Fd1Ch5 XMQs and MS

(Total: 53 marks)

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1. FD1_Sample   Q3 . 11 marks - FD1ch5 The travelling salesman problem

2. FD1_Specimen Q6 . 15 marks - FD1ch3 Algorithms on graphs

3. FD1_2020     Q3 .  9 marks - FD1ch3 Algorithms on graphs

4. FD1_2021     Q3 .  8 marks - FD1ch3 Algorithms on graphs

5. FD1_2022     Q3 . 10 marks - FD1ch3 Algorithms on graphs
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3. (a) Explain clearly the difference between the classical travelling salesperson problem and the practical travelling salesperson problem.

(2)

	A	B	C	D	E	F	G
A	–	17	24	16	21	18	41
B	17	–	35	25	30	31	x
C	24	35	–	28	20	35	32
D	16	25	28	–	29	19	45
E	21	30	20	29	–	22	35
F	18	31	35	19	22	–	37
G	41	x	32	45	35	37	–

The table shows the least distances, in km, by road between seven towns, A, B, C, D, E, F and G. The least distance between B and G is x km, where $x > 25$

Preety needs to visit each town at least once, starting and finishing at A. She wishes to minimise the total distance she travels.

(b) Starting by deleting B and all of its arcs, find a lower bound for Preety's route.

(3)

Preety found the nearest neighbour routes from each of A and C. Given that the sum of the lengths of these routes is 331 km,

(c) find x, making your method clear.

(4)

(d) Write down the smallest interval that you can be confident contains the optimal length of Preety's route. Give your answer as an inequality.

(2)

(Total for Question 3 is 11 marks)

3.

	A	B	C	D	E	F	G
A	–	17	24	16	21	18	41
B	17	–	35	25	30	31	x
C	24	35	–	28	20	35	32
D	16	25	28	–	29	19	45
E	21	30	20	29	–	22	35
F	18	31	35	19	22	–	37
G	41	x	32	45	35	37	–

Question 3 continued

	A	B	C	D	E	F	G
A	–	17	24	16	21	18	41
B	17	–	35	25	30	31	x
C	24	35	–	28	20	35	32
D	16	25	28	–	29	19	45
E	21	30	20	29	–	22	35
F	18	31	35	19	22	–	37
G	41	x	32	45	35	37	–

(Total for Question 3 is 11 marks)

Question	Scheme	Marks	AOs
3(a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited just once	B2,1,0	2.4 2.4
		(2)	
(b)	Prim's algorithm on reduced network starting at A: AD, AF, AE, CE, CG	M1	1.1b
	Lower bound = 107 + 17 + 25 = 149 (km)	M1 A1	1.1b 1.1b
		(3)	
(c)	NNA from A: A – D – F – E – C – G – B – A = $126+x$ NNA from C: C – E – A – D – F – B – G – C =$139+x$	M1 A1 A1	1.1b 1.1b 1.1b
	$(126+x)+(139+x)=331 \Rightarrow x=33$	A1	1.1b
		(4)	
(d)	$149 <$ optimal $\leqslant 159$	M1 A1	2.2b 1.1b
		(2)	
			(11 marks)

Notes:

(a)

B1: Understands the difference is connected to the number of times each vertex may be visited (but maybe incorrectly attributed). Must be an attempt at a difference (so must refer to both the classical and practical problems explicitly). Technical language (vertex/node) must be correct. Need not imply each/every/all (oe) vertices for this first mark

B1: Correctly reasons which is classical and which is practical and correctly states the difference. Must imply that each/every/all (oe) vertices are visited, so for example, 'the practical problem visits a vertex at least once while the classical visits a vertex only once' is B1B0 (note that B0B1 is not possible in (a))

(b)

M1: Correctly applying Prim's algorithm from node A for the first four arcs (or five nodes)

M1: Candidates weight of their RMST + 17 + 25 (the two smallest arcs incident to B)

A1: cao (condone lack of units)

(c)

M1: Either one route, must return to A

A1: One correct route, must return to A and corresponding length correct (do not is in part (c) if correct lengths seen but are then doubled)

A1: Both routes correct and their corresponding lengths correct

A1: cao for x

(d)

M1: Their numbers correctly used, accept any inequalities or any indication of interval from their 149 to their 159 (so 149 – 159 can score this mark). This mark is dependent on two routes seen in (c), however, neither of the two totals need to be correct. Please note that UB > LB for this mark

A1: cao (no follow through on their values) including correct inequalities or equivalent set notation (but condone $149 \leqslant$ optimal $\leqslant 159$)

6.

