Fd1Ch4 XMQs and MS

(Total: 100 marks)

1. FD1_Specimen Q2 . 6 marks - FD1ch4 Route inspection
2. FD1_Specimen Q3 . 13 marks - FD1ch3 Algorithms on graphs
3. FD1_2019 Q2 . 14 marks - FD1ch3 Algorithms on graphs
4. FD1_2020 Q6 . 11 marks - FD1ch4 Route inspection
5. FD1_2021 Q4 . 8 marks - FD1ch3 Algorithms on graphs
6. FD1_2022 Q2 . 13 marks - FD1ch3 Algorithms on graphs
7. FD1(AS)_2019 Q4 . 10 marks - FD1ch3 Algorithms on graphs
8. FD1(AS)_2020 Q3 . 11 marks - FD1ch3 Algorithms on graphs
9. FD1(AS)_2022 Q3 . 14 marks - FD1ch3 Algorithms on graphs

2.

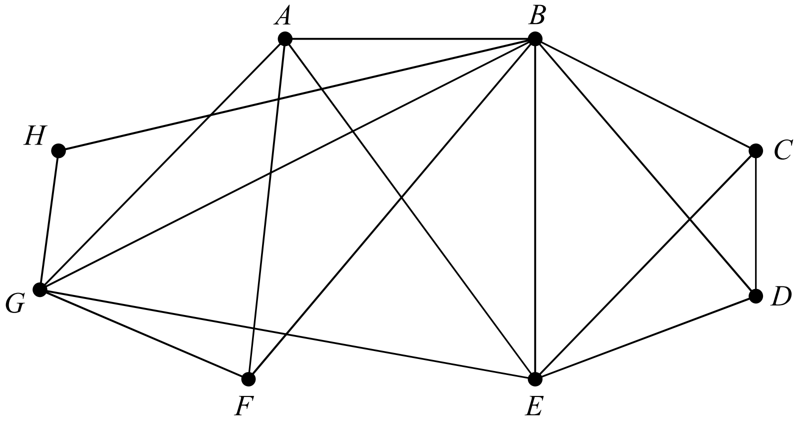


Figure 1

(a) Write down the number of arcs in a minimum spanning tree for the graph shown in Figure 1.

(1)

(b) Determine the minimum number of arcs that would need to be added to make the graph semi-Eulerian. You must justify your answer.

(2)

Taking AFGHBCDEA as the Hamiltonian cycle,

(c) use the planarity algorithm to determine whether or not the graph shown in Figure 1 is planar. You must make your working clear and justify your answer.

(3)

(Total for Question 2 is 6 marks)

2.

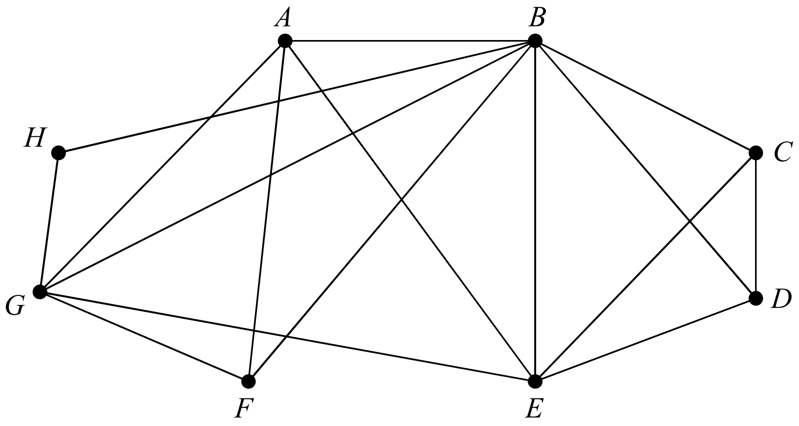


Figure 1

Question 2 continued

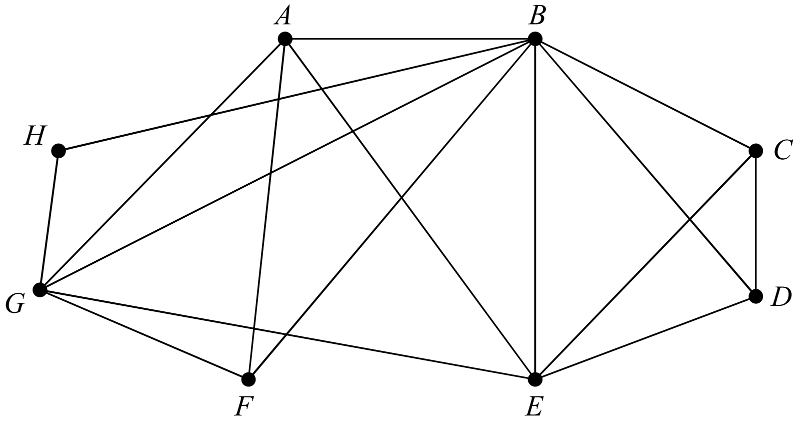


Figure 1

(Total for Question 2 is 6 marks)

Question	Scheme	Marks	AOs
2(a)	7	B1	2.2a
		(1)	
(b)	A semi-Eulerian graph requires exactly two odd nodes… …the graph has six odd nodes so only two arcs needs to be added to make the graph semi-Eulerian	B1 B1	1.2 2.2a
		(2)	
(c)	Creates two lists of arcs e.g. AB BF BE CE EF EG BG BD Since no arc appears in both lists, the graph is planar (or draws a planar version)	M1 A1 A1	2.1 1.1b 2.4
		(3)	
			(6 marks)

Notes:

(a)
B1: cao
(b)
B1: accurately recalls the fact that a semi-Eulerian graph contains exactly two odd nodes
B1: dependent on previous B mark – cao
(c)
M1: creates two list of arcs (with at least three arcs in each list) which contain no common arcs
A1: cao
A1: correct reasoning that no arc appears in both lists + so the graph is therefore planar

3.

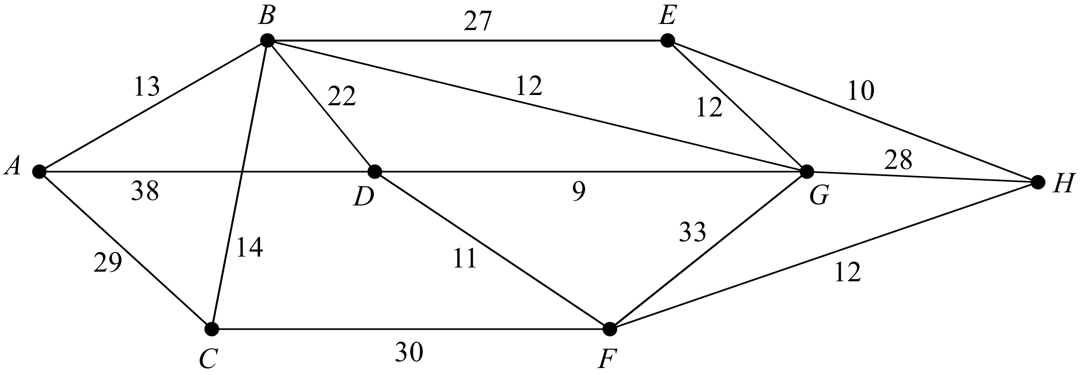


Figure 2

[*The total weight of the network is 300*]

Figure 2 represents a network of train tracks. The vertices A, B, C, D, E, F, G and H represent train stations.

(i) The number on each edge represents the time, in minutes, to travel on the corresponding track.

(a) Use Dijkstra's algorithm to find the time required to travel the quickest route from A to H.

(4)

A train is to travel from A to F via H without stopping.

(b) Find the quickest route from A to F via H. State the route and the time taken.

(2)

A train is to travel from A to H, stopping for 1 minute at each train station it passes through on its route.

(c) Explain how you would adapt the network so that Dijkstra's algorithm could be used to find the quickest route for this train. You do not need to find this route.

(2)

(ii) The number on each edge **now** represent the length, in km, of the corresponding track. A route that starts at C, finishes at G and traverses each track at least once needs to be found.

(a) Use an appropriate algorithm to find the length of the shortest route. You must make your method clear.

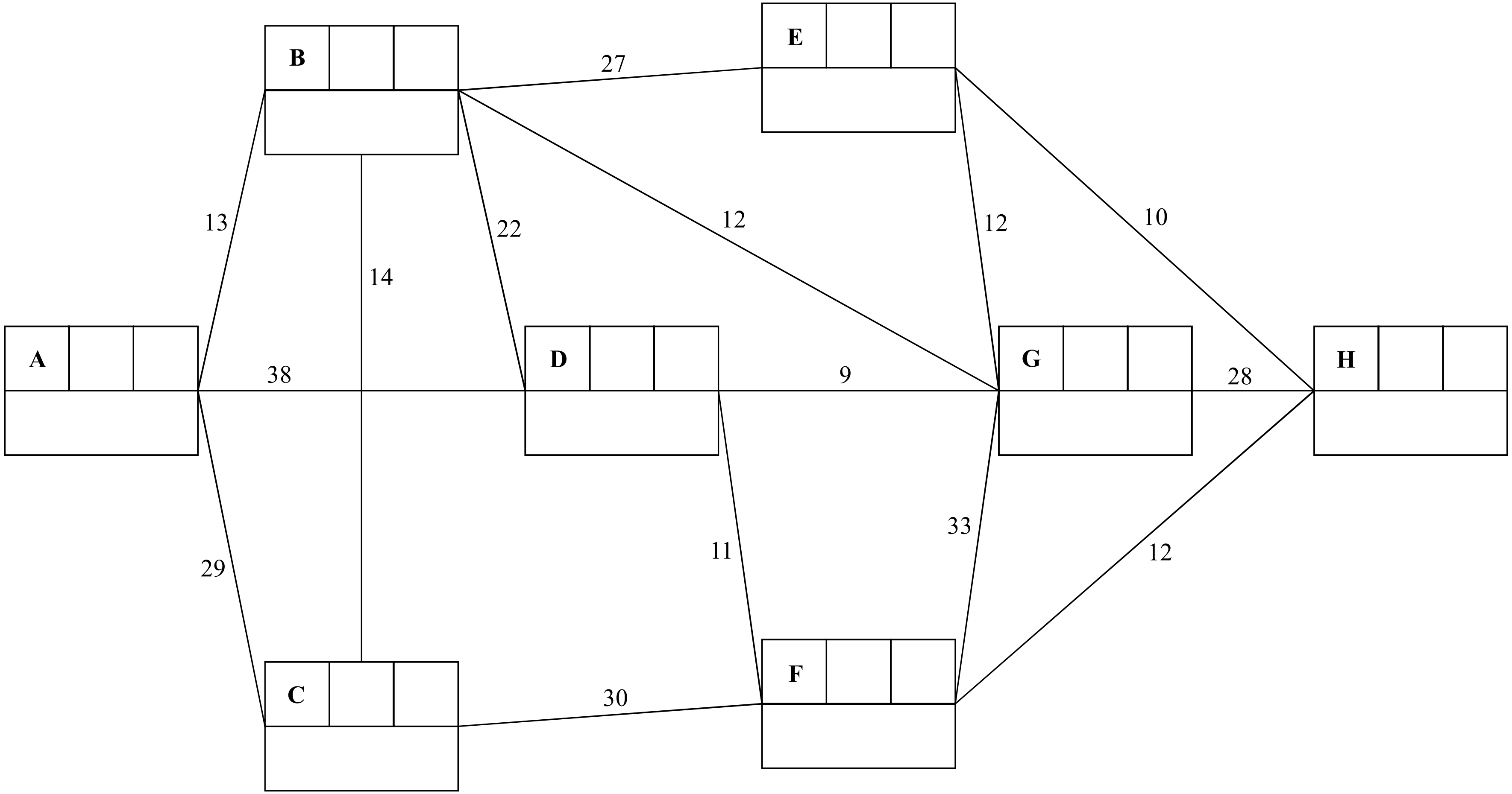
(4)

(b) State the tracks that will need to be traversed twice.

