cao – any suitable limitation to the solution in context

7.

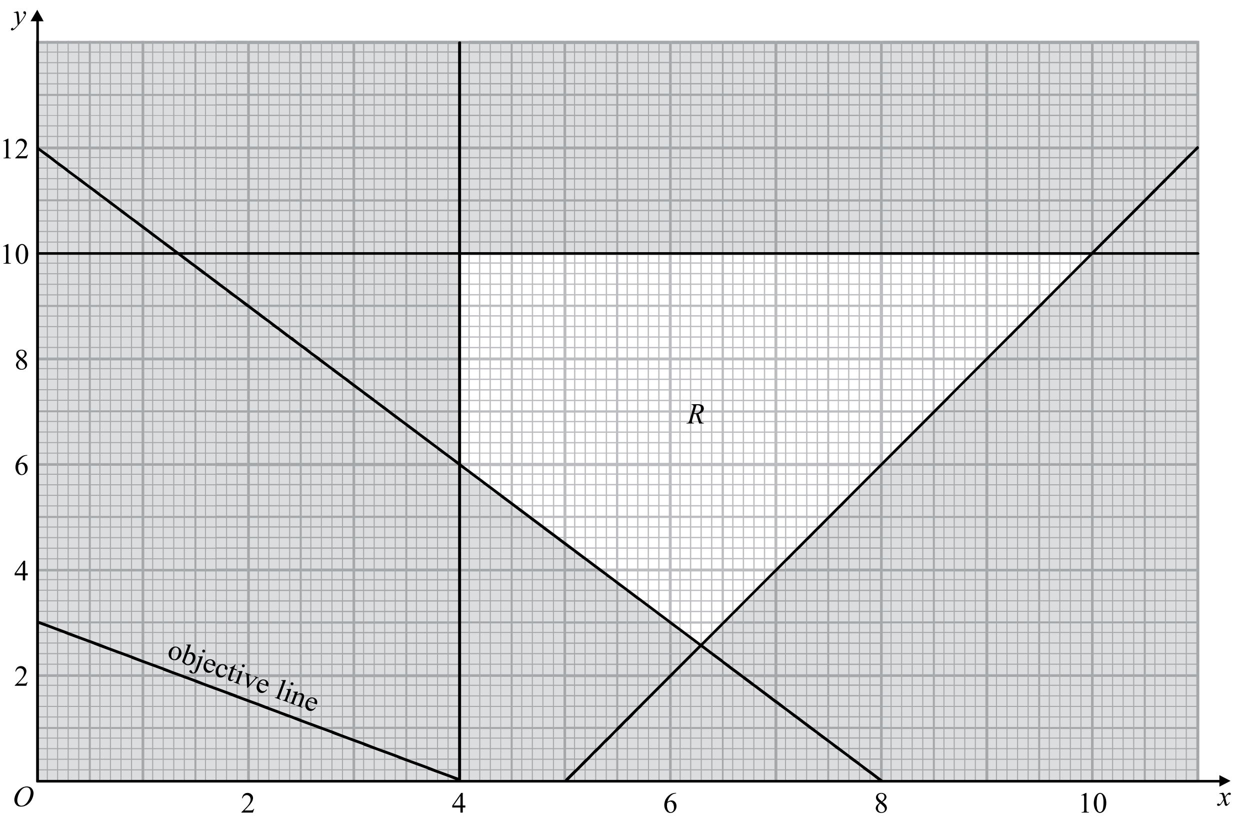

Figure 5

Figure 5 shows the constraints of a maximisation linear programming problem in x and y, where R is the feasible region. An objective line is also shown and labelled on Figure 5.

A student decides to find the optimal vertex of R by using the two-stage Simplex algorithm.

Set up an initial tableau for the two-stage Simplex algorithm. (You should not solve the linear programming problem.)

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

7.