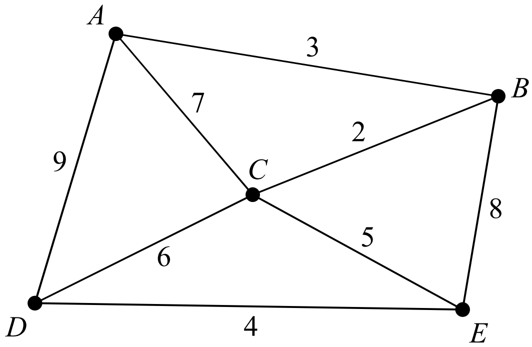


Figure 4

The network in Figure 4 shows the direct routes linking five towns, A, B, C, D and E. The edge weights represent distances, in km, between the towns.

(a) For this network, complete the initial distance and route tables provided in the answer book.

(2)

(b) Perform the first and second iterations of Floyd's algorithm. You should show the distance table and the route table after each iteration.

(4)

The final tables after five iterations of Floyd's algorithm are shown below.

Distance table

	A	**B**	**C**	**D**	**E**
A	–	3	5	9	10
B	3	–	2	8	7
C	5	2	–	6	5
D	9	8	6	–	4
E	10	7	5	4	–

Route table

	A	**B**	**C**	**D**	**E**
A	A	B	B	D	C
B	A	B	C	C	C
C	B	B	C	D	E
D	A	C	C	D	E
E	C	C	C	D	E

(c) Draw the complete network of shortest distances.

(2)

(d) (i) Use the nearest neighbour algorithm, starting at vertex E, to produce a Hamiltonian cycle for the complete network.

(ii) Write down the length of the Hamiltonian cycle.

(2)

(e) Interpret the Hamiltonian cycle from (d) in terms of the towns actually visited.

(1)

(f) Starting by deleting A from the complete network of shortest distances, and all of its edges, find a lower bound for the solution to the Travelling Salesman problem.

(2)

(g) Use your results to write down the smallest interval that you can be confident contains the optimal solution to the Travelling Salesman problem.

(2)

(Total for Question 6 is 15 marks)

6. (a)

Initial Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Initial Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

(b) 1st Iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

2nd Iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Spare tables

Distance table

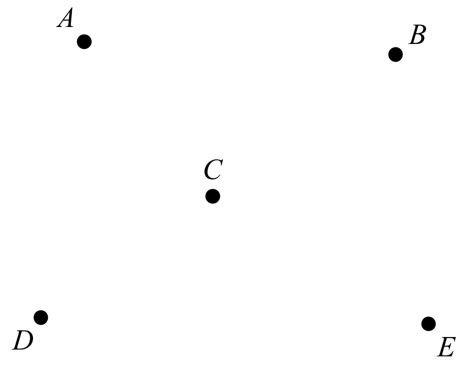
	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Question 6 continued

(c)



<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>
Distance table

	A	B	C	D	E
A	-	3	7	9	∞
B	3	-	2	∞	8
C	7	2	-	6	5
D	9	∞	6	-	4
E	∞	8	5	4	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

</td><td>B1
B1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
1st iteration:

Distance table

	A	B	C	D	E
A	-	3	7	9	∞
B	3	-	2	12	8
C	7	2	-	6	5
D	9	12	6	-	4
E	∞	8	5	4	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	A	C	A	E
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

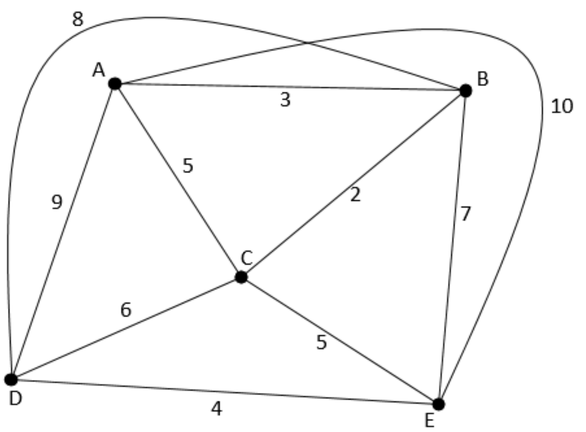
</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>
2nd iteration:

