Fd1Ch3 XMQs and MS

(Total: 82 marks)

1. FD1_Sample   Q4 . 14 marks - FD1ch3 Algorithms on graphs
2. FD1_2019   Q3 . 14 marks - FD1ch3 Algorithms on graphs
3. FD1_2020   Q1 . 6 marks - FD1ch2 Graphs and networks
4. FD1_2021   Q6 . 10 marks - FD1ch3 Algorithms on graphs
5. FD1_2022   Q6 . 12 marks - FD1ch3 Algorithms on graphs
6. FD1(AS)_2018 Q1 . 9 marks - FD1ch3 Algorithms on graphs
7. FD1(AS)_2021 Q4 . 8 marks - FD1ch3 Algorithms on graphs
8. FD1(AS)_2022 Q1 . 9 marks - FD1ch1 Algorithms

4.

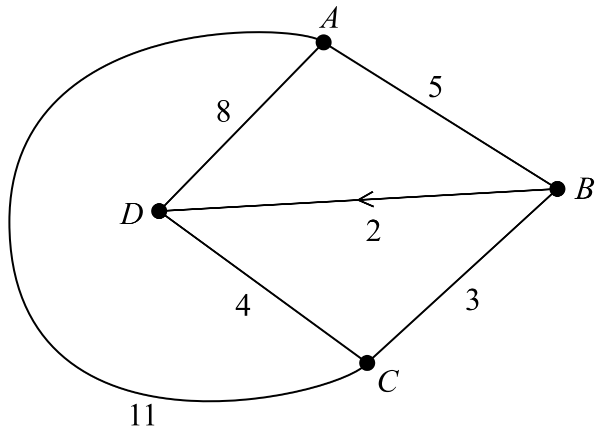


Figure 3

The network in Figure 3 shows the roads linking a depot, D, and three collection points A, B and C. The number on each arc represents the length, in miles, of the corresponding road. The road from B to D is a one-way road, as indicated by the arrow.

(a) Explain clearly if Dijkstra's algorithm can be used to find a route from D to A.

(1)

The initial distance and route tables for the network are given in the answer book.

(b) Use Floyd's algorithm to find a table of least distances. You should show both the distance table and the route table after each iteration.

(7)

(c) Explain how the final route table can be used to find the shortest route from D to B. State this route.

(2)

There are items to collect at A, B and C. A van will leave D to make these collections in any order and then return to D. A minimum route is required.

Using the final distance table and the Nearest Neighbour algorithm starting at D,

(d) find a minimum route and state its length.

(2)

Floyd's algorithm and Dijkstra's algorithm are applied to a network. Each will find the shortest distance between vertices of the network.

(e) Describe how the results of these algorithms differ.

(2)

(Total for Question 4 is 14 marks)

4.

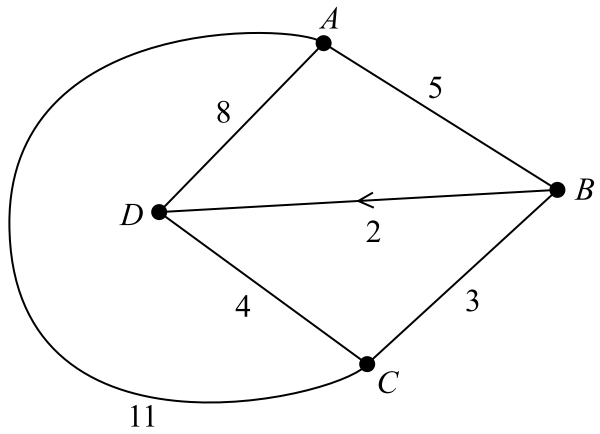


Figure 3

(b)

	A	**B**	**C**	**D**
A	–	5	11	8
B	5	–	3	2
C	11	3	–	4
D	8	∞	4	–

	A	**B**	**C**	**D**
A	A	B	C	D
B	A	B	C	D
C	A	B	C	D
D	A	B	C	D

	A	**B**	**C**	**D**
A				
B				
C				
D				

	A	**B**	**C**	**D**
A				
B				
C				
D				

	A	**B**	**C**	**D**
A				
B				
C				
D				

	A	**B**	**C**	**D**
A				
B				
C				
D				

Question 4 continued

	A	B	C	D
A				
B				
C				
D				

	A	B	C	D
A				
B				
C				
D				

	A	B	C	D
A				
B				
C				
D				

	A	B	C	D
A				
B				
C				
D				

(Total for Question 4 is 14 marks)

Question	Scheme	Marks	AOs
4(a)	Yes Dijkstra's algorithm can be applied to either a directed or undirected network	B1	3.5b
		(1)	
(b)	Initial tables $\begin{bmatrix} - & 5 & 11 & 8 \\ 5 & - & 3 & 2 \\ 11 & 3 & - & 4 \\ 8 & \infty & 4 & - \end{bmatrix} \begin{bmatrix} A & B & C & D \\ A & B & C & D \\ A & B & C & D \\ A & B & C & D \end{bmatrix}$		
	1^{st} iteration $\begin{bmatrix} - & 5 & 11 & 8 \\ 5 & - & 3 & 2 \\ 11 & 3 & - & 4 \\ 8 & [13] & 4 & - \end{bmatrix} \begin{bmatrix} A & B & C & D \\ A & B & C & D \\ A & B & C & D \\ A & [A] & C & D \end{bmatrix}$	M1 A1	1.1b 1.1b
	2^{nd} iteration $\begin{bmatrix} - & 5 & [8] & [7] \\ 5 & - & 3 & 2 \\ [8] & 3 & - & 4 \\ 8 & 13 & 4 & - \end{bmatrix} \begin{bmatrix} A & B & [B] & [B] \\ A & B & C & D \\ [B] & B & C & D \\ A & A & C & D \end{bmatrix}$	M1 A1ft	1.1b 1.1b
	3^{rd} iteration $\begin{bmatrix} - & 5 & 8 & 7 \\ 5 & - & 3 & 2 \\ 8 & 3 & - & 4 \\ 8 & [7] & 4 & - \end{bmatrix} \begin{bmatrix} A & B & B & B \\ A & B & C & D \\ B & B & C & D \\ A & [C] & C & D \end{bmatrix}$	M1 A1ft	1.1b 1.1b
	4^{th} iteration $\begin{bmatrix} - & 5 & 8 & 7 \\ 5 & - & 3 & 2 \\ 8 & 3 & - & 4 \\ 8 & 7 & 4 & - \end{bmatrix} \begin{bmatrix} A & B & B & B \\ A & B & C & D \\ B & B & C & D \\ A & C & C & D \end{bmatrix}$ no changes therefore optimal	A1	1.1b
		(7)	
(c)	Start at D (4^{th}) row and read across to the B (2^{nd}) column, there is a C there		
	so the route starts DC. Look at the C row, B column and you see B	B1	2.4
	The route is therefore DCB	B1	2.2a
		(2)	