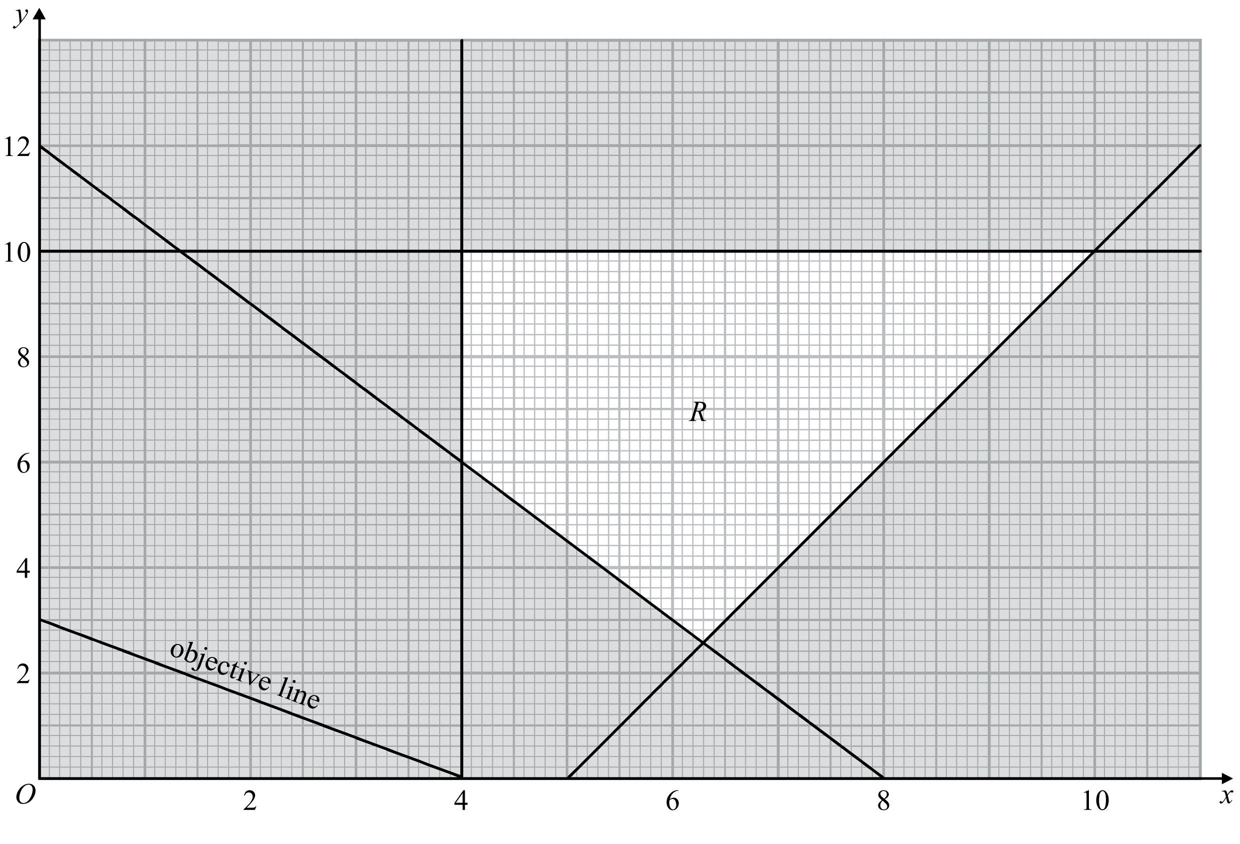

Figure 5

Question 7 continued

b.v.									Value

(Total for Question 7 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

Question	Scheme	Marks	AOs
7	Objective line $\Rightarrow$ e.g. $P-3x-4y=0$	B1	3.4
	$y \le 10$ $x \ge 4$	B1	3.4
	Line through (0, 12) and (8, 0) is $y-12=-\frac{3}{2}(x-0)$	M1	1.1b
	Line through (5, 0) and (10, 10) is $y-10=2(x-10)$	M1	1.1b
	$2x-y \le 10 \Rightarrow 2x-y+s_1=10$ $y \le 10 \Rightarrow y+s_2=10$ $x \ge 4 \Rightarrow x-s_3+a_1=4$ $3x+2y \ge 24 \Rightarrow 3x+2y-s_4+a_2=24$	M1 A1ft A1	2.1 1.1b 1.1b
	$a_1+a_2=4-x+s_3+24+s_4-3x-2y$ $\Rightarrow A=-(a_1+a_2)=4x+2y-s_3-s_4-28$ $\Rightarrow A-4x-2y+s_3+s_4=-28$	M1	2.2a
	e.g. (tableau below)	M1 A1	2.1 2.2a

b.v.	x	y	s_1	s_2	s_3	s_4	a_1	a_2	Value
s_1	2	–1	1	0	0	0	0	0	10
s_2	0	1	0	1	0	0	0	0	10
a_1	1	0	0	0	–1	0	1	0	4
a_2	3	2	0	0	0	–1	0	1	24
P	–3	–4	0	0	0	0	0	0	0
A	–4	–2	0	0	1	1	0	0	–28

(10 marks)

Notes:

B1: cao for objective function (oe e.g. $P-3x-4y=k$)
B1: cao
M1: correct method for finding the equation of the line through (0, 12) and (8, 0)
M1: correct method for finding the equation of the line through (5, 0) and (10, 10)
M1: translate all 4 inequalities into equations – must include all three types of variables (slack, surplus and artificial)
A1ft: two correct equations following their inequalities
A1: all four correct equations
M1: setting up the new objective and substituting for a_1 and a_2
M1: setting up tableau – all six lines with four basic variables
A1: cao (oe e.g. consistent P line with their objective equation)

6. A linear programming problem in x, y and z is described as follows.

Maximise $P = 2x + 2y - z$

subject to
$$3x + y + 2z \leqslant 30$$
$$x - y + z \geqslant 8$$
$$4y + 2z \geqslant 15$$
$$x, y, z \geqslant 0$$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

(b) Set up the initial tableau for solving this linear programming problem using the big-M method.