Distance table

	A	B	C	D	E
A	-	3	5	9	11
B	3	-	2	12	8
C	5	2	-	6	5
D	9	12	6	-	4
E	11	8	5	4	-

Route table

	A	B	C	D	E
A	A	B	B	D	B
B	A	B	C	A	E
C	B	B	C	D	E
D	A	A	C	D	E
E	B	B	C	D	E

</td><td>M1
A1ft</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>


</td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(d)(i)
(ii)</td><td>NNA: E – D – C – B – A – E
4 + 6 + 2 + 3 + 10 = 25 km</td><td>B1
B1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
</table>

(e)	E – D – C – B – A – B – C – E	B1	3.2a
		(1)	
(f)	Prim's algorithm on reduced network starting at B: BC, CE, DE Lower bound = 11 + 3 + 5 = 19 km	B1 B1ft	1.1b 2.2a
		(2)	
(g)	 $19 \le$ optimal ≤ 25	M1 A1	2.2b 1.1b
		(2)	
			(15 marks)

Notes:

(a)
B1: Correct distance table
B1: Correct route table
(b)
M1: No change in the first row and first column of both tables with at least one value in the distance table reduced and one value in the route table changed
A1: cao
M1: No change in the second row and second column of both tables with at least two values in the distance table reduced and two values in the route table changed
A1ft: Correct second iteration follow through from the candidate's first iteration
(c)
M1: K_5 drawn with at least one shortest distance from the final distance table present
A1: cao
(d)(i)
B1: cao
(ii)
B1: cao
(e)
B1: cao
(f)
B1: correct RMST starting at any node (except A)
B1ft: length of their RMST + 3 + 5
(g)
M1: Their numbers correctly used, accept any inequalities or any indication of interval from their 19 to their 25 (so 19 – 25 can score this mark). Please note that UB > LB for this mark
A1: cao (no follow through on their values) including correct inequalities or equivalent set notation (but condone $19 < \text{optimal} \le 25$)

3.

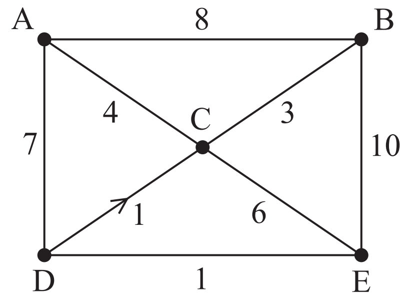


Figure 2

Direct roads between five villages, A, B, C, D and E, are shown in Figure 2. The weight on each arc is the time, in minutes, it takes to travel along the corresponding road. The road from D to C is one-way as indicated by the arrow on the corresponding arc.

Floyd's algorithm is to be used to find the complete network of shortest times between the five villages.

(a) Set up initial time and route matrices.

(2)

The matrices after two iterations of Floyd's algorithm are shown below.

Time matrix

	A	**B**	**C**	**D**	**E**
A	–	8	4	7	18
B	8	–	3	15	10
C	4	3	–	11	6
D	7	15	1	–	1
E	18	10	6	1	–

Route matrix

	A	**B**	**C**	**D**	**E**
A	A	B	C	D	B
B	A	B	C	A	E
C	A	B	C	A	E
D	A	A	C	D	E
E	B	B	C	D	E

(b) Perform the next two iterations of Floyd's algorithm that follow from the tables above. You should show the time and route matrices after each iteration.

(4)

The final time matrix after completion of Floyd's algorithm is shown below.

Final time matrix

	A	**B**	**C**	**D**	**E**
A	–	7	4	7	8
B	7	–	3	10	9
C	4	3	–	7	6
D	5	4	1	–	1
E	6	5	2	1	–

(c) (i) Use the nearest neighbour algorithm, starting at A, to find a Hamiltonian cycle in the complete network of shortest times.

(ii) Find the time taken for this cycle.

(iii) Interpret the cycle in terms of the actual villages visited.