Question	Scheme	Marks	AOs
(d)	D – C –B – A – **B** – D	M1	2.2a
	Length 19 (miles)	A1	1.1b
		(2)	
(e)	Dijkstra's algorithm finds the shortest distances from **one** vertex to **all** the others. Floyd's algorithm finds the shortest distance between **every pair** of vertices.	B1 B1	2.5 2.5
		(2)	
		(14 marks)	

Question 4 notes:

(a)
B1: cao (must include mention of 'directed' network)
M1: No change in the first row and first column of both tables with at least one value in the distance table reduced and one value in the route table changed

(b)
A1: cao
M1: No change in the second row and second column of both tables with at least two values in the distance table reduced and two values in the route table changed
A1ft: Correct second iteration follow through from the candidate's first iteration
M1: No change in the third row and third column of both tables with at least one value in the distance table reduced and one value in the route table changed
A1ft: Correct third iteration follow through from the candidate's second iteration
A1: cao (no change after the fourth iteration) – all previous marks must have been awarded in this part

(c)
B1: Clear indication of how the final route table can be used to get from D to B (therefore must mention the correct rows and columns in their reasoning)
B1: Completely correct argument + correct route (DCB)

(d)
M1: Deduce correctly their minimum route from their final distance table (dependent on all M marks in (a)) must begin and end at D
A1: cao (length of 19)

(e)
B1: cao – must use correct language '**one** vertex to **all** other vertices'
B1: cao – must use correct language '**every pair** of vertices'

3.

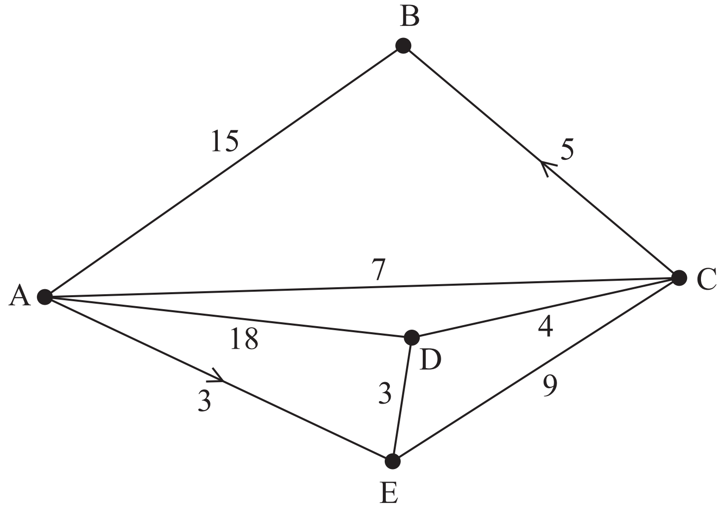


Figure 2

The network in Figure 2 shows the direct roads linking five villages, A, B, C, D and E. The number on each arc represents the length, in miles, of the corresponding road. The roads from A to E and from C to B are one-way, as indicated by the arrows.

(a) Complete the initial distance and route tables for the network provided in the answer book.
(2)

(b) Perform the first three iterations of Floyd's algorithm. You should show the distance table and the route table after each of the three iterations.
(5)

After five iterations of Floyd's algorithm the final distance table and partially completed final route table are shown below.

Distance table

	A	**B**	**C**	**D**	**E**
A	–	12	7	6	3
B	15	–	22	21	18
C	7	5	–	4	7
D	11	9	4	–	3
E	14	12	7	3	–

Route table

	A	**B**	**C**	**D**	**E**
A	A				
B	A	B			
C	A	B	C		
D	C	C	C	D	
E	D	D	D	D	E

(c) (i) Explain how the partially completed final route table can be used to find the shortest route from E to A.

(ii) State this route.
(3)

Mabintou decides to use the distance table to try to find the shortest cycle that passes through each vertex. Starting at D, she applies the nearest neighbour algorithm to the final distance table.

(d) (i) State the cycle obtained using the nearest neighbour algorithm.

(ii) State the length of this cycle.

(iii) Interpret the cycle in terms of the actual villages visited.

(iv) Prove that Mabintou's cycle is not optimal.
(4)

(Total for Question 3 is 14 marks)

3. (a)

Initial distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Initial route table

	A	B	C	D	E
A					
B					
C					
D					
E					

(b) 1st iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

2nd iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

3rd iteration

Distance table

	A	B	C	D	E
A					
B					
C					
D					
E					

Route table

	A	B	C	D	E
A					
B					
C					
D					
E					

Turn to page 11 for spare copies of these tables if you need to correct your work.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Only use these tables if you need to correct your work.

(a)

Initial distance table

	A	**B**	**C**	**D**	**E**
A					
B					
C					
D					
E					

Initial route table

	A	**B**	**C**	**D**	**E**
A					
B					
C					
D					
E					

(b) 1st iteration

Distance table

	A	**B**	**C**	**D**	**E**
A					
B					
C					
D					
E					

Route table

	A	**B**	**C**	**D**	**E**
A					
B					
C					
D					
E					

2nd iteration

Distance table

	A	**B**	**C**	**D**	**E**
A					
B					
C					
D					
E					

Route table

	A	**B**	**C**	**D**	**E**
A					
B					
C					
D					
E					

3rd iteration

Distance table

	A	**B**	**C**	**D**	**E**
A					
B					
C					
D					
E					

Route table

	A	**B**	**C**	**D**	**E**
A					
B					
C					
D					
E					

(Total for Question 3 is 14 marks)

Qu	Scheme	Marks	AOs
3(a)	Distance table and Route table (below)	B1 B1	1.1b 1.1b
		(2)	

3(a)

Distance table

	A	B	C	D	E
A	-	15	7	18	3
B	15	-	∞	∞	∞
C	7	5	-	4	9
D	18	∞	4	-	3
E	∞	∞	9	3	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	C	D	E
C	A	B	C	D	E
D	A	B	C	D	E
E	A	B	C	D	E

(b)

1st iteration: (M1 1.1b, A1 1.1b)

Distance table

	A	B	C	D	E
A	-	15	7	18	3
B	15	-	22	33	18
C	7	5	-	4	9
D	18	33	4	-	3
E	∞	∞	9	3	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

2nd iteration: (A1ft 1.1b)

Distance table

	A	B	C	D	E
A	-	15	7	18	3
B	15	-	22	33	18
C	7	5	-	4	9
D	18	33	4	-	3
E	∞	∞	9	3	-

Route table

	A	B	C	D	E
A	A	B	C	D	E
B	A	B	A	A	A
C	A	B	C	D	E
D	A	A	C	D	E
E	A	B	C	D	E

3rd iteration (**SEE NOTES FOR VALID ALTERNATIVE**): (M1 1.1b, A1 1.1b)