(1)

(Total for Question 3 is 13 marks)

3.

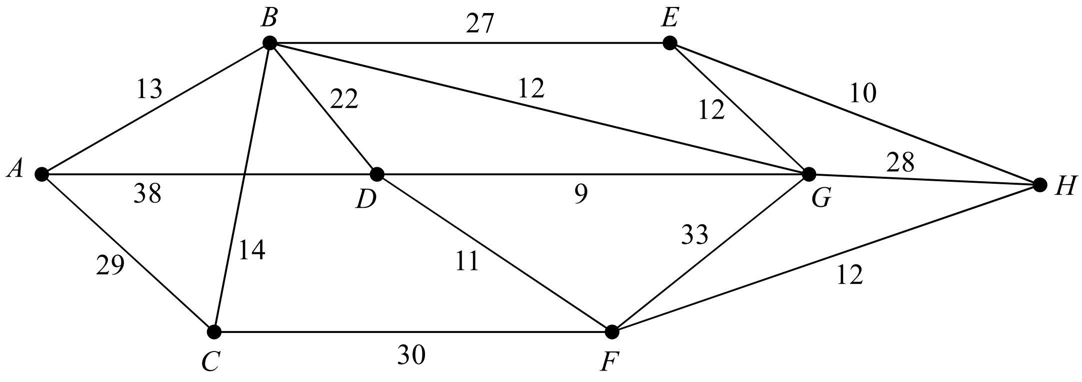


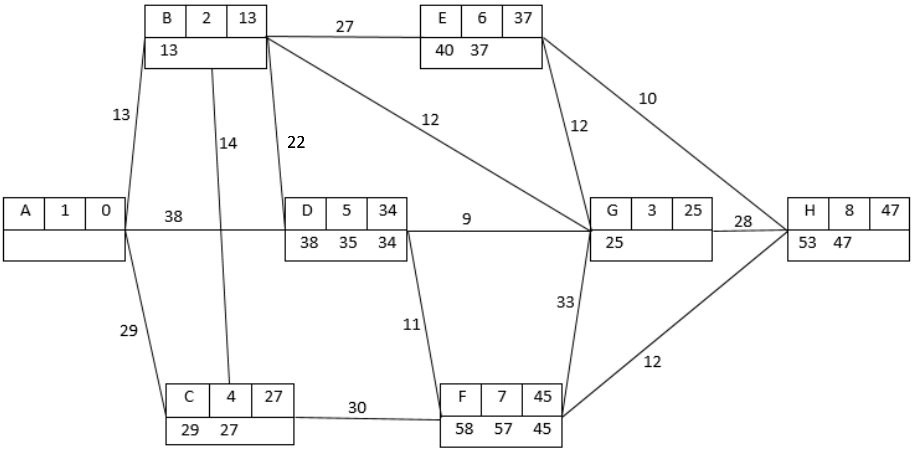
Key:

Vertex	Order of labelling	Final value
Working values		

Time required to travel quickest route from A to H: ______________________

Question 3 continued



Question	Scheme	Marks	AOs
3(i)(a)	 Length of quickest route from A to H is 47 minutes	M1 A1 A1 A1ft	1.1b 1.1b 1.1b 2.2a
		(4)	
(b)	Shortest path from A to F via H: ABGEHF Length: 47 + 12 = 59 minutes	B1 B1ft	1.1b 2.2a
		(2)	
(c)	e.g. add 1 to each arc	M1	3.5c
	except AB, AD, AC (or EH, GH, FH)	A1	2.3
		(2)	
(ii)(a)	AB + EH = 13 + 10 = 23* A(BG)E + B(GE)H = 37 + 34 = 71 A(BGE)H + B(G)E = 47 + 24 = 71 Length of the shortest route is 300 + 23 = 323 km	M1 A1ft A1 A1ft	2.1 1.1b 1.1b 2.2a
		(4)	
(b)	Repeat arcs: AB, EH	B1	2.2a
		(1)	
			(13 marks)

Notes:

(i) (a)

M1: for a larger number replaced by a smaller one in the working values boxes at C, D, E, F or H

A1: for all values correct (and in correct order) at A, B, G and C

A1: for all values correct (and in correct order) at D, E, F and H

A1ft: for 47 or ft their final value at H

(b)
B1: cao
B1ft: for 59 or ft their final at H + 12
(c)
M1: valid general method – any mention of adding 1 to the weight of the arcs
A1: cao – so adding 1 to each arc except {AB, AD, AC} or {EH, GH, FH}

(ii)(a)
M1: correct three pairings of the required four odd nodes
A1ft: at least two pairings and totals correct (ft their values from (a))
A1: all three pairings and totals correct
A1ft: for 323 or 300 + their shortest repeat
(b)
B1: selecting the shortest pairing, and stating that these arcs should be repeated

2.

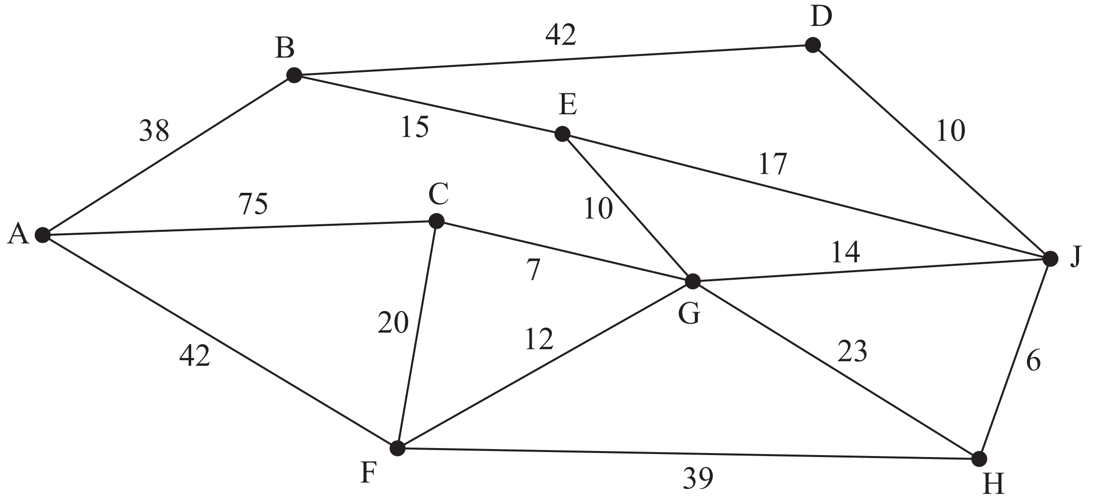


Figure 1

[*The total weight of the network is* 370]

Figure 1 represents a network of corridors in a building. The number on each arc represents the length, in metres, of the corresponding corridor.

(a) Use Dijkstra's algorithm to find the shortest path from A to D, stating the path and its length.

(6)

On a particular day, Naasir needs to check the paintwork along each corridor. Naasir must find a route of minimum length. It must traverse each corridor at least once, starting at B and finishing at G.

(b) Use an appropriate algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.

(4)

(c) Find the length of Naasir's route.

(1)

On a different day, all the corridors that start or finish at B are closed for redecorating. Naasir needs to check all the remaining corridors and may now start at any vertex and finish at any vertex. A route is required that excludes all those corridors that start or finish at B.

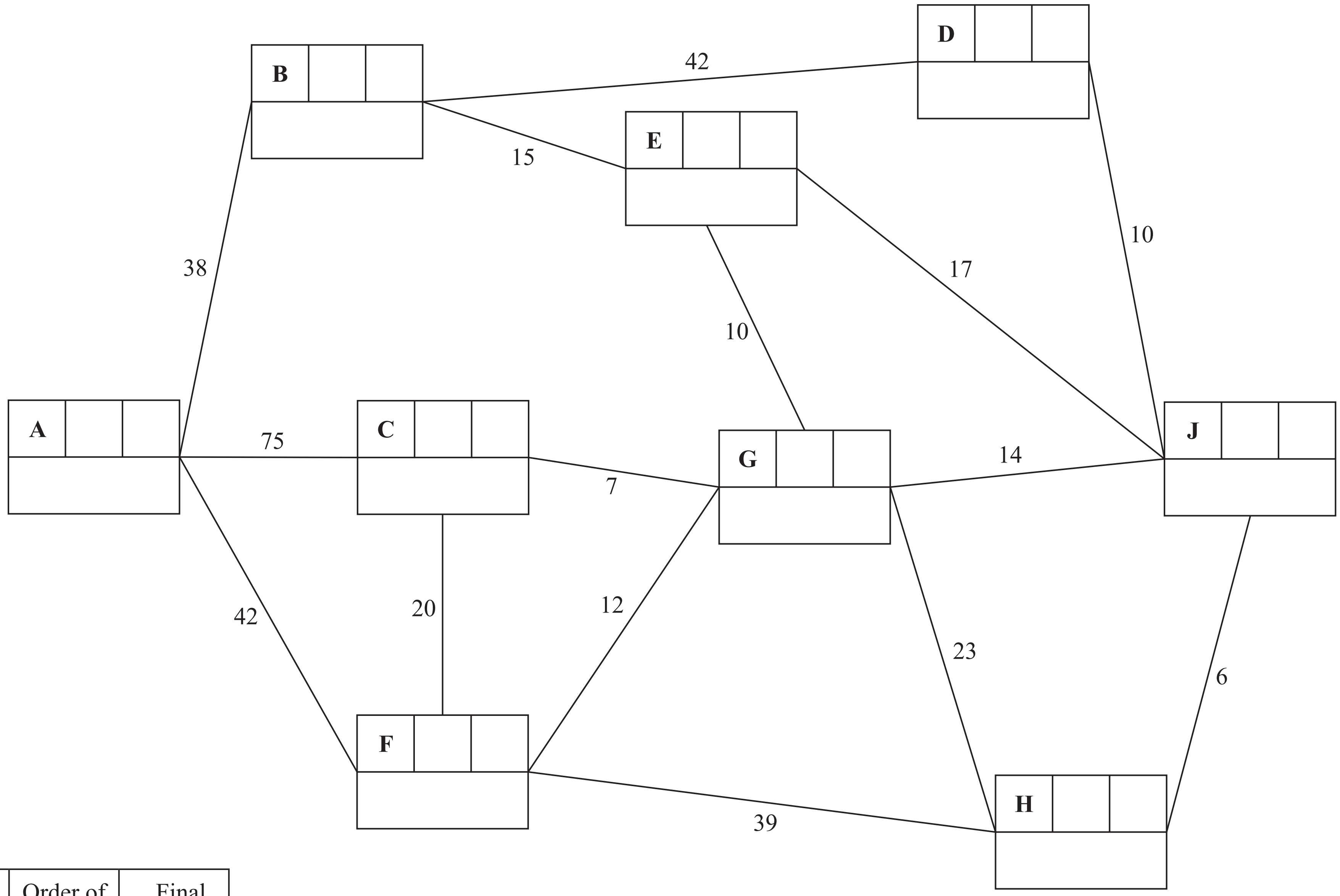
(d) (i) Determine the possible starting and finishing points so that the length of Naasir's route is minimised. You must give reasons for your answer.