(7)

After a first iteration of the big-M method, the tableau is

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	0	1.5	1	0	0.25	0	−0.25	26.25
a_1	1	0	1.5	0	−1	−0.25	1	0.25	11.75
y	0	1	0.5	0	0	−0.25	0	0.25	3.75
P	$-(2 + M)$	0	$2 - 1.5M$	0	M	$-0.5 + 0.25M$	0	$0.5 + 0.75M$	$7.5 - 11.75M$

(c) State the value of each variable after the first iteration.

(1)

(d) Explain why the solution given by the first iteration is not feasible.

(1)

Taking the most negative entry in the profit row to indicate the pivot column,

(e) obtain the most efficient pivot for a second iteration. You must give reasons for your answer.

(2)

(Total for Question 6 is 12 marks)

Qu	Scheme	Marks	AOs
6(a)	Simplex can only be applied when the non-negativity constraints are $\leqslant$	B1	3.5b
		(1)	
(b)	$3x+y+2z\le 30 \Rightarrow 3x+y+2z+s_1=30$	B1	1.1b
	$x-y+z\ge 8 \Rightarrow x-y+z-s_2+a_1=8$	B1	2.5
	$4y+2z\ge 15 \Rightarrow 4y+2z-s_3+a_2=15$	B1	1.1b
	$P=2x+2y-z \Rightarrow P=2x+2y-z-M\left(a_1+a_2\right)$ together with $a_1+a_2=23-x-3y-3z+s_2+s_3$	M1	2.1
	$P-\left(2+M\right)x-\left(2+3M\right)y-\left(-1+3M\right)z+Ms_2+Ms_3=-23M$	A1	1.1b
	(tableau below)	M1 A1	3.3 2.2a
		(7)	
(c)	$s_1=26.25, a_1=11.75, y=3.75, x=z=s_2=s_3=a_2=0$	B1	3.4
		(1)	
(d)	The solution after the 1st iteration is not feasible because $a_1=11.75$ is an artificial variable which must be zero in a feasible solution	B1	2.4
		(1)	
(e)	The most negative value in the objective row is $2-1.5M$ so the pivot is a value from the *z*-column	B1	2.4
	The 0.5 in the *y* row is the pivot because $\frac{3.75}{0.5}$ is less than both $\frac{26.25}{1.5}$ and $\frac{11.75}{1.5}$	dB1	2.2a
		(2)	
		(12 marks)	

Tableau for (b):

b.v	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	1	2	1	0	0	0	0	30
a_1	1	-1	1	0	-1	0	1	0	8
a_2	0	4	2	0	0	-1	0	1	15
P	$-\left(2+M\right)$	$-\left(2+3M\right)$	$-\left(3M-1\right)$	0	M	M	0	0	$-23M$

Notes for Question 6
(a) **B1:** CAO – e.g. not all of the constraints are $\leq$, the origin is not a (basic feasible) solution of the LP **(b)** **B1:** CAO $3x + y + 2z + s_1 = 30$ (may be seen in the simplex tableau – allow any s_i (or *s*) for s_1) **B1:** CAO $x - y + z - s_2 + a_1 = 8$ (may be seen in the simplex tableau – allow any consistent s_i for s_2 (or *t* say) but not the same s_i as in the previous mark and allow any a_i for a_1) **B1:** CAO $4y + 2z - s_3 + a_2 = 15$ (may be seen in the simplex tableau – same conditions as above) **M1:** setting up the new objective which must be $P = 2x + 2y - z - M\left(a_1 + a_2\right)$ and substituting for their a_1 and a_2 (if no working then the **correct** objective line in the tableau implies this mark) **A1:** CAO $P - \left(2 + M\right)x - \left(2 + 3M\right)y - \left(-1 + 3M\right)z + Ms_2 + Ms_3 = -23M$ (any equivalent form – need not be factorised and does not need to be re-arranged into this form - if no working then the **correct** objective line in the tableau implies this mark) **M1:** setting up initial tableau – all four rows complete with two correct rows (but ignore b.v. column for this mark) **A1:** CAO (any equivalent correct form) **(c)** **B1:** CAO $s_1 = 26.25, a_1 = 11.75, y = 3.75, x = z = s_2 = s_3 = a_2 = 0$ (ignore expression for *P* if given) **(d)** **B1:** correct reasoning of why the solution is not feasible e.g. a_1 is not zero but B0 for just stating that the artificial variable is non-zero (so must see either a_1 or 11.75 being stated as non-zero) **(e)** **B1:** correct reasoning of why the pivot comes from a value from the *z*-column so must say that the most negative value (in the objective row) is 2 – 1.5*M* (or this expression clearly implied) **dB1:** correct justification of why the 0.5 in the third row is the next pivot (dependent on previous B mark) – so must compare or state that $\frac{3.75}{0.5}$ or 7.5 is less than both $\frac{26.25}{1.5}$ or 17.5 and $\frac{11.75}{1.5}$ or 7.8(3333….) – just stating that the 0.5 in the third row is the next pivot without reasoning is no marks in this part

7. A maximisation linear programming problem in x, y and z is to be solved using the two-stage simplex method.

The partially completed initial tableau is shown below.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	1	2	3	1	0	0	0	0	45
a_1	3	2	0	0	–1	0	1	0	9
a_2	–1	0	4	0	0	–1	0	1	4
P	–2	–1	–3	0	0	0	0	0	0
A									

(a) Using the information in the above tableau, formulate the linear programming problem. State the objective and list the constraints as inequalities.