Distance table

	A	B	C	D	E
A	-	12	7	11	3
B	15	-	22	26	18
C	7	5	-	4	9
D	11	9	4	-	3
E	16	14	9	3	-

Route table

	A	B	C	D	E
A	A	C	C	C	E
B	A	B	A	C	A
C	A	B	C	D	E
D	C	C	C	D	E
E	C	C	C	D	E

Qu	Scheme	Marks	AOs
(b)	1st iteration	M1 A1	1.1b 1.1b
	2nd iteration	A1ft	1.1b
	3rd iteration	M1 A1	1.1b 1.1b
		(5)	
(c)(i)	Start at E(5th row) and read across to the A (1st column), there is a D there so the route from E to A is via D	B1	2.4
	Now consider both E to D and D to A – for E reading across to the D (4th column), there is a D indicating that the shortest path from E to D is ED. For D reading across to the A (1st column), there is a C indicating that the shortest path from D to A is via C	dB1	2.4
(ii)	EDCA	B1	2.2a
		(3)	

(d)(i)	NNA: D – E – C – B – A – D	B1	1.1b
(ii)	3 + 7 + 5 + 15 + 6 = 36 miles	B1	1.1b
(iii)	D – E – D – C – B – A – E – D	B1	3.2a
(iv)	e.g. the cycle A – E – D – C – B – A has a length of 30 miles < 36 miles so Mabintou's route is not optimal	B1	2.4
		(4)	
			(14 marks)

Notes for Question 3

IN THE DISTANCE AND ROUTE TABLE FOR PARTS (a) and (b) IGNORE WHATEVER IS WRITTEN IN THE LEAD DIAGONAL (TOP LEFT TO BOTTOM RIGHT)

(a) B1: Correct distance table (condone dashes, crosses, etc. for infinity but do not condone a 'large' number in these cells)
B1: Correct route table

(b) M1: No change in the first row and first column of both tables with at least two values in the distance table correctly reduced and two letters in the route table correctly changed – all cells complete
A1: CAO (condone dashes, etc. in cells EA and EB)
A1ft: No change from candidate's first iteration to second iteration for either table or ft from candidate's first iteration
M1: No change in the third row and third column of both tables with at least two values in the distance table correctly reduced from their second iteration and two values in the route table correctly changed
A1: CAO for third iteration (**note that the entry in row B column D for the route table could be an A**)

(c) B1: Row E column A is D so the route is E to A via D (or implies that the order of the nodes in the route is EDA) **or** D implied from general argument **or** Row E column A is D therefore the route begins ED (in all cases must clearly imply **row** E and **column** A)
dB1: Row D column A is C therefore the route goes via C (before A) **or** complete general argument that allows the route from D to A to be found **or** allow those who say that row D column A is C so the route is EDC and then row C column A is A
B1: CAO (EDCA)

(d)(i) B1: CAO (D – E – C – B – A – D)
(ii) B1: CAO (36 – no units required)
(iii) B1: CAO (D – E – D – C – B – A – E – D) **or** mentions that the cycle would visit E twice and D three times (or visit D before the end of the cycle – if D visited once stated and it is not clear that this isn't the start or finish then B0) **or** mention of E to C via D and A to D via E
(iv) B1: A correct cycle stated (e.g. a cyclic permutation of A – E – D – C – B – A) with corresponding correct length – **dependent on second B mark in this part** (so must have had 36 in **(d)(ii)**)

1. The table below shows the lengths, in km, of the roads in a network connecting seven towns, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	24	–	22	35	–	–
B	24	–	25	27	–	–	–
C	–	25	–	33	31	36	26
D	22	27	33	–	–	42	–
E	35	–	31	–	–	37	29
F	–	–	36	42	37	–	40
G	–	–	26	–	29	40	–

(a) By adding the arcs from vertex D along with their weights, complete the drawing of the network on Diagram 1 in the answer book.

(2)

(b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(c) State the weight of the minimum spanning tree.

(1)

(Total for Question 1 is 6 marks)

1.

	A	B	C	D	E	F	G
A	–	24	–	22	35	–	–
B	24	–	25	27	–	–	–
C	–	25	–	33	31	36	26
D	22	27	33	–	–	42	–
E	35	–	31	–	–	37	29
F	–	–	36	42	37	–	40
G	–	–	26	–	29	40	–

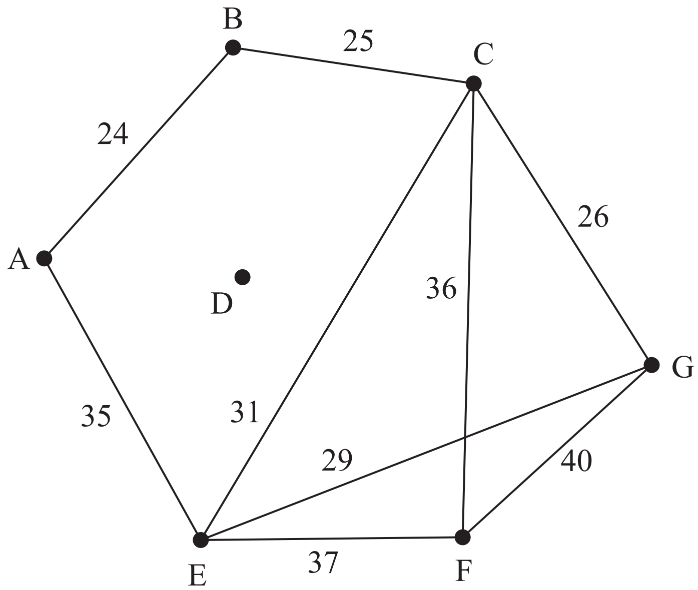
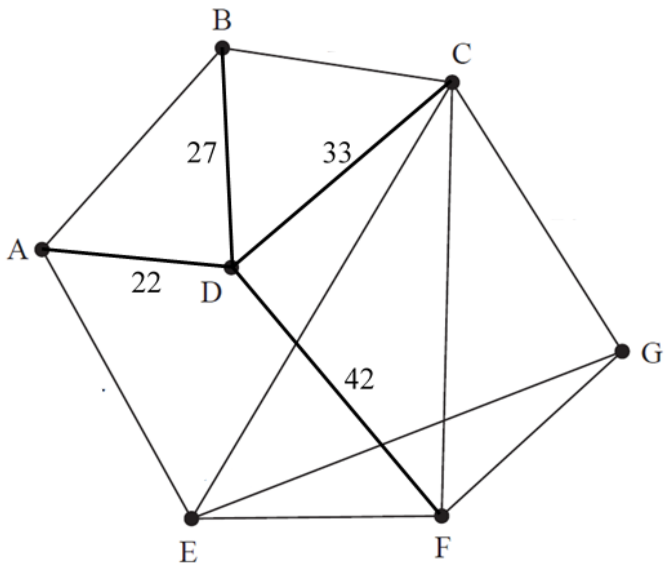


Diagram 1

Question	Scheme	Marks	AOs
1(a)		M1 A1	1.1b 1.1b
		(2)	
(b)	Kruskal's algorithm: AD, AB, BC, CG, reject BD, EG, reject CE, reject CD, reject AE, CF (reject EF, reject FG, reject DF)	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(c)	Weight of MST: 162 (km)	B1	1.1b
		(1)	
			(6 marks)

Notes for Question 1

(a)
M1: Either all arcs correct (ignore weights) or two arcs correct (including correct weights)
A1: CAO
(b)
M1: Kruskal's algorithm – first three arcs correctly chosen and at least one rejection seen at some point
A1: All six arcs selected correctly AD, AB, BC, CG, EG, CF only
A1: CSO – all selections and rejections correct (in correct order and at the correct time)
(c)
B1: CAO (condone lack of units)

6.