(ii) Find the length of Naasir's new route.

(3)

(Total for Question 2 is 14 marks)

2.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to D: ________________

Length of shortest path from A to D: ________________

Question 2 continued

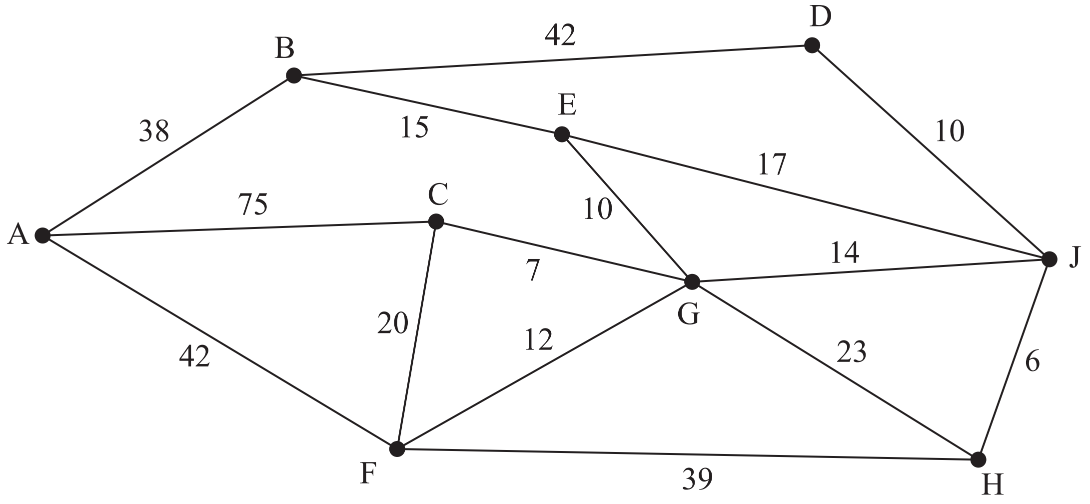
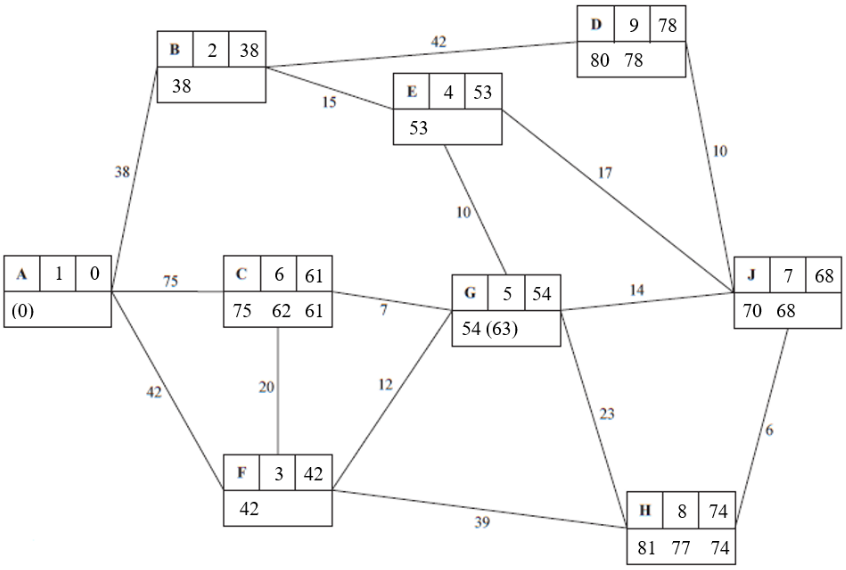


Figure 1

Qu	Scheme	Marks	AOs
2(a)		M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b
	Path from A to D is AFGJD	A1	2.2a
	Length of path from A to D is 78 metres	A1ft	2.2a
		(6)	
(b)		M1	3.1b
	A(FG)C + E(J)H = 61 + 23 = 84 A(B)E + C(GJ)H = 53 + 27 = 80* A(FGJ)H + C(G)E = 74 + 17 = 91	A1ft A1	1.1b 1.1b
	Repeat arcs: AB, BE, CG, GJ and JH	A1	2.2a
		(4)	
(c)	Length of the route is 370 + 80 = 450 metres	B1ft	2.2a
		(1)	
(d)(i)	If node B is removed this makes D, C, G and H odd	M1	3.1b
	The shortest path between any two odd nodes is CG (so repeat CG) so the route should start at D and finish at H (or vice-versa)	A1	2.2a
(ii)	Length of new route is 370 – 38 – 42 – 15 + 7 = 282 metres	B1	2.2a
		(3)	
			(14 marks)

Notes for Question 2

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 81 77 74 in that order (so 81 74 77 is incorrect)
It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

(a) M1: A larger value replaced by a smaller value in at least two of the working boxes at either C or D or E or G or J

A1: All values in A, B, F and E correct. Condone lack of 0 in A's working value

A1: All values G, C and J correct and the working values in the correct order. Penalise order of labelling only once per question (G, C and J must be labelled in that order and G must be labelled after A, B, F and E). Note that an additional working value of 63 at G after the 54 is not an error so 54 63 is fine, however, any other number or 63 54 in this order is incorrect and scores A0 in this part

A1ft: All values in H and D correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through H check that the working value at H follows from the candidate's final values from their feeds into H (which will come from nodes F, G and/or J (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for D (which will possibly have working values from B and J with the order of these values determined by the candidate's order of labelling at B and J)

A1: CAO - correct path from A to D (AFGJD)

A1ft: ft their final value at D only (if 78 stated and 78 is not the final value at D then A0)

(b) M1: correct three pairings of the correct four odd nodes (A, C, E and H)

A1ft: any row correct including pairing **and** total (ft the final values from **(a)** for their shortest paths from A to the three other nodes C, E or H only (so the pairing that does not include A must be correct))

A1: all three rows correct including pairings **and** totals

A1: selecting the shortest pairing, and stating that these arcs (AB, BE, CG, GJ and JH) should be repeated. Must be these arcs and not e.g. ABE, CGJH or AE via B, etc.

(c) B1ft: For 370 + their smallest repeat out of a choice of at least **two** totals seen in **(b)** – this mark is dependent on M1 in (b)

(d) M1: Mention of the fact that these four nodes D, C, G and H only are now odd **or** clear consideration of these four nodes only

A1: CAO D and H **and** must have clearly indicated that the shortest path is from C to G (or vice-versa) – but A0 if clearly selecting the shortest pairing first before selecting the shortest path (as the shortest path is embedded in the shortest pairing)

B1: CAO (282)

6.

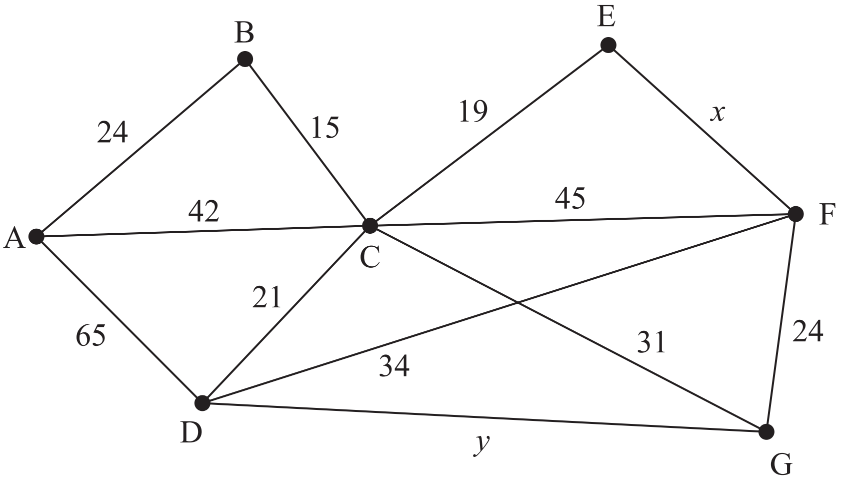


Figure 4

[*The total weight of the network is 320 + x + y*]

(a) State, with justification, whether the graph in Figure 4 is Eulerian, semi-Eulerian or neither.