(3)

(Total for Question 3 is 9 marks)

3. (a)

Initial time matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

Initial route matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

(b)

Time matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

Route matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

Time matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

Route matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

If you make an error there are spare copies of these matrices on Page 9.

Question 3 continued

Only use these matrices if you have made an error in your working.

(a)

Spare copy of initial time matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

Spare copy of initial route matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

(b)

Spare copy of time matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

Spare copy of route matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

Spare copy of time matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

Spare copy of route matrix

	A	B	C	D	E
A					
B					
C					
D					
E					

(Total for Question 3 is 9 marks)

Question	Scheme	Marks	AOs
3(a)	See time matrix and route matrix below	B1 B1	1.1b 1.1b
		(2)	

Time matrix

	A	B	C	D	E
A	-	8	4	7	∞
B	8	-	3	∞	10
C	4	3	-	∞	6
D	7	∞	1	-	1
E	∞	10	6	1	-

Route matrix

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

(b)

Time matrix

	A	B	C	D	E
A	-	7	4	7	10
B	7	-	3	14	9
C	4	3	-	11	6
D	5	4	1	-	1
E	10	9	6	1	-

Route matrix

	A	B	C	D	E
A	A	C	C	D	C
B	C	B	C	C	C
C	A	B	C	A	E
D	C	C	C	D	E
E	C	C	C	D	E

Time matrix

	A	B	C	D	E
A	-	7	4	7	8
B	7	-	3	14	9
C	4	3	-	11	6
D	5	4	1	-	1
E	6	5	2	1	-

Route matrix

	A	B	C	D	E
A	A	C	C	D	D
B	C	B	C	C	C
C	A	B	C	A	E
D	C	C	C	D	E
E	D	D	D	D	E

Question	Scheme	Marks	AOs
(b)		M1 A1 M1 A1	1.1b 1.1b 1.1b 1.1b
		(4)	
(c)(i)	NNA: A – C – B – E – D – A	B1	1.1b
(ii)	4 + 3 + 9 + 1 + 5 = 22 minutes	dB1	1.1b
(iii)	A – C – B – C – E – D – C – A	B1	3.2a
		(3)	
	(9 marks)		

Notes for Question 3

(a)
B1: Correct time matrix
B1: Correct route matrix

(b)

M1: No change in the third row and third column of both matrices with at least one value in the time matrix reduced correctly and one value in the route matrix changed to C

A1: CAO

M1: No change in the fourth row and fourth column of both matrices with at least one value in the time matrix reduced correctly (follow through their first iteration) and one value in the route matrix changed to D

A1: CAO

(c)(i)

B1: CAO

(c)(ii)

dB1: CAO – not from A – C – D – E – B – A

(c)(iii)

B1: CAO

3.

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

Table 1

Table 1 shows the shortest distances, in miles, between eight towns, A, B, C, D, E, F, G and H.

(a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this table of distances. You must clearly state the order in which you select the edges of your tree.

(3)

(b) State the weight of the minimum spanning tree.

(1)

	A	B	C	D	E	F	G	H
J	31	27	50	29	43	25	49	35

Table 2

Table 2 shows the distances, in miles, between town J and towns A, B, C, D, E, F, G and H.

Pranav needs to visit all of the towns, starting and finishing at J, and wishes to minimise the total distance he travels.

(c) Starting at J, use the nearest neighbour algorithm to obtain an upper bound for the length of Pranav's route. You must state your route and its length.

(2)

(d) Starting by deleting J, and all of its edges, find a lower bound for the length of Pranav's route.

(2)

(Total for Question 3 is 8 marks)

3.

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

Question 3 continued

	A	B	C	D	E	F	G	H
A	–	24	42	48	34	37	32	22
B	24	–	40	35	30	41	39	44
C	42	40	–	21	26	45	38	36
D	48	35	21	–	32	37	29	27
E	34	30	26	32	–	34	40	28
F	37	41	45	37	34	–	43	41
G	32	39	38	29	40	43	–	38
H	22	44	36	27	28	41	38	–

(Total for Question 3 is 8 marks)