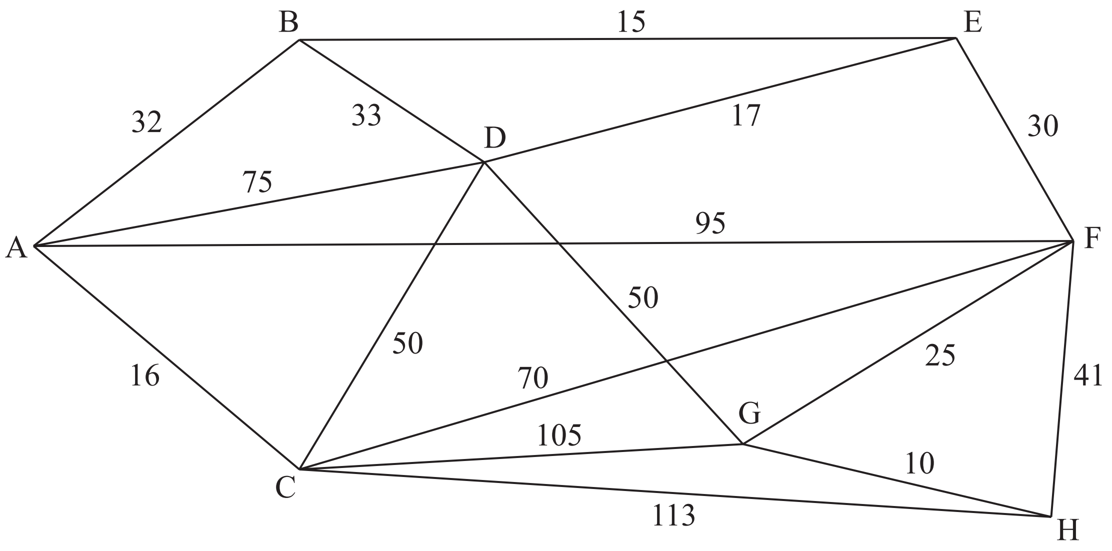


Figure 4

In Figure 4 the weights on the arcs represent distances.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to H.

(ii) State the length of the shortest path from A to H.

(6)

One application of Dijkstra's algorithm has order n^2, where n is the number of nodes in the network. A computer produces a table of shortest distances between any two different nodes by repeatedly applying Dijkstra's algorithm from each node of the network.

It takes the computer 0.082 seconds to produce a table of shortest distances for a network of 10 nodes.

(b) Calculate approximately how long it will take, in seconds, for the computer to produce a table of shortest distances for a network with 200 nodes. You must give a reason for your answer.

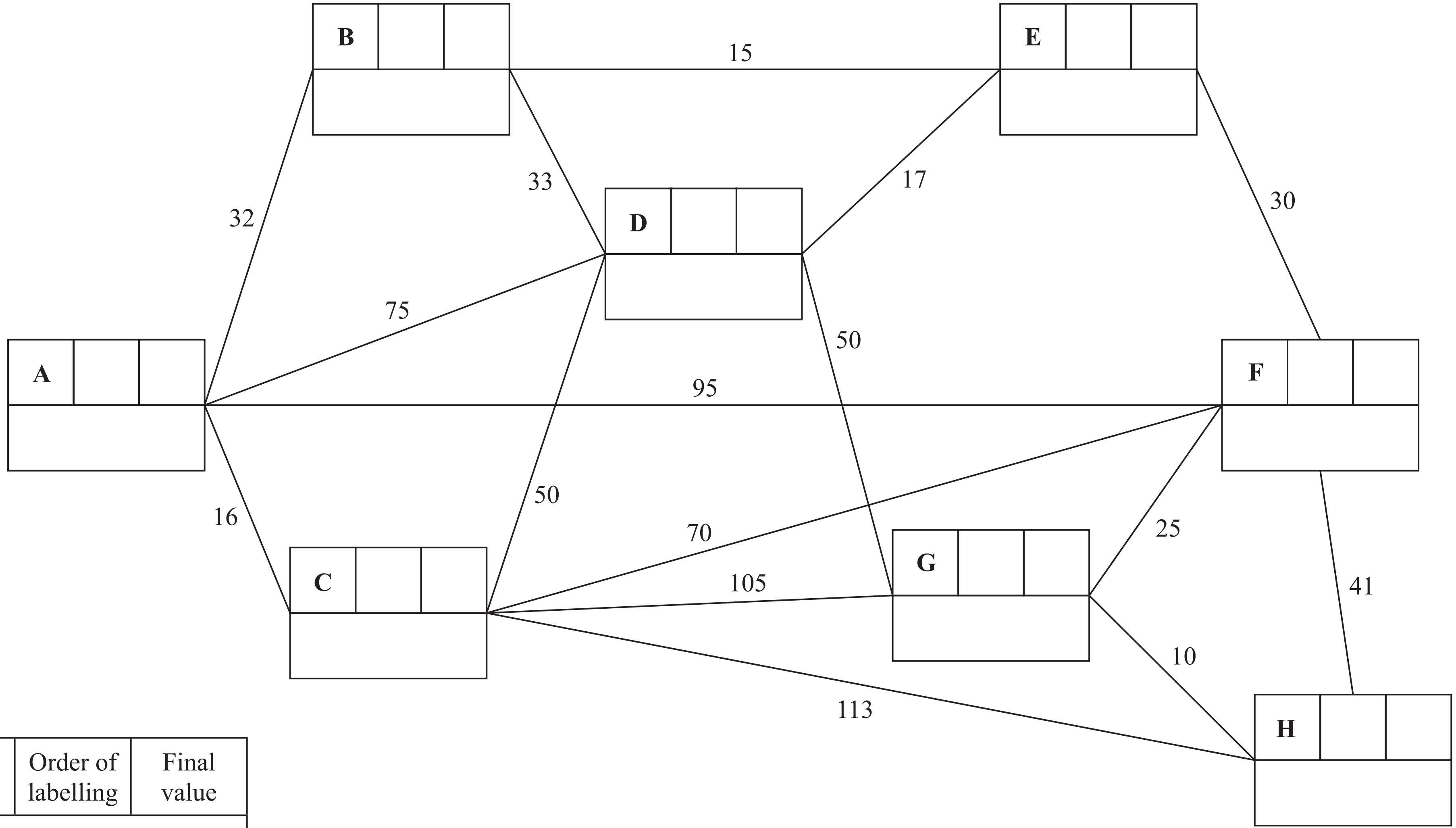
(3)

(c) Explain why your answer to part (b) can only be an approximation.