(2)

The weights on the arcs in Figure 4 represent distances. The weight on arc EF is x where $12 < x < 26$ and the weight on arc DG is y where $0 < y < 10$

An inspection route of minimum length that traverses each arc at least once is found.
The inspection route starts and finishes at A and has a length of 409

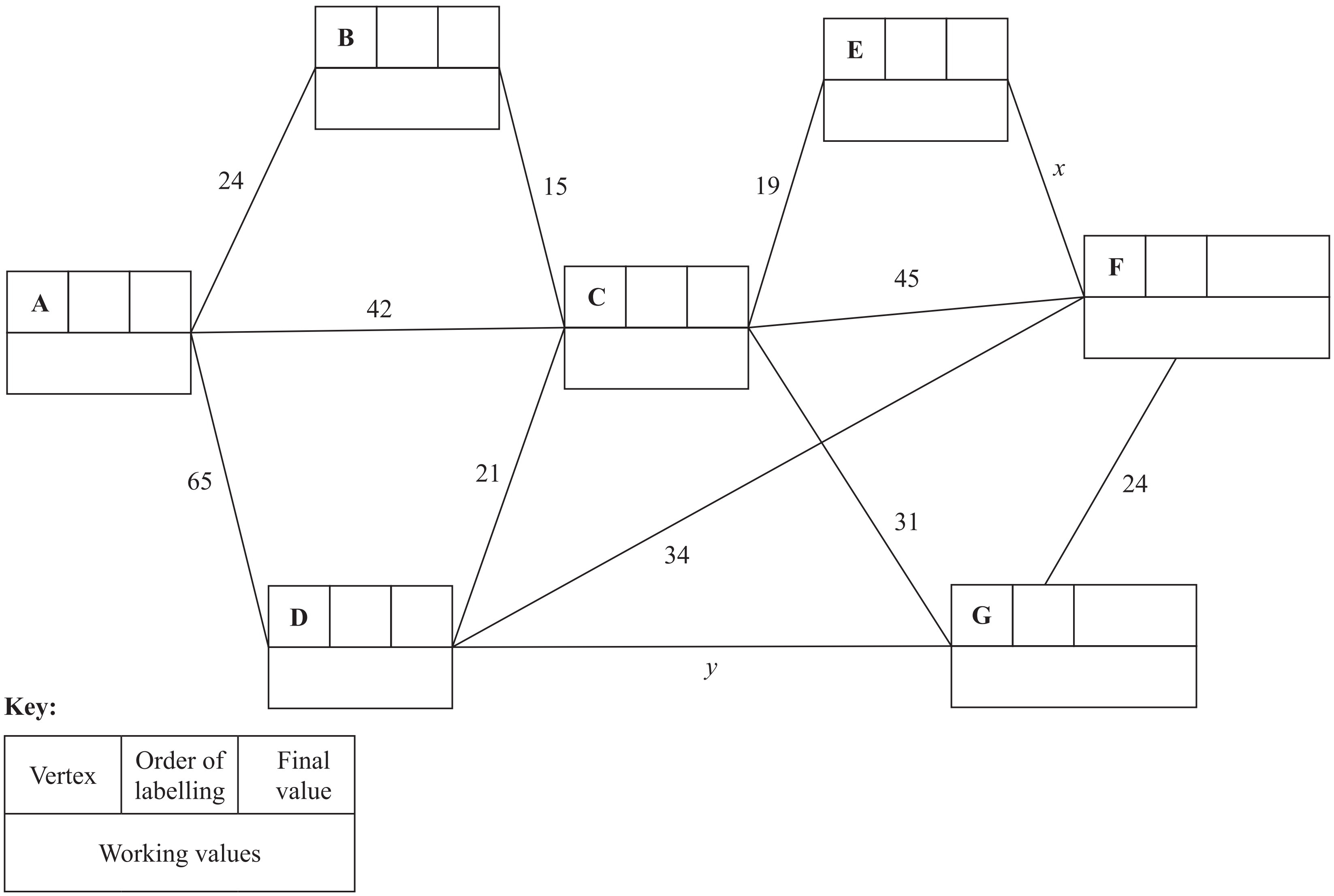
It is also given that the length of the shortest route from F to G via A is 140

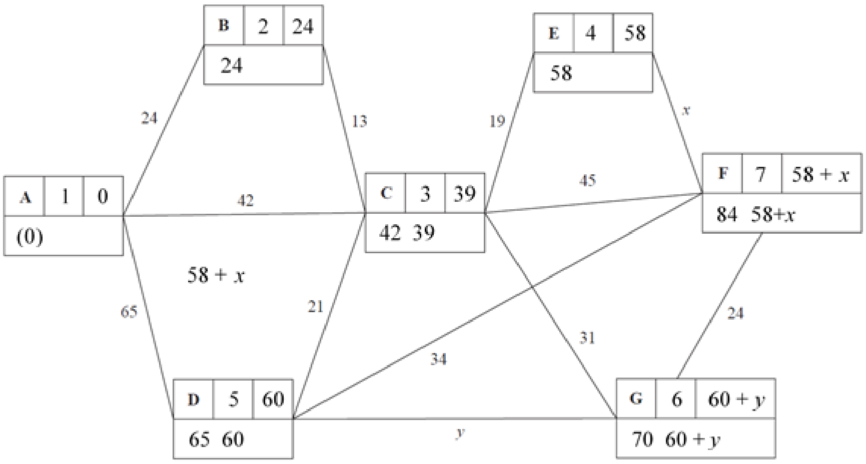
(b) Using appropriate algorithms, find the value of x and the value of y.

(9)

(Total for Question 6 is 11 marks)

6. (a)


A
B
C
D
E
F
G
24
15
42
65
21
19
45
x
34
31
24
y
Key:
Vertex
Order of labelling
Final value
Working values

Question	Scheme	Marks	AOs
6(a)	The graph has exactly two odd nodes and so the graph is semi-Eulerian	B1 dB1	2.4 2.2a
		(2)	
(b)		M1 A1 A1 A1	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to F is 58 + x and shortest path from A to G is 60 + y	A1ft	2.2a
	$58 + x + 60 + y = 140$	M1	2.1
	The only odd nodes in the network are A and G	B1	2.2a
	Route inspection algorithm: Shortest route between A and G is 60 + y $\Rightarrow$ $320 + x + y + 60 + y = 409$	M1	3.1b
	$x = 15$ and $y = 7$	A1	2.2a
		(9)	
			(11 marks)

Notes for Question 6

(a)

B1: Explanation which consists of the graph having two odd nodes or stating graph is semi-Eulerian

dB1: Exactly two odd nodes (or two odd nodes and five even nodes or the rest even) together with the deduction that therefore the graph is semi-Eulerian

(b)

M1: For a larger number replaced by a smaller one in two working value boxes at C, D, G or F

A1: For all values correct (and in correct order) at A, B and C

A1: For all values correct (and in correct order) at E and D

A1: For all values correct (and in correct order) at G and F

A1ft: Length of shortest path from A to F or A to G stated (may be seen in an equation(s))

M1: (length of shortest path from A to F) + (length of shortest path from A to G) = 140 – linear equation in x and y

B1: Correctly stating the two odd nodes (A and G) – could be implied by subsequent working

M1: For an equation based on the route from A to G (320 + x + y + final value at G (in y) = 409)

A1: CAO for x and y

4.

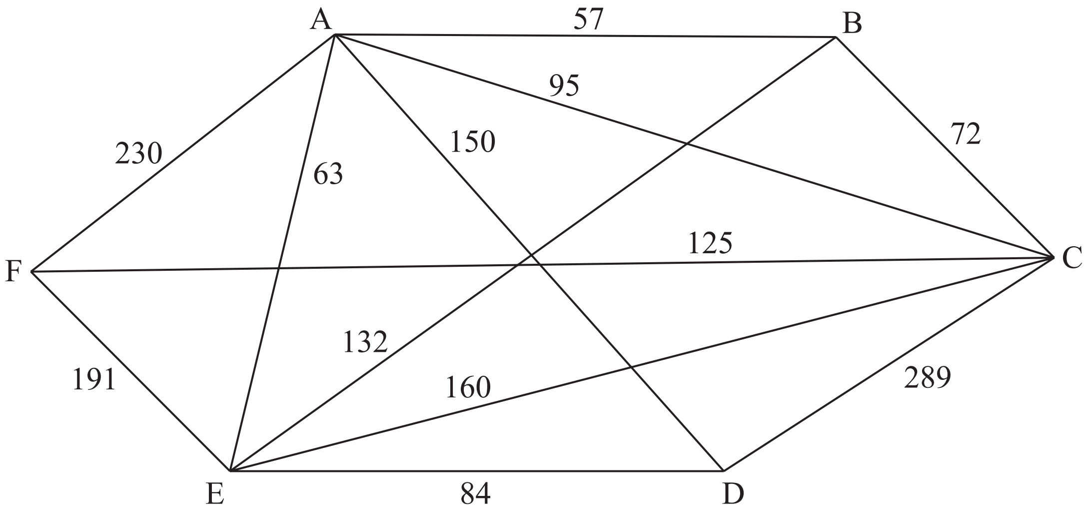


Figure 3

[*The total weight of the network is* 1648]

Direct roads between six cities, A, B, C, D, E and F, are represented in Figure 3. The weight on each arc is the time, in minutes, required to travel along the corresponding road.

Floyd's algorithm is to be used to find the complete network of shortest times between the six cities.

An initial route matrix is given in the answer book.

(a) Set up the initial time matrix.

(1)

(b) Perform the first iteration of Floyd's algorithm. You should show the time and route matrices after this iteration.

(2)

The final time matrix after completion of Floyd's algorithm is shown below.

	A	**B**	**C**	**D**	**E**	**F**
A	–	57	95	147	63	220
B	57	–	72	204	120	197
C	95	72	–	242	158	125
D	147	204	242	–	84	275
E	63	120	158	84	–	191
F	220	197	125	275	191	–

A route is needed that minimises the total time taken to traverse each road at least once.

The route must start at B and finish at E.

(c) Use an appropriate algorithm to find the roads that will need to be traversed twice. You should make your method and working clear.

(4)

(d) Write down the length of the route.

(1)

(Total for Question 4 is 8 marks)

4. (a)