(4)

(b) Complete the bottom row of Table 1 in the answer book. You should make your method and working clear.

(2)

The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	0	$\frac{5}{6}$	0	1	$\frac{7}{12}$	$\frac{3}{4}$	$-\frac{7}{12}$	$-\frac{3}{4}$	$\frac{147}{4}$
x	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	3
z	0	$\frac{1}{6}$	1	0	$-\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{7}{4}$
P	0	$\frac{5}{6}$	0	0	$-\frac{11}{12}$	$-\frac{3}{4}$	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{45}{4}$
A	0	0	0	0	0	0	1	1	0

(c) (i) Explain how the above tableau shows that a basic feasible solution has been found for the original linear programming problem.

(ii) Write down the basic feasible solution for the second stage.

(3)

(d) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method, to obtain a new tableau, T. Make your method clear by stating the row operations you use.

(5)

(e) (i) Explain, using T, whether or not an optimal solution to the original linear programming problem has been found.

(ii) Write down the value of the objective function.

(iii) State the values of the basic variables.

(3)

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

7.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	1	2	3	1	0	0	0	0	45
a_1	3	2	0	0	–1	0	1	0	9
a_2	–1	0	4	0	0	–1	0	1	4
P	–2	–1	–3	0	0	0	0	0	0
A									

Table 1

Question 7 continued

b.v.	x	y	z	s_1	s_2	s_3	Value
s_1	0	$\frac{5}{6}$	0	1	$\frac{7}{12}$	$\frac{3}{4}$	$\frac{147}{4}$
x	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	3
z	0	$\frac{1}{6}$	1	0	$-\frac{1}{12}$	$-\frac{1}{4}$	$\frac{7}{4}$
P	0	$\frac{5}{6}$	0	0	$-\frac{11}{12}$	$-\frac{3}{4}$	$\frac{45}{4}$

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
P								

Spare copy

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
P								

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7(a)</td><td>Maximise $(P=)2x+y+3z$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>$x+2y+3z\le 45$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>$3x+2y\ge 9$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>$-x+4z\ge 4$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>$A=-(a_1+a_2)\Rightarrow -(9-3x-2y+s_2+4+x-4z+s_3)$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$A-2x-2y-4z+s_2+s_3=-13$ therefore bottom row of the table is
<table><tr><td>A</td><td>–2</td><td>–2</td><td>–4</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>–13</td></tr></table></td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)(i)</td><td>In the given tableau the value of the objective A is equal to zero indicating that a basic feasible solution has been found</td><td>B1</td><td>2.4</td></tr>
<tr><td>(c)(ii)</td><td>$x=3, y=0, z=\frac{7}{4}, s_1=\frac{147}{4}, s_2=s_3=0$</td><td>B1
B1</td><td>3.4
1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>
<table>
<tr><th>b.v</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>Value</th><th>Row Ops</th></tr>
<tr><td>s_2</td><td>0</td><td>$\frac{10}{7}$</td><td>0</td><td>$\frac{12}{7}$</td><td>1</td><td>$\frac{9}{7}$</td><td>63</td><td>R1 $\div\frac{7}{12}$</td></tr>
<tr><td>x</td><td>1</td><td>$\frac{8}{7}$</td><td>0</td><td>$\frac{4}{7}$</td><td>0</td><td>$\frac{3}{7}$</td><td>24</td><td>R2 + $\frac{1}{3}$R1</td></tr>
<tr><td>z</td><td>0</td><td>$\frac{2}{7}$</td><td>1</td><td>$\frac{1}{7}$</td><td>0</td><td>$-\frac{1}{7}$</td><td>7</td><td>R3 + $\frac{1}{12}$R1</td></tr>
<tr><td>P</td><td>0</td><td>$\frac{15}{7}$</td><td>0</td><td>$\frac{11}{7}$</td><td>0</td><td>$\frac{3}{7}$</td><td>69</td><td>R4 + $\frac{11}{12}$R1</td></tr>
</table></td><td>M1
A1
M1
A1
A1</td><td>2.1
1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(e)(i)</td><td>Yes, an optimal solution has been found as there are no negative values in the objective (P) row</td><td>B1</td><td>2.4</td></tr>
<tr><td>(e)(ii)</td><td>$P=69$</td><td>B1ft</td><td>3.4</td></tr>
<tr><td>(e)(iii)</td><td>$s_2=63, x=24, z=7$</td><td>B1ft</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4">(17 marks)</td></tr>
</table>