(1)

(Total for Question 6 is 10 marks)

6.

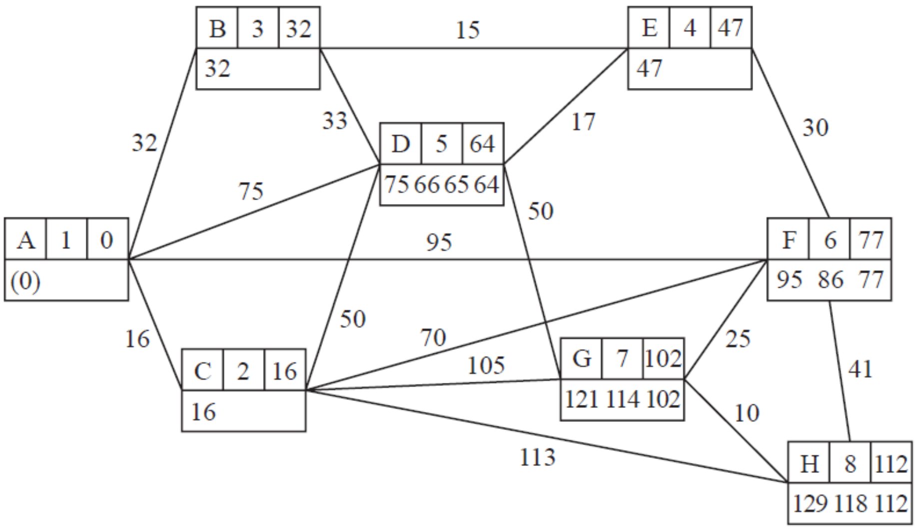


Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to H: ______________________________

Length of shortest path from A to H: ______________________________

Question	Scheme	Marks	AOs
6(a)(i)		M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to H: ABEFGH	A1	2.2a
(a)(ii)	Length of shortest path from A to H is 112	A1ft	2.2a
		(6)	
(b)	Applying Dijkstra repeatedly to n nodes implies that the order is $n\left(n^2\right)=n^3$	B1	3.1b
	$t=0.082\left(\frac{200}{10}\right)^3$	M1	3.4
	$= 656$ (seconds)	A1	2.2a
		(3)	
(c)	e.g. order of n^3 does not mean that the order is proportional to n^3 (which is the assumption behind the answer in (b)) but merely means that the dominant term is of order n^3	B1	3.2b
		(1)	
			(10 marks)

Notes:

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 129 118 112 in that order (so 129 112 118 is incorrect)
It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

(a)

M1: A larger value replaced by a smaller value in at least two of the working boxes at either D or F or G or H

A1: All values in A, B, C and E correct. Condone lack of 0 in A's working value

A1: All values D and F correct and the working values in the correct order. Penalise order of labelling only once per question (D and F must be labelled in that order and D must be labelled after A, B, C and E)

A1ft: All values in G and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working value at G follows from the candidate's final values from their feeds into G (which will most likely come from nodes C, D and/or F (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will possibly have working values from C, F and/or G with the order of these values determined by the candidate's order of labelling at C, F and G)

A1: cao - correct path from A to H (ABEFGH)

A1ft: Follow through their final value at H only (if 112 stated and 112 is not the final value at H then **A0**)

(b)

B1: Any indication that repeated application of Dijkstra has cubic order

M1: Complete method – allow 10/200 – allow slips in values e.g. 0.82 for 0.082 and accept 200/10 (or 10/200) either squared or cubed **only**

A1: cao

(c)

B1: Any indication that the run-time is not <u>exactly</u> proportional to n^3 e.g., may suggest that there are other terms ($n^3 + \ldots$), or that n^3 is the dominant term, or that order does not imply proportionality. Do not accept only that 'n^3 is not exact'. Condone use of n^2 (oe) for n^3

6. The following algorithm determines the number of comparisons made when Prim's algorithm is applied to K_n

Step 1	Start
Step 2	Input the value of n
Step 3	Let $a = 1$
Step 4	Let $b = n - 2$
Step 5	Let $c = b$
Step 6	Let $a = a + 1$
Step 7	Let $b = b - 1$
Step 8	Let $c = c + (a \times b) + (a - 1)$
Step 9	If $b > 0$ go to Step 6
Step 10	Output c
Step 11	Stop

(a) For K_5, complete the table in the answer book to show the results obtained at each step of the algorithm.

(3)

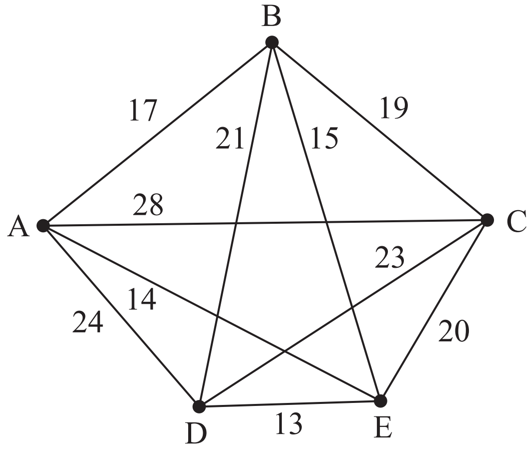


Figure 4

The weights of the ten arcs in Figure 4 are

17 21 24 14 23 13 15 19 28 20

(b) (i) Starting at the left-hand end of the above list, sort the list into ascending order using bubble sort. You need only write down the state of the list at the end of each pass.

(ii) Find the total number of comparisons performed during the sort.

(5)

(c) Find the maximum total number of comparisons required to sort the weights of the 10 arcs of K_5 into ascending order using bubble sort.

(1)

It is given that the maximum total number of comparisons required to sort the weights of the arcs of K_n into ascending order using bubble sort is

$$\lambda n(n-1)(n+1)(n-2)$$

where λ is a constant.

(d) Determine the maximum total number of comparisons required to sort the weights of the arcs of K_{50} into ascending order using bubble sort. You must make your method and working clear.