Initial time matrix

	A	**B**	**C**	**D**	**E**	**F**
A						
B						
C						
D						
E						
F						

Initial route matrix

	A	**B**	**C**	**D**	**E**	**F**
A	A	B	C	D	E	F
B	A	B	C	D	E	F
C	A	B	C	D	E	F
D	A	B	C	D	E	F
E	A	B	C	D	E	F
F	A	B	C	D	E	F

(b) 1st iteration

Time matrix

	A	**B**	**C**	**D**	**E**	**F**
A						
B						
C						
D						
E						
F						

Route matrix

	A	**B**	**C**	**D**	**E**	**F**
A						
B						
C						
D						
E						
F						

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>4(a)</td><td>Initial time matrix
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr>
<tr><td>A</td><td>-</td><td>57</td><td>95</td><td>150</td><td>63</td><td>230</td></tr>
<tr><td>B</td><td>57</td><td>-</td><td>72</td><td>∞</td><td>132</td><td>∞</td></tr>
<tr><td>C</td><td>95</td><td>72</td><td>-</td><td>289</td><td>160</td><td>125</td></tr>
<tr><td>D</td><td>150</td><td>∞</td><td>289</td><td>-</td><td>84</td><td>∞</td></tr>
<tr><td>E</td><td>63</td><td>132</td><td>160</td><td>84</td><td>-</td><td>191</td></tr>
<tr><td>F</td><td>230</td><td>∞</td><td>125</td><td>∞</td><td>191</td><td>-</td></tr>
</table></td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>1st iteration:
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr>
<tr><td>A</td><td>-</td><td>57</td><td>95</td><td>150</td><td>63</td><td>230</td></tr>
<tr><td>B</td><td>57</td><td>-</td><td>72</td><td>207</td><td>120</td><td>287</td></tr>
<tr><td>C</td><td>95</td><td>72</td><td>-</td><td>245</td><td>158</td><td>125</td></tr>
<tr><td>D</td><td>150</td><td>207</td><td>245</td><td>-</td><td>84</td><td>380</td></tr>
<tr><td>E</td><td>63</td><td>120</td><td>158</td><td>84</td><td>-</td><td>191</td></tr>
<tr><td>F</td><td>230</td><td>287</td><td>125</td><td>380</td><td>191</td><td>-</td></tr>
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<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr>
<tr><td>A</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>C</td><td>A</td><td>A</td><td>A</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>A</td><td>A</td><td>F</td></tr>
<tr><td>D</td><td>A</td><td>A</td><td>A</td><td>D</td><td>E</td><td>A</td></tr>
<tr><td>E</td><td>A</td><td>A</td><td>A</td><td>D</td><td>E</td><td>F</td></tr>
<tr><td>F</td><td>A</td><td>A</td><td>C</td><td>A</td><td>E</td><td>F</td></tr>
</table></td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>Route must start and finish at B, E therefore need to consider pairings of the other four odd nodes (A, C, D and F)</td><td>M1</td><td>3.1b</td></tr>
<tr><td></td><td>AC + DF = 95 + 275 = 370
AD + CF = 147 + 125 = 272*
AF + CD = 220 + 242 = 462</td><td>A1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>Repeat arcs: AE, DE and CF</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(d)</td><td>Length: 1648 + 272 = 1920 (minutes)</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
</table>

Notes:

(a)

B1: Correct distance table (condone dashes, crosses, etc. for infinity but do not condone a ‘large’ number in these cells or these cells left blank)

(b)

M1: No change in the first row and first column of both tables with at least two values in the distance table correctly reduced and two letters in the route table correctly changed – all cells complete

A1: cao

(c)

M1: Either the correct three pairings of the correct four nodes A, C, D and F **or** recognises that as the route begins at B and finishes at E that only the nodes A, C, D and F need to be considered

A1: Two rows correct including pairings **and** totals

A1: All three rows correct including pairings **and** totals

A1: selecting the shortest pairing, and stating that these arcs (AE, DE and CF) should be repeated. Must be these arcs and not e.g., AED or AD via E, etc.

(d)

B1: cao

2.

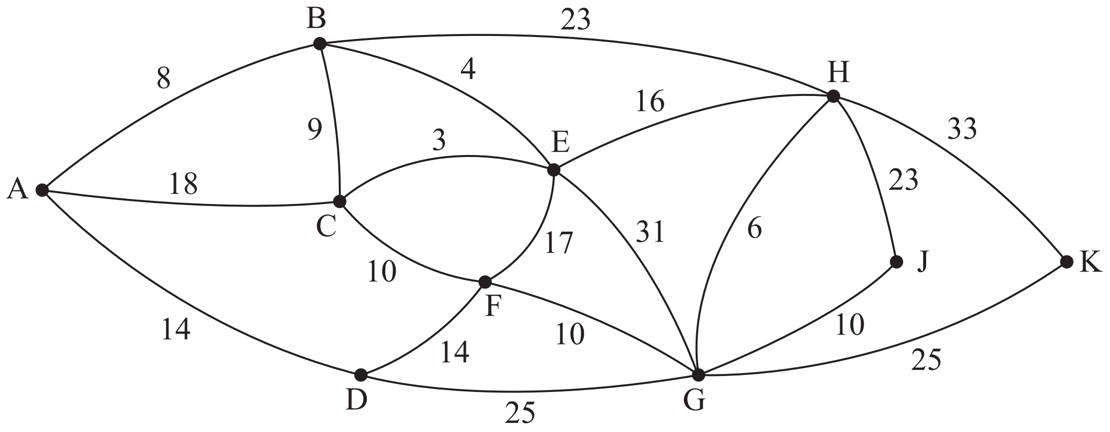


Figure 1

[*The total weight of the network is* 299]

Figure 1 represents a network of cycle tracks between 10 landmarks, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in kilometres, of the corresponding track.

One day, Blanche wishes to cycle from A to K. She wishes to minimise the distance she travels.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to K.

(ii) State the length of the shortest path from A to K.

(6)

The cycle tracks between the landmarks now need to be inspected. Blanche must travel along each track at least once. She wishes to minimise the length of her inspection route. Blanche will start her inspection route at D and finish at E.

(b) (i) State the edges that will need to be traversed twice.

(ii) Find the length of Blanche's route.

(2)

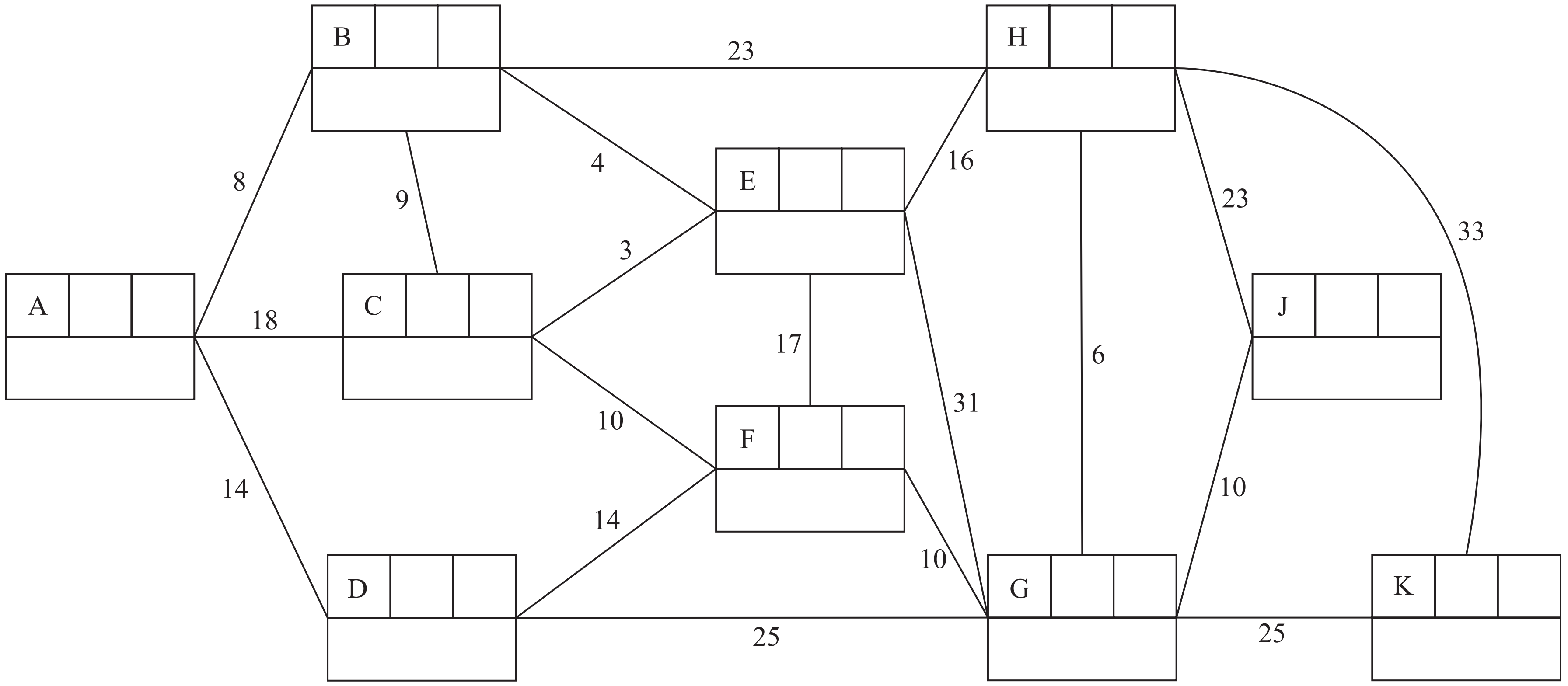
It is now decided to start the inspection route at A and finish at K. Blanche must minimise the length of her route and travel along each track at least once.

(c) By considering the pairings of all relevant nodes, find the length of Blanche's new route. You must make your method and working clear.

(5)

(Total for Question 2 is 13 marks)

2.

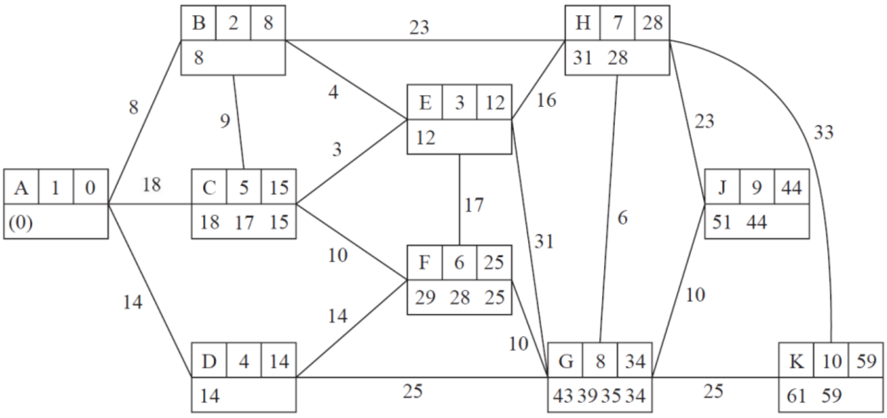


Key:

<table>
<tr><td>Vertex</td><td>Order of labelling</td><td>Final value</td></tr>
<tr><td colspan="3">Working value</td></tr>
</table>

Shortest path from A to K: ______________________________

Length of shortest path from A to K: ______________________________

Question	Scheme	Marks	AOs
2(a)	 **(i)** **(ii)** Path from A to K: ABEHGK **(iii)** Length of shortest path from A to K is 59 (km)	**M1** **A1** **A1** **A1ft** **A1** **A1ft**	1.1b 1.1b 1.1b 1.1b 2.2a 2.2a
		(6)	
(b)(i)	Repeated edges: AB, BE, EH	**B1**	2.2a
(b)(ii)	Length of route = 299 + 28 = 327 (km)	**B1ft**	2.2a
		(2)	
(c)	Route must start at A and finish at K therefore need to consider pairings of the nodes D, E, H and K	**M1**	3.1b
	D(AB)E + H(G)K = 26 + 31 = 57 D(FG)H + E(HG)K = 30 + 47 = 77 D(FG)K + EH = 49 + 16 = 65	**A1** **A1** **A1**	1.1b 1.1b 1.1b
	Length of new route = 299 + 57 = 356 (km)	**A1**	2.2a
		(5)	