Question	Scheme	Marks	AOs
3(a)	Prim's starting at A: AH, AB, DH; CD, CE; DG, EF	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(b)	Weight of MST is 183 (miles)	B1	2.2a
		(1)	
(c)	NNA: J – F – E – C – D – H – A – B – G – J	B1	1.1b
	Upper bound is 267 (miles)	B1	2.2a
		(2)	
(d)	183 + 27 + 25 =…	M1	3.1b
	… = 235 (miles)	A1	2.2a
		(2)	
			(8 marks)

Notes:

(a)

M1: First three arcs correctly chosen in order {AH, AB, DH, …} or first four nodes correctly chosen in order{A, H, B, D,…}. If any rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of the matrix {1, 3, -, 4, -, -, -, 2} so please check the top of the matrix carefully

A1: First five arcs correctly chosen in order {AH, AB, DH, CD, CE, …} or all eight nodes correctly chosen in order{A, H, B, D, C, E, G, F}. Order of nodes may be seen at the top of the matrix so for the first two marks accept {1, 3, 5, 4, 6, 8, 7, 2} (do not condone any missing numbers e.g. the number 8 must be above F)

A1: cso – all arcs correct stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

(b)

B1: cao (183)

(c)

B1: cao (for route – must return to J)

B1: cao (for upper bound of 267)

(d)

M1: Their answer to **(b)** + 27 + 25 (the two smallest arcs incident to J)

A1: cao (235)

3. The initial distance matrix (Table 1) shows the lengths, in metres, of the corridors connecting six classrooms, A, B, C, D, E and F, in a school. For safety reasons, some of the corridors are one-way only.

	A	**B**	**C**	**D**	**E**	**F**
A	–	12	32	24	29	11
B	12	–	17	8	∞	∞
C	32	17	–	4	12	∞
D	24	∞	4	–	∞	13
E	∞	∞	12	18	–	12
F	11	∞	∞	13	12	–

Table 1

(a) By adding the arcs from vertex A, along with their weights, complete the drawing of this network on Diagram 1 in the answer book.

(2)

Floyd's algorithm is to be used to find the complete network of shortest distances between the six classrooms.

The distance matrix after **two** iterations of Floyd's algorithm is shown in Table 2.

	A	**B**	**C**	**D**	**E**	**F**
A	–	12	29	20	29	11
B	12	–	17	8	41	23
C	29	17	–	4	12	40
D	24	36	4	–	53	13
E	∞	∞	12	18	–	12
F	11	23	40	13	12	–

Table 2

(b) Perform the next two iterations of Floyd's algorithm that follow from Table 2. You should show the distance matrix after each iteration.

(4)

The final distance matrix after completion of Floyd's algorithm is shown in Table 3.

	A	B	C	D	E	F
A	–	12	24	20	23	11
B	12	–	12	8	24	21
C	28	17	–	4	12	17
D	24	21	4	–	16	13
E	23	29	12	16	–	12
F	11	23	17	13	12	–

Table 3

Yinka must visit each classroom. He will start and finish at E and wishes to minimise the total distance travelled.

(c) (i) Use the nearest neighbour algorithm, starting at E, to find two Hamiltonian cycles in the completed network of shortest distances.

(ii) Find the length of each of the two cycles.

(iii) State, with a reason, which of the two cycles provides the better upper bound for the length of Yinka's route.

(4)

(Total for Question 3 is 10 marks)

3.

(a)

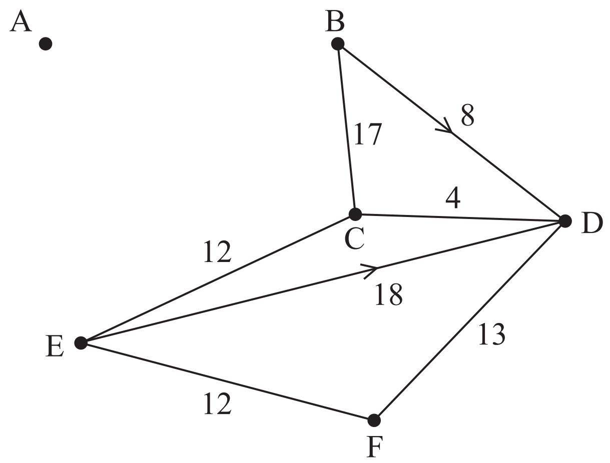


Diagram 1

(b)

There are spare copies of these tables, if required, on Page 9.