(3)

(Total for Question 6 is 12 marks)

6. *You may not need to use all the rows in this table.*

It may not be necessary to complete all the boxes in each row.

n	a	b	c

Output: ______________________________

Question 6 continued

17 21 24 14 23 13 15 19 28 20

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>
<table>
<tr><th>n</th><th>a</th><th>b</th><th>c</th></tr>
<tr><td>5</td><td>1</td><td>3</td><td>3</td></tr>
<tr><td></td><td>2</td><td>2</td><td>8</td></tr>
<tr><td></td><td>3</td><td>1</td><td>13</td></tr>
<tr><td></td><td>4</td><td>0</td><td>16</td></tr>
</table>

Output: 16</td><td>M1
A1

A1</td><td>1.1b
1.1b

2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(b)
(i)</td><td>17 21 24 14 23 13 15 19 28 20

17 21 14 23 13 15 19 24 20 28
17 14 21 13 15 19 23 20 24 28
14 17 13 15 19 21 20 23 24 28
14 13 15 17 19 20 21 23 24 28
13 14 15 17 19 20 21 23 24 28
13 14 15 17 19 20 21 23 24 28</td><td>M1
A1
A1ft
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td>(b)
(ii)</td><td>Total number of comparisons: 9 + 8 + 7 + 6 + 5 + 4 = 39</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(c)</td><td>45</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>e.g. when n = 3 the total number of comparisons is 3
therefore $\lambda(3)(3-1)(3+1)(3-2)=3$
e.g. when n = 5 the total number of comparisons is 45 (from (c))
therefore $\lambda(5)(5-1)(5+1)(5-2)=45$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>For K_{50} total number of comparisons is $'\lambda'(50)(50-1)(50+1)(50-2)$</td><td>dM1</td><td>3.4</td></tr>
<tr><td></td><td>749 700</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(12 marks)</td></tr>
</table>

Notes for Question 6
a1M1: At least three rows of cells completed with a correct first row – condone repeated values in all columns or a single value in each row **a1A1:** cao – the values in the second and third rows correct **a2A1:** cao – correct output following a correct fourth row (with no extra rows) – the output **must** either be stated on the given answer line or 'output 16' must be clearly written somewhere near the table (do not bod the 16 circled, underlined, written twice, etc.) **bi1M1:** Bubble sort. Consistent direction, end number (28) in place, the list containing ten numbers with the list beginning with the correct first five numbers (17 21 14 23 13). Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 28 appears at the end of the list is the end of their first pass **bi1A1:** Second and third passes correct – so end three numbers in place **bi2A1ft:** Fourth and fifth passes correct following through from the candidate's third pass – so end five numbers in place. After their third pass their list must contain the correct 10 numbers **bi3A1:** cso (correct solution only – so previous three marks must have been awarded in this part). Must show a 6th pass showing no swaps/changes (give bod if the passes are not labelled but do not award this mark if it is clear that after the 5th pass the list is simply being written out again (rather than a genuine 6th pass taking place)). Condone if the sort continues until a 9th pass has been completed (but there must be no changes in the 6th to 9th passes) **bii1B1:** cao (for total number of comparisons) In **(b)** starting at the right-hand end of the list is M0. Quick sort (or any other sorting algorithm e.g. shuttle) is M0. No misreads in this part – mark exactly to the scheme. If sorting into descending order, then award M1 for 21 24 17 23 14 15 19 28 20 13 and the first A1 for **both** 24 21 23 17 15 19 28 20 14 13 **and** 24 23 21 17 19 28 20 15 14 13 **ONLY** (so two out of the first four marks) even if the list is re-ordered after the sort is complete **c1B1:** cao (45) **d1M1:** Considering the total number of comparisons for any positive integer value of n (greater than 2) and substitute into the given expression (**if** correct $\lambda = \frac{1}{8}$). The correct value of λ implies this mark. If using n = 5 then follow through their value from **(c)**. If any other value of n used, then the maximum number of comparisons must be correct e.g. *n*: 3, 4, 5, 6, 7, 8, 9, 10 Comparisons: 3, 15, 45, 105, 210, 378, 630, 990 **d2dM1:** Using their λ and n = 50 to calculate the maximum total number of comparisons (dependent on the previous M mark). Writing $\frac{1}{8}(50)(50-1)(50+1)(50-2)$ implies the first two M marks **d1A1:** cao (749 700) – no marks for the correct answer with **no** working

Alternative solution to (d) for those who are clearly using an algebraic method to derive the quartic expression for the maximum total number of comparisons:

If there are N values then
1st pass of bubble sort requires $N - 1$ comparisons
2nd pass of bubble sort requires $N - 2$ comparisons
3rd pass of bubble sort requires $N - 3$ comparisons and so on

Therefore, the total number of comparisons is $\sum_{r=1}^{N-1} r = \frac{1}{2}(N-1)N$ M1

Maximum number of comparisons in K_n is therefore

$$\frac{1}{2}\left(\frac{1}{2}n(n-1)-1\right)\left(\frac{1}{2}n(n-1)\right)$$

$$=\frac{1}{8}n(n-1)\left[n(n-1)-2\right]$$

dM1

$$=\frac{1}{8}n(n-1)(n^2-n-2)=\frac{1}{8}n(n-1)(n-2)(n+1) \text{ so } k=\frac{1}{8}$$

Therefore, maximum number of comparisons for K_{50} is 749 700 A1

M1: Considering the total number of comparisons that are required to sort a list containing N values (possibly will see $1 + 2 + 3 + \ldots + (n - 1)$) and using the standard series result that $\sum r = \frac{1}{2}n(n+1)$ to get the correct quadratic expression for the total number of comparisons when sorting N values (allow any letter)

dM1: Dependent on previous M mark – substituting the correct quadratic expression into the correct quadratic expression with correct algebraic working leading to $\frac{1}{8}n(n-1)(n-2)(n+1)$

A1: Correct answer of 749 700

Answer ALL questions. Write your answers in the Answer Book provided.

1.