(13 marks)

Notes for Question 2

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded, for example at F the working values must be 29 28 25 in that order (so 29 25 28 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a 0 label at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4,… will be penalised once (see notes below) but 1, 2, 3, 5, 6,… is fine. Errors in the final values and working values are penalised before errors in the order of labelling. Condone crossed out working values if they are still legible

a1M1: A larger value replaced by a smaller value in at least **two** different Working Value boxes at either C or F or G or H or J or K

a1A1: All values at A, B, E, D and C correct. Condone lack of 0 in A's working value

a2A1: All values at F, H and G correct and the working values in the correct order. Penalise order of labelling only once per question (F, H and G must be labelled in that order and F must be labelled after A, B, E, D and C)

a3A1ft: All values in J and K correct on the follow through and the working values in the correct order. To follow through J check that the working values at J follow from the candidate's final values for the nodes that are directly attached to J (which are G and H). For example, **if** correct then the order of labelling of nodes G and H are 8 and 7 respectively so the working values at J should come from H and G in that order. The first working value at J should be their 28 (the Final value at H) + 23 (the weight of the arc HJ), the second working value at J should be their 34 (the Final value at G) + 10 (the weight of the arc GJ). Repeat the process for K (which will have working values from G and H with the order of these nodes determined by the candidate's order of labelling at G and H)

a4A1: cao (correct path from A to K – ABEHGK). Do not accept KGHEBA. Condone the length of the shortest path and the path interchanged on the answer lines in the answer book. Allow the paths written in terms of arcs, e.g. AB, BE, EH, HG, GK

a5A1ft: ft their final value at K only (if 59 stated and 59 is not the final value at K then A0)

bi1B1: cao (AB, BE, EH only but in any order) – must be stated as edges so B0 for ABEH

bii1B1ft: 327 or follow through their final value at H from **(a)** + 299

c1M1: The correct three pairings of the correct four nodes (D, E, H and K)

c1A1: One row correct including pairing **and** total

c2A1: Two rows correct including pairings **and** totals

c3A1: All three rows correct including pairings **and** totals

c4A1: cao (356) – dependent on all previous marks in this part

Condone lack of units throughout

4.

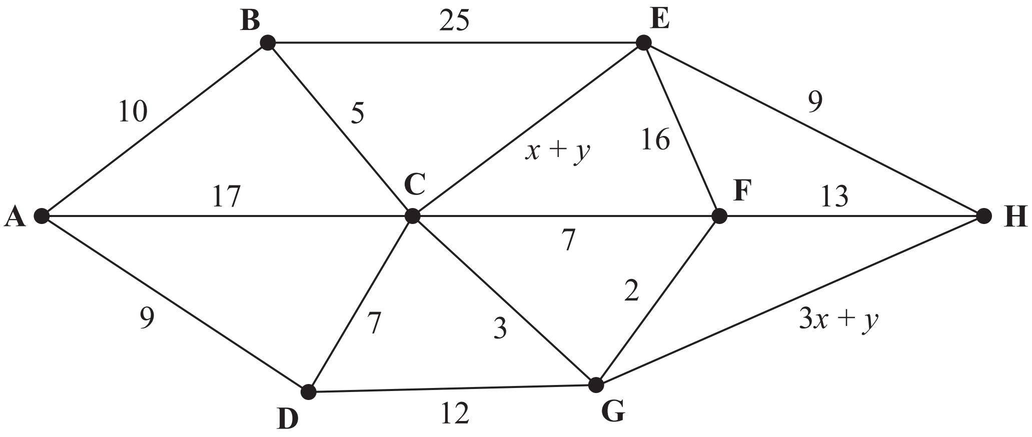


Figure 1

[The total weight of the network is $135 + 4x + 2y$]

The weights on the arcs in Figure 1 represent distances. The weights on the arcs CE and GH are given in terms of x and y, where x and y are positive constants and $7 < x + y < 20$

There are three paths from A to H that have the same minimum length.

(a) Use Dijkstra's algorithm to find x and y.

(7)

An inspection route starting at A and finishing at H is found. The route traverses each arc at least once and is of minimum length.

(b) State the arcs that are traversed twice.

(1)

(c) State the number of times that vertex C appears in the inspection route.

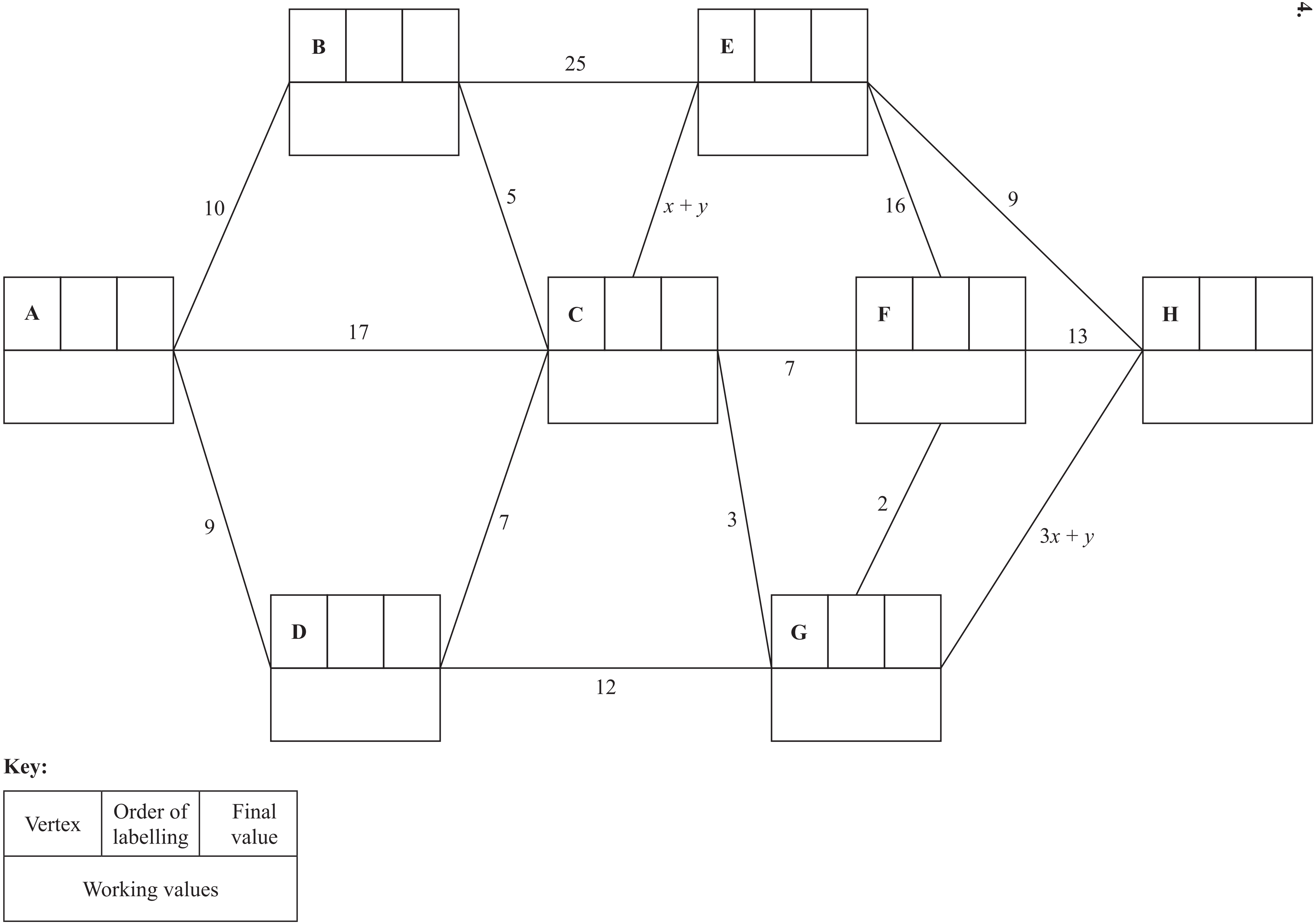
(1)

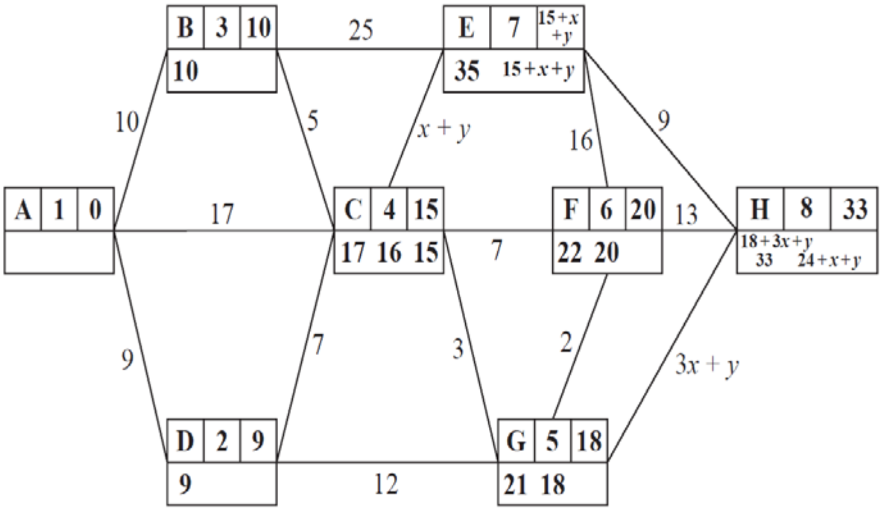
(d) Determine the length of the inspection route.

(1)

(Total for Question 4 is 10 marks)

4.