Notes for Question 7
(a) **B1:** CAO – including maximise (or max) **B1:** CAO (oe) **B1:** CAO (oe) **B1:** CAO (oe) **(b)** **M1:** Setting up the new objective and substituting for a_1 and a_2 **A1:** Correct values substituted into Table 1 **(c)** **B1:** CAO – mention that $A = 0$ **B1:** At least three values stated correctly **B1:** All six values correct (ignore values stated for a_1, a_2 and P) **(d)** **M1:** Correct pivot located, attempt to divide row **A1:** Pivot row correct including change of b.v. **M1: All** values in one of the non-pivot rows correct **or** one of the non zero and one columns (y, s_1, s_3 or value) correct following through their choice of pivot from column s_2 or s_3 **A1:** Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (y, s_1, s_3 or value) correct **A1:** CAO all values and row operations correctly stated – allow if row operations given in terms of old row 1 – **ignore b.v. column for this mark** **(e)(i)** **B1:** Correct reasoning of why solution is optimal or using $P = 69 - \frac{15}{7}y - \frac{11}{7}s_1 - \frac{3}{7}s_3$ and mentioning increasing y, s_1, s_3 would decrease P (oe) **(e)(ii)** **B1ft:** their value of P – dependent on both M marks in (d) **(e)(iii)** **B1ft:** their values of the basic variables **only** – dependent on both M marks in (d)

8. Susie is preparing for a triathlon event that is taking place next month. A triathlon involves three activities: swimming, cycling and running.

Susie decides that in her training next week she should

- maximise the total time spent cycling and running
- train for at most 39 hours
- spend at least 40% of her time swimming
- spend a total of at least 28 hours of her time swimming and running

Susie needs to determine how long she should spend next week training for each activity. Let

- x represent the number of hours swimming
- y represent the number of hours cycling
- z represent the number of hours running

(a) Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(5)

Susie decides to solve this linear programming problem by using the two-stage Simplex method.

(b) Set up an initial tableau for solving this problem using the two-stage Simplex method.

As part of your solution you must show how

- the constraints have been made into equations using slack variables, exactly one surplus variable and exactly one artificial variable
- the rows for the two objective functions are formed

(6)

The following tableau T is obtained after one iteration of the second stage of the two-stage Simplex method.

b.v.	x	y	z	s_1	s_2	s_3	Value
y	0	1	0	1	0	1	11
s_2	0	0	5	−2	1	−5	62
x	1	0	1	0	0	−1	28
P	0	0	−1	1	0	1	11

(c) Obtain a suitable pivot for a second iteration. You must give reasons for your answer.

(2)

(d) Starting from tableau T, solve the linear programming problem by performing one further iteration of the second stage of the two-stage Simplex method. You should make your method clear by stating the row operations you use.

(5)

(Total for Question 8 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS

Question 8 continued

b.v.	x	y	z	s_1	s_2	s_3	a_1	Value
P								
I								

Question 8 continued

b.v.	x	y	z	s_1	s_2	s_3	Value
y	0	1	0	1	0	1	11
s_2	0	0	5	−2	1	−5	62
x	1	0	1	0	0	−1	28
P	0	0	−1	1	0	1	11

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
P								

Spare copy

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
P								

Question	Scheme	Marks	AOs
8(a)	$x+y+z \leq 39$	B1	3.3
	$\frac{2}{5}(x+y+z) \leq x \quad (\Rightarrow -3x+2y+2z \leq 0)$	M1 A1	3.3 1.1b
	$x+z \geq 28$	B1	1.1b
	Maximise $P = y+z \quad (\Rightarrow P-y-z=0)$	B1	3.3
		(5)	
(b)	$x+y+z \leq 39 \Rightarrow x+y+z+s_1 = 39$ $-3x+2y+2z \leq 0 \Rightarrow -3x+2y+2z+s_2 = 0$	M1 A1	2.1 1.1b
	$x+z \geq 28 \Rightarrow x+z-s_3+a_1 = 28$	B1	2.5
	$I = -a_1 \Rightarrow I-x-z+s_3 = -28$	M1	2.1
	e.g. (see tableau below)	M1 A1	3.3 2.2a

b.v	x	y	z	s_1	s_2	s_3	a_1	Value
s_1	1	1	1	1	0	0	0	39
s_2	–3	2	2	0	1	0	0	0
a_1	1	0	1	0	0	–1	1	28
P	0	–1	–1	0	0	0	0	0
I	–1	0	–1	0	0	1	0	–28

Question	Scheme	Marks	AOs
		(6)	
(c)	The only negative in the objective row is the -1 so the pivot is from the z-column	B1	2.4
	The 5 in the s_2 row is the pivot because $\frac{62}{5}$ is less than $\frac{28}{1}$	B1	2.2a
		(2)	