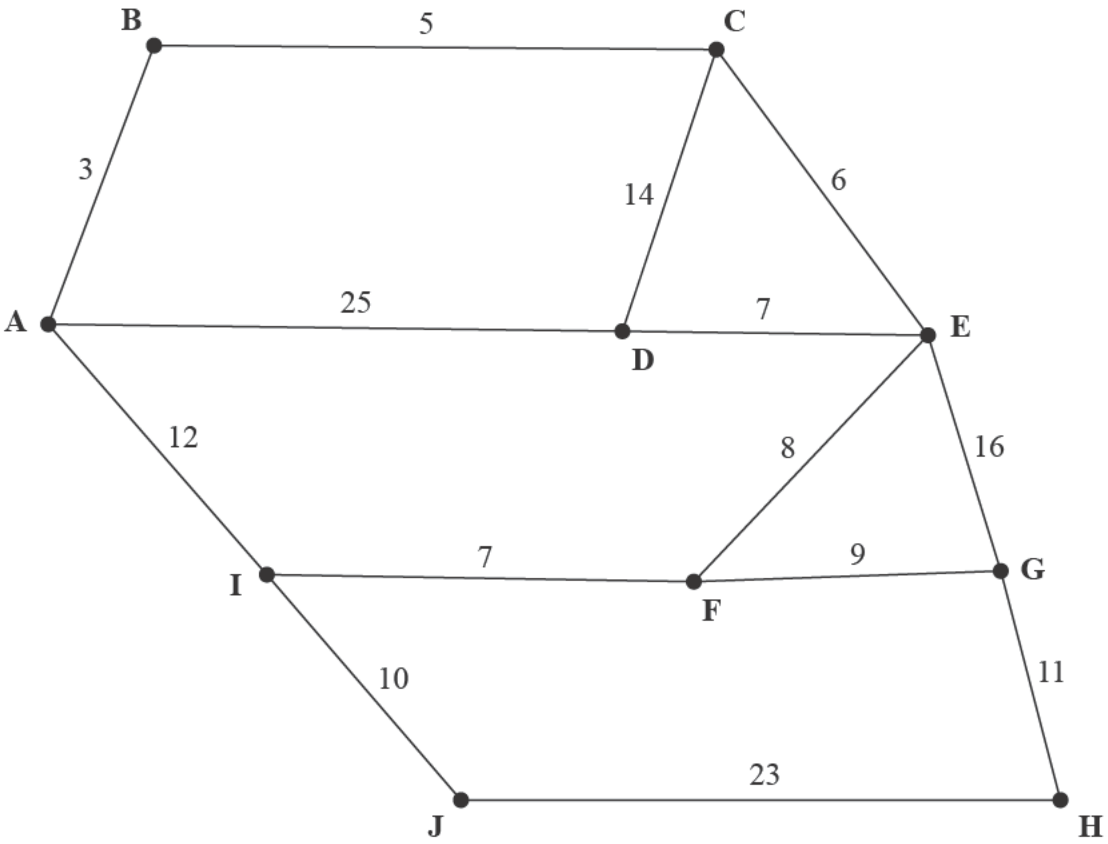


Figure 1

Figure 1 represents a network of roads.
The number on each arc represents the time taken, in minutes, to drive along the corresponding road.

(a) (i) Use Dijkstra's algorithm to find the shortest time needed to travel from A to H.

(ii) State the quickest route.

(6)

For a network with n vertices, Dijkstra's algorithm has order n^2

(b) If it takes 1.5 seconds to run the algorithm when $n = 250$, calculate approximately how long it will take, in seconds, to run the algorithm when $n = 9500$. You should make your method and working clear.

(2)

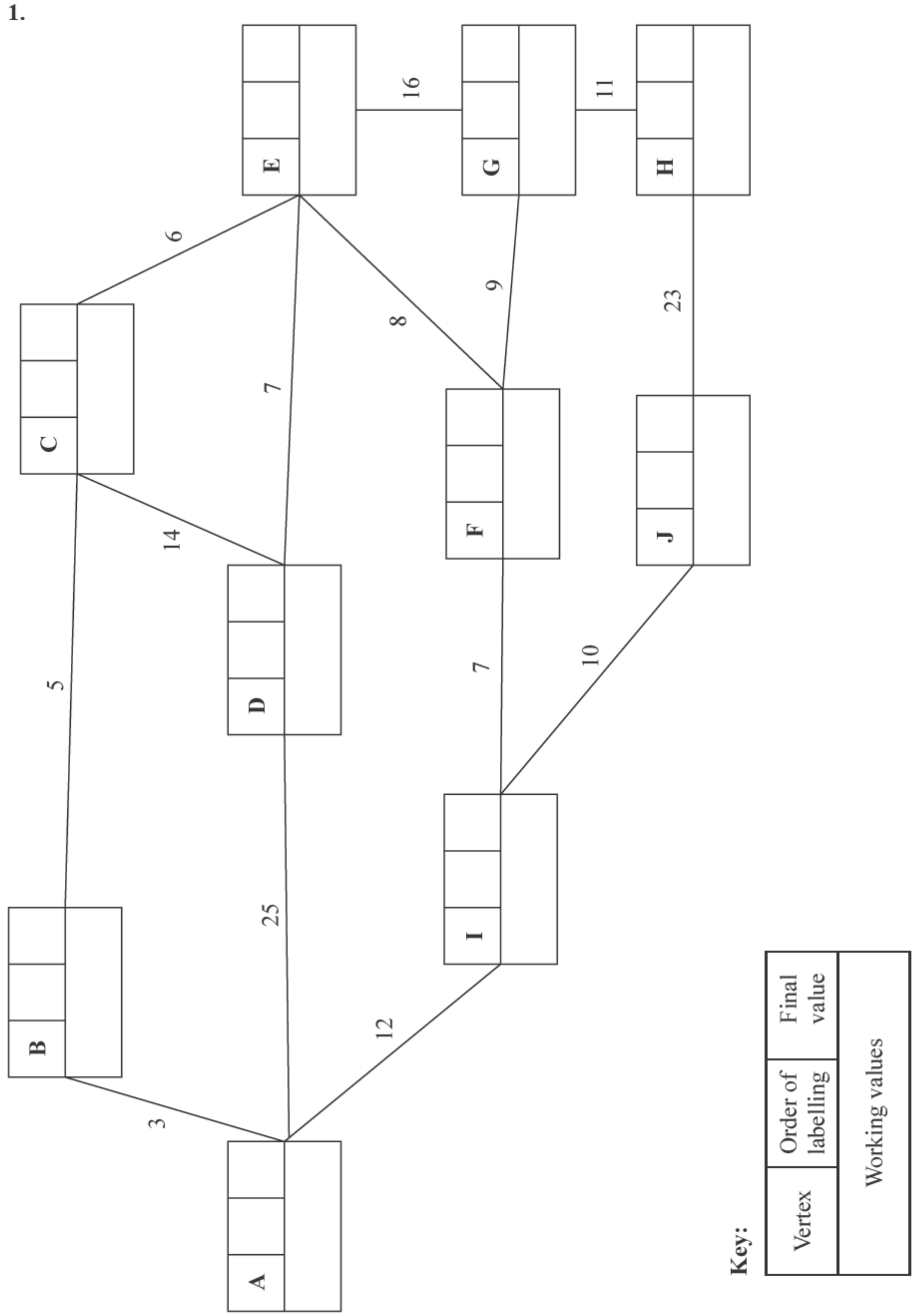
(c) Explain why your answer to part (b) is only an approximation.

(1)

(Total for Question 1 is 9 marks)

Leave blank

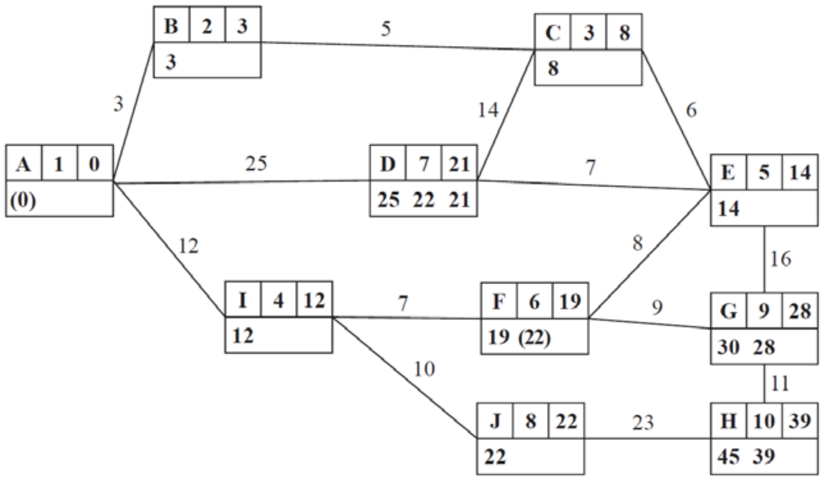
1.