A
B
C
D
E
F
G
H
10
17
9
5
7
25
x + y
12
3
7
16
9
2
13
3x + y
Key:
Vertex
Order of labelling
Final value
Working values

Question	Scheme	Marks	AOs
4(a)	 Attempt to form a pair of simultaneous equations using their three working values from H e.g. $3x + y = 15$ or $x + y = 9$ $x = 3, y = 6$	M1 A1 A1ft A1 M1dep A1 A1	1.1b 1.1b 1.1b 1.1b 3.1a 2.1 2.2a
		(7)	
(b)	Arcs BC and CD need to be traversed twice	B1	1.1b
		(1)	
(c)	Vertex C would appear 4 times	B1	2.2a
		(1)	
(d)	135 + 4(3) + 2(6) + 12 = 171	B1ft	1.1b
		(1)	
			(10 marks)

Notes

(a)

M1: For a larger number replaced by a smaller one in the working value boxes at C, F or G

A1: For all values correct (and in correct order) at A, D, B and C (condone order of labelling starting at A with 0)

A1ft: For all values correct (and in correct order) at G and F following through from A, D, B and C

A1: For all **working values** correct at E and H (order of working values must be correct at E but condone any order of working values at H) **however, at H if only one working value is seen e.g. 18 + 3*x* + *y* then both 33 and 24 + *x* + *y* must be seen (or clearly implied) in later working for this mark to be awarded (e.g. 3*x* + *y* = 15 and *x* + *y* = 9 would imply this). Similarly, if only two working values seen (e.g. 18 + 3*x* + *y* and 33) at H then the third (24 + *x* + *y*) must be implied by later working. Any incorrect working values seen at H though will score A0**

M1dep: Forming two equations from the candidate's three working values at H (so two of their $18+3x+y=24+x+y$, $18+3x+y=33$ and $24+x+y=33$) – allow all three working values stated anywhere in their solution – dependent on previous M mark. Must be a complete method – so for those finding *x* from $18+3x+y=24+x+y$ they must also either state or use one of the other two equations (so candidates must be interacting with all three paths from A to H)

A1: Two correct equations formed (dependent on correct working values either seen at H or in their subsequent working) – can be unsimplified but must come from correct working

A1: CAO for *x* and *y* (*x* = 3 and *y* = 6) – must come from correct working

If all three correct working values at H are seen (either at H or subsequent working) together with both correct answers (with no other working) then award M1A1A1.

(b)

B1: CAO (arcs BC and CD)

(c)

B1: CAO (4 times)

(d)

B1ft: Follow through only for 135 + 12 + 4*x* + 2*y* (for their *x* and *y* values provided $7 < x + y < 20$ and *x* and *y* are positive constants)

3.

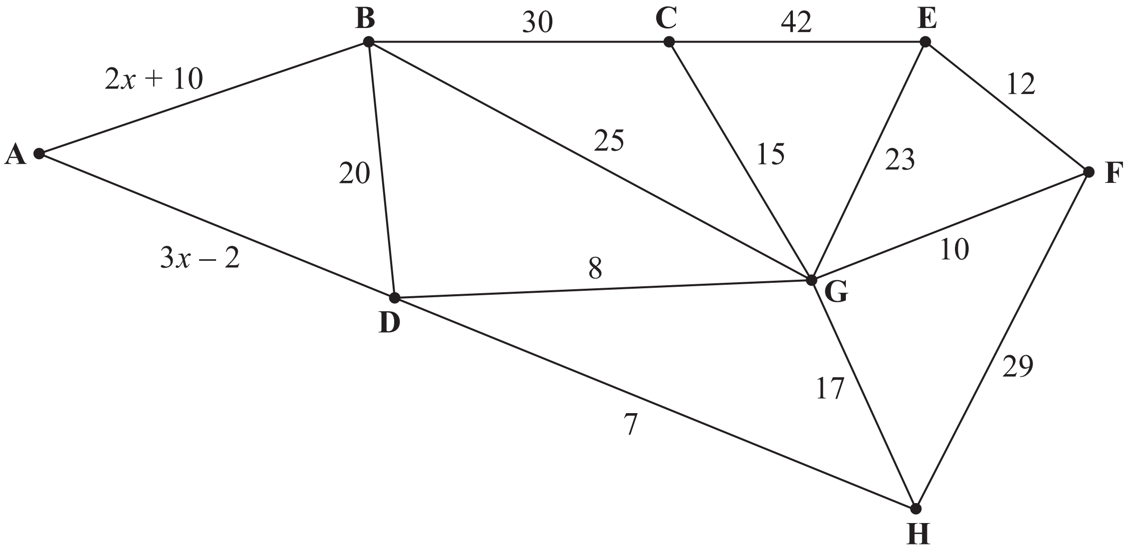


Figure 2

[*The weight of the network is* $5x + 246$]

(a) Explain why it is not possible to draw a graph with an odd number of vertices of odd valency.

(2)

Figure 2 represents a network of 14 roads in a town. The expression on each arc gives the time, in minutes, to travel along the corresponding road.

Prim's algorithm, starting at A, is applied to the network. The order in which the arcs are selected is AD, DH, DG, FG, EF, CG, BD. It is given that the order in which the arcs are selected is unique.

(b) Using this information, find the smallest possible range of values for x, showing your working clearly.

(3)

A route that minimises the total time taken to traverse each road at least once is required. The route must start and finish at the same vertex.

Given that the time taken to traverse this route is 318 minutes,

(c) use an appropriate algorithm to determine the value of x, showing your working clearly.

(6)

(Total for Question 3 is 11 marks)

3.

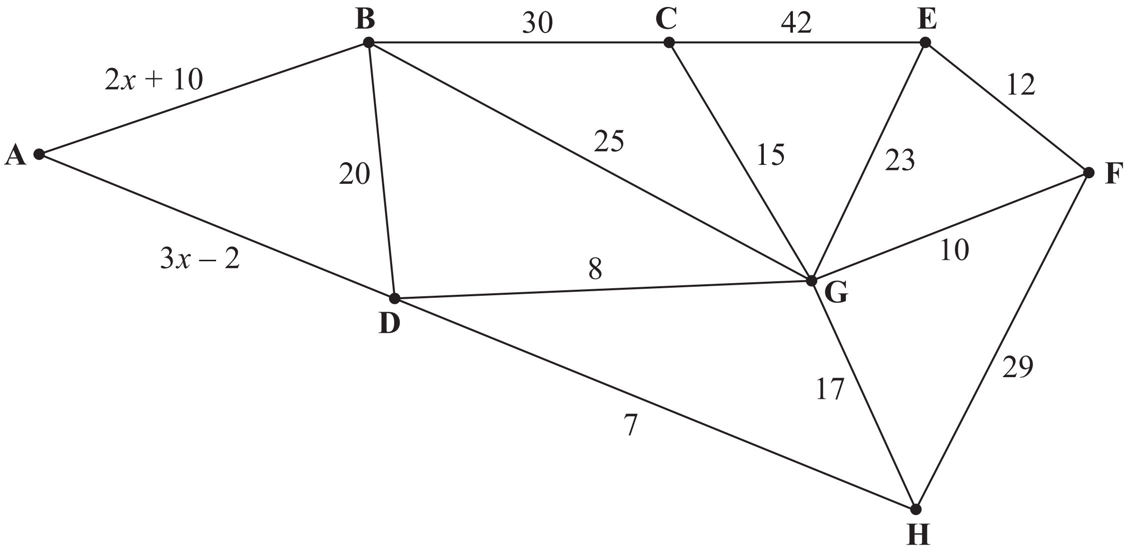


Figure 2

[*The weight of the network is* $5x + 246$]

Question	Scheme	Marks	AOs
3(a)	e.g. (each arc contributes 1 to the orders of two nodes, and so) the sum of the orders of all the nodes is equal to twice the number of arcs	B1	1.2
	Which implies that the sum of the orders of all the nodes is even and therefore there must be an even (or zero) number of vertices of odd order hence there cannot be an odd number of vertices of odd order	B1dep	2.4
		(2)	
3(b)	Either $2x+10>3x-2$ or $2x+10>20$	M1	3.1b
	$x < 12$	A1	1.1b
	$x > 5$	A1	1.1b
		(3)	
3(c)	Applies the route inspection algorithm to this non-standard case	M1	3.1b
	C(GF)E + F(GD)H = 37 + 25 = 62	A1	1.1b
	C(G)F + E(FGD)H = 25 + 37 = 62	A1	1.1b
	C(GD)H + EF = 30 + 12 = 42*	A1	1.1b
	$5x + 246 + 42 = 318$	M1dep	3.1a
	$x = 6$	A1	2.2a
		(6)	
			(11 marks)

Notes

(a)

B1: For one of the following points:

- ‘Sum of the order/valencies of the nodes/vertices = 2(number of arcs/edges)’
- ‘Each arc/edge contributes 1 to the order/valency of two nodes/vertices’
- ‘Sum of the order/valencies of the nodes/vertices is even’

But condone for B1 only

- ‘sum of the valencies = 2(number of arcs/edges)’ **or** ‘sum of the nodes/vertices = 2(number of arcs/edges)’ **or** ‘sum of the orders = 2(number of arcs/edges)
- ‘sum of the valencies is even’ **or** ‘sum of the nodes/edges is even’

B1dep: Stating that ‘the sum of the order (or valencies) of the nodes/vertices = 2(number of arcs/edges) therefore the sum of the order (of the nodes/vertices) is even which implies that there must be an even number of nodes/vertices of odd order (or there cannot be an odd number of nodes/vertices of odd order) **OR** each arc/edge contributes 1 to the order of two nodes/vertices therefore the sum of the order (of the nodes/vertices) is even which implies that there must be an even number of nodes/vertices of odd order (or there cannot be an odd number of nodes/vertices of odd order)

So in summary the first B mark should be awarded for a broadly correct statement (but allow bod as shown in the last two bullet-points above) but for both B marks a fully correct explanation must be given without any bod (please note therefore it is not possible to score B0B1). Do not accept non-technical language for nodes/arcs for either B1B0 or B1B1

(b)

M1: Either comparing arc AB with AD or BD with AB – accept any inequality symbol or equals

A1: CAO ($x < 12$)

A1: CAO ($x > 5$)

(c)

M1: Correct three pairings of the required four odd nodes (C, E, F and H)

A1: Any one correct pairing **and** total

A1: Any two correct pairings **and** totals

A1: All three correct pairings **and** totals

M1dep: Setting up an equation using the given values and their smallest pairing (dependent on the previous M mark) – must have three totals from application of route inspection

A1: CAO ($x = 6$)

3.

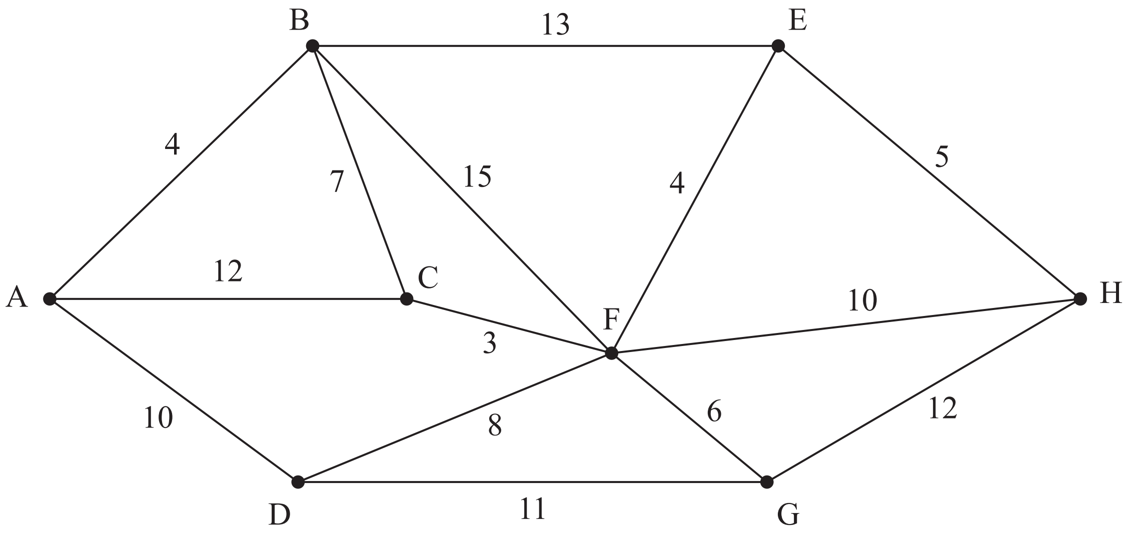


Figure 2

[*The total weight of the network is* 120]

(a) Explain what is meant by the term “path”.