3rd iteration

	A	B	C	D	E	F
A	–					
B		–				
C			–			
D				–		
E					–	
F						–

4th iteration

	A	B	C	D	E	F
A	–					
B		–				
C			–			
D				–		
E					–	
F						–

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 3 continued

(b)

Spare copy for 3rd iteration

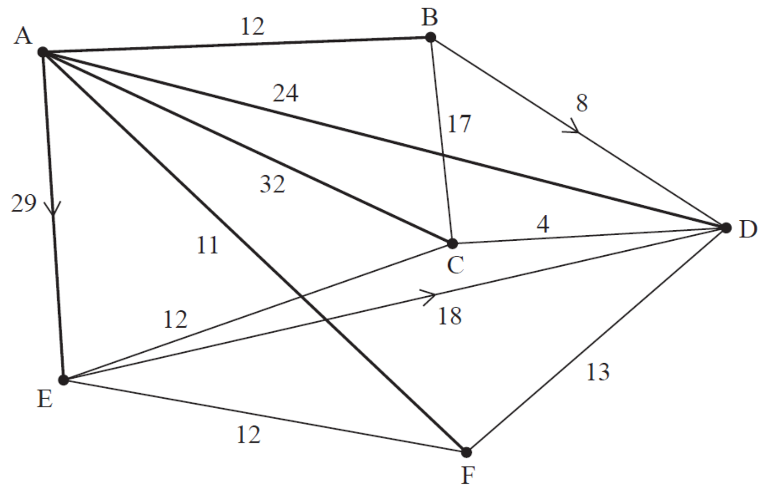
	A	B	C	D	E	F
A	–					
B		–				
C			–			
D				–		
E					–	
F						–

Spare copy for 4th iteration

	A	B	C	D	E	F
A	–					
B		–				
C			–			
D				–		
E					–	
F						–

(Total for Question 3 is 10 marks)

Question	Scheme	Marks	AOs
3(a)	(diagram below)	M1 A1	1.1b 1.1b
		(2)	



(b)

Third iteration:

	A	B	C	D	E	F
A	-	12	29	20	29	11
B	12	-	17	8	**29**	23
C	29	17	-	4	12	40
D	24	**21**	4	-	**16**	13
E	**41**	**29**	12	**16**	-	12
F	11	23	40	13	12	-

M1 1.1b

A1 1.1b

Fourth iteration:

	A	B	C	D	E	F
A	-	12	**24**	20	29	11
B	12	-	**12**	8	**24**	**21**
C	**28**	17	-	4	12	**17**
D	24	21	4	-	16	13
E	**40**	29	12	16	-	12
F	11	23	**17**	13	12	-

M1 1.1b

A1 1.1b

(4)

Question	Scheme	Marks	AOs
(c)	Nearest neighbour routes:	M1	3.4
	E – C – D – F – A – B – E Length = 76	A1	1.1b
	E – F – A – B – D – C – E Length = 59	A1	1.1b
	The cycle that begins E – F –… is the better upper bound as the value is the smaller of the two	A1	2.4
		(4)	
			(10 marks)

Notes for Question 3
a1M1: Correct five arcs from A **a1A1:** cao no additional arcs or arrows, correct weights and arc AE directed from A to E In the distance matrix for part (b) ignore whatever is written in the lead diagonal (top left to bottom right) and just consider the values (ignore shading etc.) – check bottom of page 9 for replaced matrices **b1M1:** No change in the third row and third column with at least two values reduced correctly (no blank entries – apart from the lead diagonal) **b1A1:** cao for the third iteration **b2M1:** No change in their fourth row and fourth column (following their third iteration) with at least two values correctly reduced follow through from their previous iteration (no blank entries – apart from the lead diagonal) **b2A1:** cso for both iterations (no follow through from an incorrect third iteration even if the fourth iteration is 'correct' so M1A0M1A1 is not possible in this part) **c1M1:** Either cycle correct (must include returning to E) – nodes must be in the correct order and therefore not reversed but allow if stated in terms of arcs. If only one cycle correct then M1 only **c1A1:** Both cycles correct – nodes must be in the correct order but allow if stated in terms of arcs **c2A1:** Both cycles and corresponding lengths correct **c3A1:** Correct reasoning that the smallest (oe) value is the better upper bound – dependent on all previous marks in this part (cso)