(d)

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
y	0	1	0	1	0	1	11	R1
z	0	0	1	$-\frac{2}{5}$	$\frac{1}{5}$	-1	$\frac{62}{5}$	$\frac{1}{5}$ R2
x	1	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	0	$\frac{78}{5}$	R3 – R2
P	0	0	0	$\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{117}{5}$	R4 + R2

(d)		B1 M1 A1 A1	1.1b 2.1 1.1b 1.1b
	Spend 15.6 hours swimming, 11 hours cycling and 12.4 hours running	A1	3.2a
		(5)	
			(18 marks)

Notes:

(a)

B1: cao ($x+y+z \le 39$)

M1: $\frac{2}{5}(x+y+z) \square x$ where $\square$ is any inequality or equals

A1: cao

B1: cao ($x+z \ge 28$)

B1: Correct objective function ($P = y+z$) plus 'maximise' or 'max' but not 'maximum'

(b)

M1: One ≤ constraint re-formulated as an equation using slack variables – dependent on either the first **B** mark in **(a)** or the **M** mark in **(a)**

A1: cao (both ≤ constraints)

B1: ≥ constraint re-formulated as an equation using one surplus and one artificial variable

M1: Formulates second objective with $I = -a_1$ and their expression for a_1

M1: Setting up the initial tableau – all five rows complete with two correct rows (but ignore b.v. column for this mark)

A1: cao (any equivalent correct form)

(c)

B1: Correct reasoning that the pivot is a value from the *z*-column – condone any mention of negative value in *P* row

B1: Correct justification of why the 5 in the s_2 row is the next pivot – so must compare or state that 12.4 is less than 28 (not sufficient to just say that 12.4 (oe) is the least)

(d)

B1: Pivot row correct including change of b.v.

M1: All values in one of the non-pivot rows correct **or** one of the non zero and one columns (s_1, s_2 or value) correct (from their choice of pivot)

A1: Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (s_1, s_2 or value)

A1: For all values and row operations correctly stated – do not penalise lack of correct b.v. in pivot row twice. Condone blank Row Ops in the first row only

A1: Correct allocation of training times – must be in context (so not just in terms of *x*, *y* and *z*)

4. A linear programming problem in x, y and z is to be solved using the big-M method. The initial tableau is shown below.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	–2	2	0	–1	0	1	0	8
a_2	3	0	–4	0	0	–1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

(a) Using the information in the above tableau, formulate the linear programming problem. You should

- list each of the constraints as an inequality
- state the two possible objectives

(4)

(b) Obtain the most efficient pivot for a first iteration of the big-M method. You must give reasons for your answer.

(2)

(Total for Question 4 is 6 marks)

4.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	–2	2	0	–1	0	1	0	8
a_2	3	0	–4	0	0	–1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

Question	Scheme	Marks	AOs
4(a)	Constraints: $2x+3y+4z \leqslant 13$ $x-2y+2z \geqslant 8$ $3x-4z \geqslant 12$ $(x, y, z \geqslant 0)$	B1 B1	3.4 2.5
	Objective functions: Maximise $-2x+3y+z$ Minimise $2x-3y-z$	M1 A1	3.1a 2.2a
		(4)	
(b)	(Because *M* is big) the only negative in the objective row is the $2-4M$ so the pivot is from the *x*-column	B1	2.4
	The 3 in the a_2 row is the pivot as $\frac{12}{3}$ is less than both $\frac{8}{1}$ and $\frac{13}{2}$	B1	2.2a
		(2)	
			(6 marks)

Notes for Question 4

a1B1: One correct non-trivial inequality (allow strict inequality provided direction of inequality sign is correct) – equations with slack variables etc. scores no marks unless replaced with correct inequalities

a2B1: All three non-trivial inequalities correct

a1M1: Either expression stated correctly (allow equal to (or an inequality with) any letter e.g. $P=-2x+3y+z$ but not equal to a value e.g. = 0) – ignore any mention of maximum/minimum for this mark

a1A1: Both expressions correct including max/min correctly matched with each expression (allow equal to any letter only) – do not isw if they continue and place their expression(s) equal to a value(s)

b1B1: Correct reasoning that the pivot is a value from the *x*-column – as a minimum must state that the <u>2 – 4*M*</u> is the <u>only</u> <u>negative</u> (condone <u>most</u> <u>negative</u>) in the <u>objective row</u> (allow <u>profit row</u> or <u>*P* row</u>, condone '<u>bottom row</u>')

b2B1: Correct justification of why the 3 in the a_2 row or the 3 in the *x* column is the pivot – so **must** state the correct pivot in a clear unambiguous way (so just saying the pivot is 'the 3' is B0) **and** comparing or stating that $\frac{12}{3}$ or 4 is less than/least positive for both $\frac{8}{1}$ or 8 **and** $\frac{13}{2}$ or 6.5 – **must** see all three values so do check the table for possibly stating the θ values there. However, just stating that the 3 is the pivot because it is the smallest θ value (without seeing anywhere these θ values) is B0

7.