(2)

(b) State, with a reason, whether the network in Figure 2 is Eulerian, semi-Eulerian or neither.

(1)

Figure 2 represents a network of cycle tracks between eight villages, A, B, C, D, E, F, G and H. The number on each arc represents the length, in km, of the corresponding track. Samira lives in village A, and wishes to visit her friend, Daisy, who lives in village H.

(c) Use Dijkstra’s algorithm to find the shortest path that Samira can take.

(5)

An extra cycle track of length 9 km is to be added to the network. It will either go directly between C and D or directly between E and G.

Daisy plans to cycle along every track in the new network, starting and finishing at H.

Given that the addition of either track CD or track EG will not affect the final values obtained in (c),

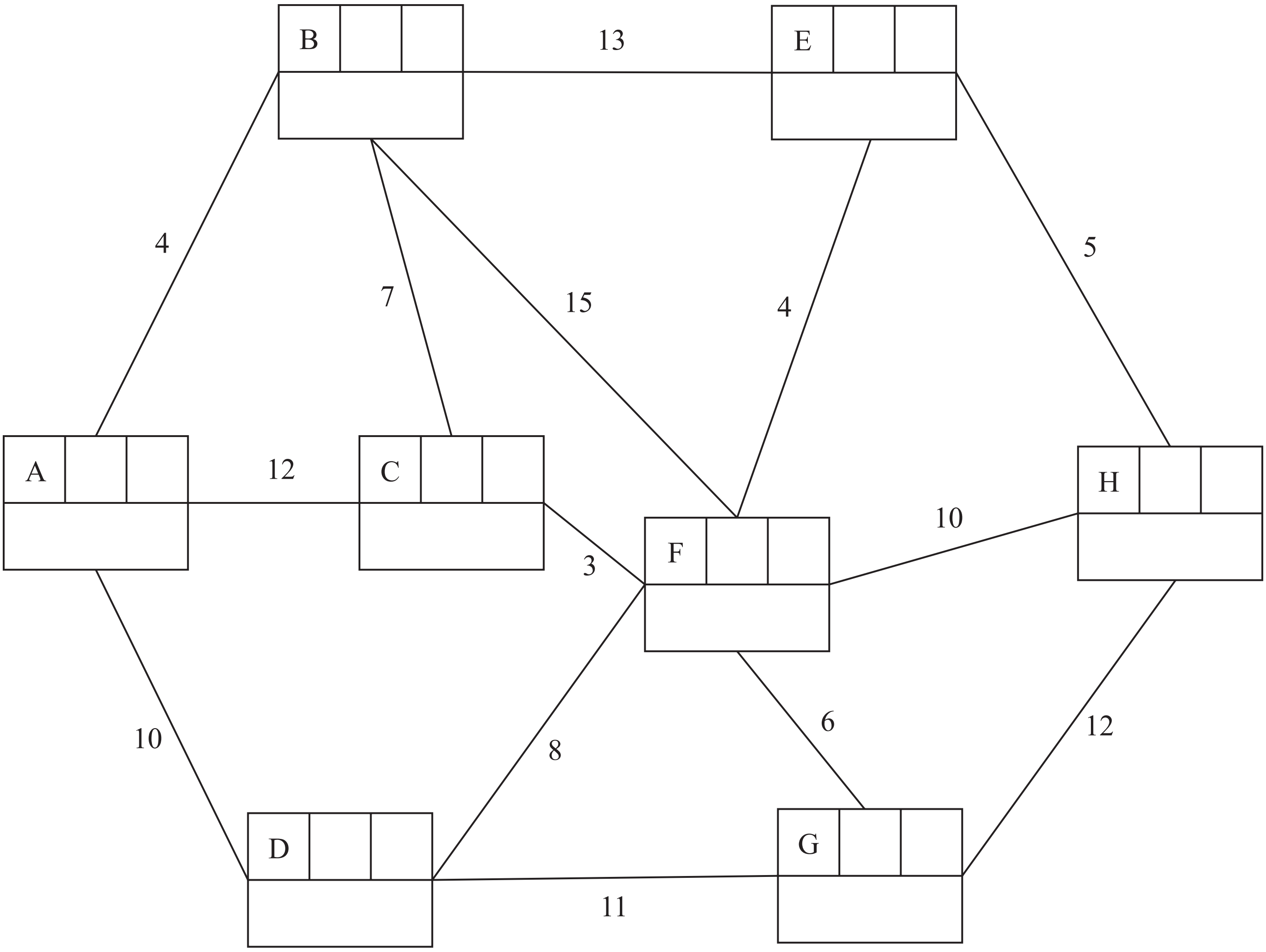
(d) use a suitable algorithm to find out which of the two possible extra tracks will give Daisy the shortest route, making your method and working clear. You must

- state which tracks Daisy will repeat in her route
- state the total length of her route

(6)

(Total for Question 3 is 14 marks)

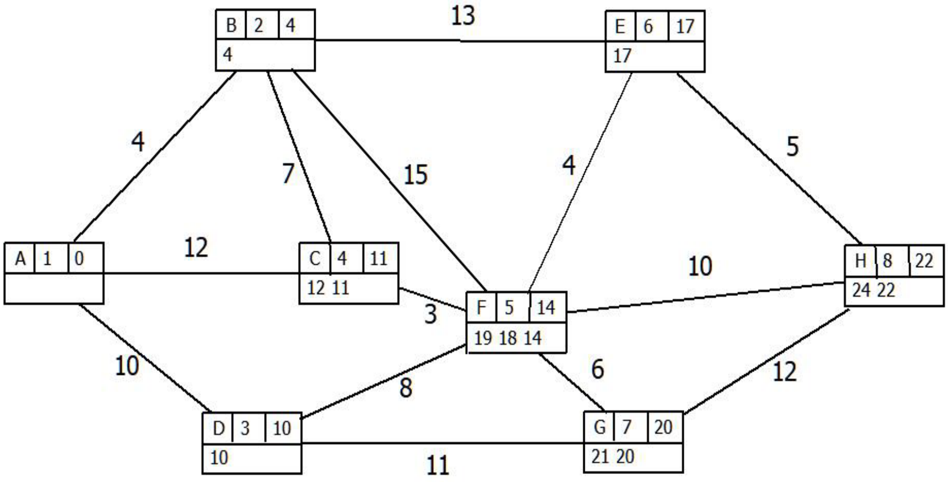
3.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path ________________________________

Qu	Scheme	Marks	AOs
3(a)	A path is a (i) finite sequence of edges, such that (ii) the end vertex of one edge in the sequence is the start vertex of the next, and in which (iii) no vertex appears more than once	B2,1,0	1.2 1.2
		(2)	
(b)	Graph is neither Eulerian nor semi-Eulerian because it has six odd vertices.	B1	2.4
		(1)	
(c)	 Shortest path: ABEH	M1 A1 (ABDC) A1(FE) A1ft (GH) A1	1.1b 1.1b 1.1b 1.1b 2.2a
		(5)	
(d)	If arc CD included: AE + GH = 17 + 12 = 29 AG + EH = 20 + 5 = 25 AH + EG = 22 + 10 = 32 If arc EG included: AC + DH = 11 + 17 = 28 AD + CH = 10 + 12 = 22* AH + CD = 22 + 11 = 33 Track EG with repeated arcs AD, CF, FE, EH Length = 120 + 9 + 22 = 151 (km)	M1 A1 depM1 A1 A1 A1	3.1b 1.1b 1.1b 1.1b 2.2a 2.2a
		(6)	
(14 marks)			
Notes for Question 3			

a1B1: One of the three points made clearly (‘finite, edges’, ‘end vertex of one edge is the start vertex of the next’, ‘no vertex appears more than once’ – condone ‘a vertex cannot appear twice’ but not ‘a vertex cannot be repeated more than once’)

a2B1: All three points made clearly. Candidates who state that a path is a walk in which no vertex appears more than once can score B1B0 only

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b1B1: Correct statement (neither) with correct reason. Either states that there are more than two odd nodes **or** does not have exactly zero or two odd nodes **or** that there are six odd nodes. Their argument must be convincing that the graph cannot be Eulerian or semi-Eulerian (e.g. ‘the network does not have two odd nodes’ is B0). Do not ISW (or BOD) if any incorrect reasoning given

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In (c) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 24 22 in that order (so 22 24 is incorrect)

It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

---

c1M1: A larger value replaced by a smaller value at least twice in the working values at either C, F, G, H

c1A1: All values at A, B, D and C correct and the working values in the correct order

c2A1: All values at F and E correct and working values in the correct order. Penalise order of labelling only once per question. Condone an additional working value of 18 after the 17 at E

c3A1ft: All values in G and H correct on the follow through and the working values in the correct order. To follow through G check that the working values at G follow from the candidate’s final values for the nodes that are directly attached to G (which are D and F). For example, **if** correct then the order of labelling of nodes D and F are 3 and 5 respectively so the working values at G should come from D and F in that order. The first working value at G should be their 10 (the Final value at D) + 11 (the weight of the arc DG), the second working value at G should be their 14 (the Final value at F) + 6 (the weight of the arc FG). Repeat the process for H (which will have working values from F, E and G with the order of these nodes determined by the candidate’s order of labelling at F, E and G). Condone an additional working value of 32 after the 22 at H

c4A1: cao for shortest path (ABEH)

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d1M1: One correct set (either AEGH or ACDH) of three distinct pairings of the correct four odd nodes (so must have AE + GH, AG + EH and AH + EG **or** AC + DH, AD + CH and AH + CD)

d1A1: **Any** three rows correct including pairings **and** totals, from either set AEGH or set ACDH

d2dM1: All six distinct pairings for nodes AEGH and ACDH – dependent on first M mark

d2A1: All six rows correct including pairings **and** totals

d3A1: cao correct edges clearly stated and not just in their working. **Must** be edges AD, CF, FE, EH **and** clearly selecting track EG

d4A1: cao (151) from correct working – **dependent on first four marks in this part**