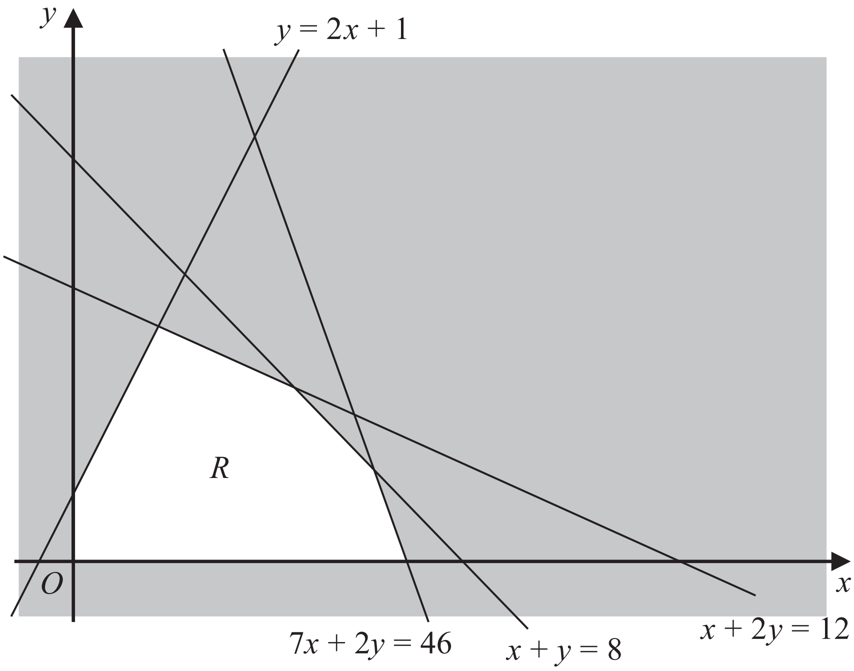

Figure 5

Figure 5 shows the constraints of a linear programming problem in x and y where R is the feasible region.

The objective is to maximise $P = x + ky$, where k is a positive constant.

The optimal vertex of R is to be found using the Simplex algorithm.

(a) Set up an initial tableau for solving this linear programming problem using the Simplex algorithm.

(5)

After two iterations of the Simplex algorithm a possible tableau T is

b.v.	x	y	s_1	s_2	s_3	s_4	Value
s_1	0	0	1	$-\frac{3}{5}$	0	$\frac{1}{5}$	1
x	1	0	0	$\frac{1}{5}$	0	$-\frac{2}{5}$	2
s_3	0	0	0	$-\frac{11}{5}$	1	$\frac{12}{5}$	22
y	0	1	0	$\frac{2}{5}$	0	$\frac{1}{5}$	5
P	0	0	0	$\frac{1}{5}+\frac{2}{5}k$	0	$-\frac{2}{5}+\frac{1}{5}k$	$5k+2$

(b) State the value of each variable after the second iteration.

(1)

It is given that T does not give an optimal solution to the linear programming problem.

After a third iteration of the Simplex algorithm the resulting tableau does give an optimal solution to the problem.

(c) Perform the third iteration of the Simplex algorithm and hence determine the range of possible values for P. You should state the row operations you use and make your method and working clear.

(9)

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

7.

b.v.	x	y					Value
P							

(b)

DO NOT WRITE IN THIS AREA

Question 7 continued

b.v.	x	y	s_1	s_2	s_3	s_4	Value
s_1	0	0	1	$-\frac{3}{5}$	0	$\frac{1}{5}$	1
x	1	0	0	$\frac{1}{5}$	0	$-\frac{2}{5}$	2
s_3	0	0	0	$-\frac{11}{5}$	1	$\frac{12}{5}$	22
y	0	1	0	$\frac{2}{5}$	0	$\frac{1}{5}$	5
P	0	0	0	$\frac{1}{5}+\frac{2}{5}k$	0	$-\frac{2}{5}+\frac{1}{5}k$	$5k+2$

b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row Ops
P								

Spare copy

b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row Ops
P								

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7
(a)</td><td>$x+y \leqslant 8 \Rightarrow x+y+s_1=8$
$x+2y \leqslant 12 \Rightarrow x+2y+s_2=12$
$7x+2y \leqslant 46 \Rightarrow 7x+2y+s_3=46$
$y \leqslant 2x+1 \Rightarrow -2x+y+s_4=1$
$P=x+ky \Rightarrow P-x-ky=0$</td><td>M1
A1

B1</td><td>3.4
1.1b

1.1b</td></tr>
<tr><td></td><td>e.g.
<table>
<tr><th>b.v.</th><th>x</th><th>y</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>Value</th></tr>
<tr><td>s_1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>8</td></tr>
<tr><td>s_2</td><td>1</td><td>2</td><td>0</td><td>1</td><td>0</td><td>0</td><td>12</td></tr>
<tr><td>s_3</td><td>7</td><td>2</td><td>0</td><td>0</td><td>1</td><td>0</td><td>46</td></tr>
<tr><td>s_4</td><td>–2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>P</td><td>–1</td><td>$-k$</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table></td><td>M1

A1</td><td>3.3

2.2a</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(b)</td><td>$x=2, y=5, s_1=1, s_2=0, s_3=22, s_4=0$ ($P = 5k + 2$)</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><th>b.v.</th><th>x</th><th>y</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>Value</th><th>Row Ops</th></tr>
<tr><td>s_4</td><td>0</td><td>0</td><td>5</td><td>–3</td><td>0</td><td>1</td><td>5</td><td>5r1</td></tr>
<tr><td>x</td><td>1</td><td>0</td><td>2</td><td>–1</td><td>0</td><td>0</td><td>4</td><td>r2 + 0.4R1</td></tr>
<tr><td>s_3</td><td>0</td><td>0</td><td>–12</td><td>5</td><td>1</td><td>0</td><td>10</td><td>r3 – 2.4R1</td></tr>
<tr><td>y</td><td>0</td><td>1</td><td>–1</td><td>1</td><td>0</td><td>0</td><td>4</td><td>r4 – 0.2R1</td></tr>
<tr><td>P</td><td>0</td><td>0</td><td>$2-k$</td><td>$k-1$</td><td>0</td><td>0</td><td>$4k+4$</td><td>r5 $-(0.2k-0.4)$ R1</td></tr>
</table></td><td>B1

M1

A1

A1

B1</td><td>1.1b

2.1

1.1b

1.1b

2.4</td></tr>
<tr><td></td><td>Optimal value of P is $4k+4$ (at $x = y = 4$)</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td>Second iteration not optimal $\Rightarrow -\frac{2}{5}+\frac{1}{5}k<0 \therefore k<2$</td><td>B1</td><td>3.1a</td></tr>
<tr><td></td><td>Third iteration optimal $\Rightarrow 2-k \geqslant 0$ and $k-1 \geqslant 0$ $(\therefore k \geqslant 1)$</td><td>dM1</td><td>3.4</td></tr>
<tr><td></td><td>$1 \leqslant k<2 \Rightarrow 8 \leqslant P<12$</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(9)</td><td></td></tr>
<tr><td colspan="4" align="right">(15 marks)</td></tr>
</table>

Notes for Question 7

a1M1: Correctly re-writing any two inequalities as equations with slack variables (can be implied by two correct rows in the Simplex tableau ignoring b.v. column). Or correctly stating all four constraints as inequalities

a1A1: Correctly re-writing all inequalities as equations with slack variables (can be implied by the four correct constraint rows in the Simplex tableau ignoring b.v. column)

a1B1: Correctly re-writes objective function (can be implied by correct row in tableau)

a2M1: **Any** two rows correct including consistent b.v. column entries **or** any three rows correct (ignoring b.v. column)

a2A1: cao (including consistent b.v. column) – **note that the candidate's order in which the rows appear in the tableau (and choice of letter to represent the slack variable) may be different. A correct tableau implies full marks in this part**

b1B1: cao for x, y, s_1, s_2, s_3 and s_4 only (ignore any mention of P)

c1B1: Pivot row completely correct including change of b.v.

c1M1: All **values** in one of the non-pivot rows correct (so ignore b.v. column and 'Row Ops' column) **or** one of the 'non zero and one' columns (which are s_1, s_2 or Value) correct (must have pivoted on the correct value)

c1A1: Row operations used correctly at least twice, i.e. two of the 'non zero and one' columns (s_1, s_2 or Value) correct

c2A1: cao **all** values including b.v. column – ignore 'Row Ops' column for this mark

c2B1: Correct row operations stated

(alternatives row operations are 5r1; r2 + 2r1; r3 – 12r1; r4 – r1; r5 – (k – 2)r1)

c2M1: Their optimal value (as a linear expression in k) stated correctly following their third iteration (must have pivoted on a positive value from the s_4 column and completed the bottom row of the tableau). Condone this expression ($4k + 4$ **if** correct) being stated as part of an equation/inequality (or as part of their final answer) – sight of this expression (but must be seen **outside** of the tableau) scores this mark

c3B1: Correctly inferring that $k < 2$ either from $-\frac{2}{5} + \frac{1}{5}k < 0$ or from $4k + 4 > 5k + 2$ – just stating $k < 2$ without it being clear where this comes from is B0

c3dM1: Considering (at least) two of their linear expressions in k from their objective row (not including the Value column) after the third iteration $\geqslant 0$ **(dependent on the previous M mark)** – note that working may be minimal here so please follow through their expressions in k from their objective row (so **if** correct, stating $k \leqslant 2$ and $k \geqslant 1$ implies this mark)

c3A1: cao for the range of values for P - this mark is dependent on a correct objective row in the tableau and the previous three marks in this part