Shortest time: ____________________

Quickest route: ____________________

P 6 0 2 0 8 A 0 2 1 2

Question	Scheme	Marks	AOs
1(a)(i)		M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b
	Shortest time to travel from A to H is 39 minutes	A1ft	1.1b
(a)(ii)	Quickest route is AIFGH	A1	1.1b
		(6)	
(b)	$1.5\times\left(\frac{9500}{250}\right)^2$	M1	1.1a
	= 2166 seconds	A1	1.1b
		(2)	
(c)	order of n^2 does not mean that the order is proportional to n^2 (which is the assumption behind the answer in (b)) but merely means that the dominant term is of order n^2	B1	3.2b
		(1)	
			(9 marks)

Notes
(a)(i) **M1:** For a larger number replaced by a smaller number in the working value boxes at either D, G or H **A1:** For all values correct (and in correct order) at A, B, C, I and E **A1:** For all values correct (and in correct order) at F and D **A1ft:** For all values correct (and in correct order) on the follow through at J, G and H **A1ft:** Follow through their final value at H (condone lack of units) **(a)(ii)** **A1:** Cao (AIFGH)
(b) **M1** Complete method – allow 250/9500 (but must be squared) – allow slips in values e.g. 950 for 9500 **A1:** Cao (accept 2170 but only with correct working) – accept 2166 with no working for M1 only
(c) **B1:** Any indication that the run-time is not <u>exactly</u> proportional to n^2 e.g. may suggest that there are other terms (n^2 + …), or that n^2 is the dominant term, or that order does not imply proportionality. Do not accept only that 'n^2 is not exact'.

4.

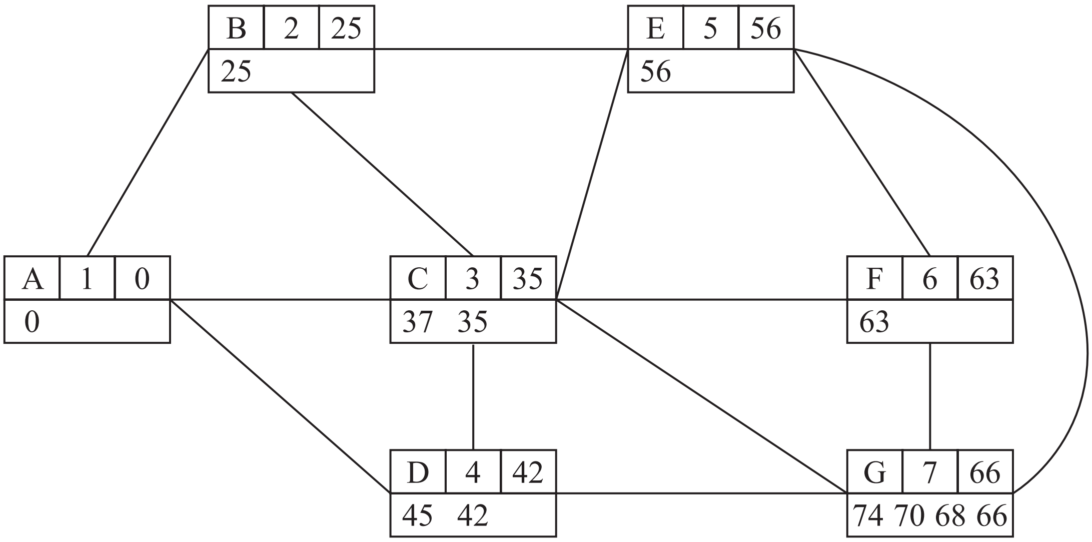


Figure 2

Dijkstra's algorithm has been applied to the network in Figure 2.

A working value has only been replaced at a node if the new working value is smaller.

(a) State the length of the shortest path from A to G.

(1)

(b) Complete the table in the answer book giving the weight of each arc listed.
(Note that arc CE and arc EF are not in the table.)

(3)

(c) State the shortest path from A to G.

(1)

It is now given that

- when Prim's algorithm, starting from A, is applied to the network, the order in which the arcs are added to the tree is AB, BC, CD, CE, EF and FG
- the weight of the corresponding minimum spanning tree is 80
- the shortest path from A to F via E has weight 67

(d) Determine the weight of arc CE and the weight of arc EF, making your reasoning clear.

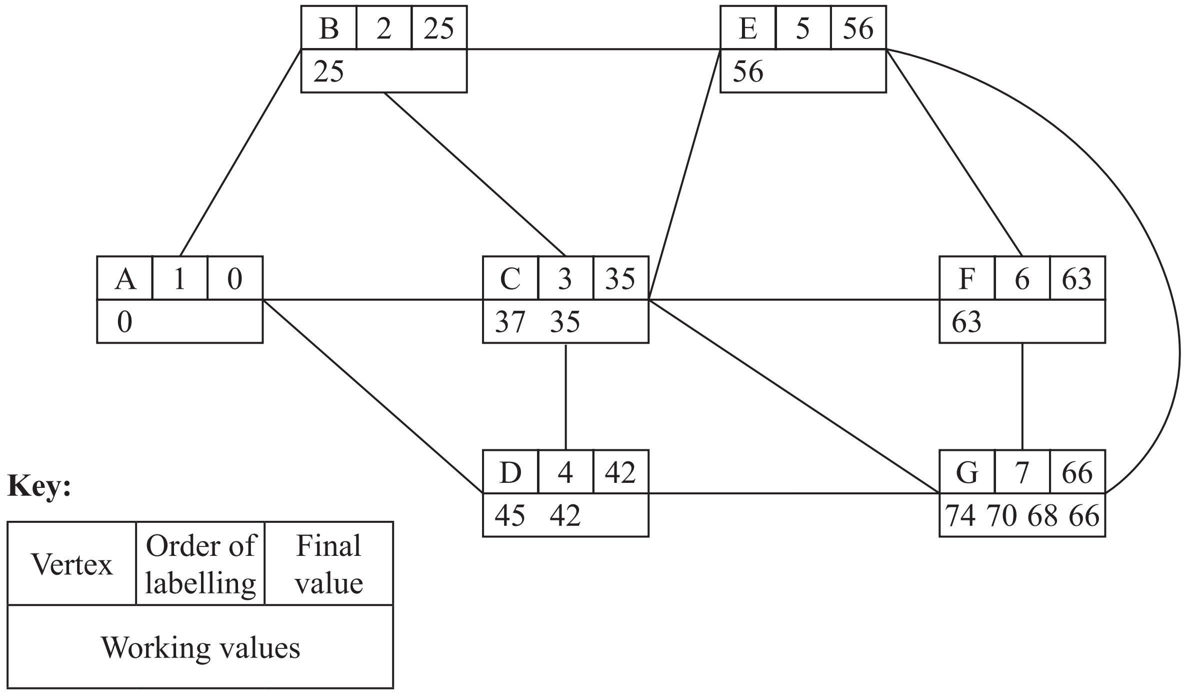
(3)

(Total for Question 4 is 8 marks)

TOTAL FOR DECISION MATHEMATICS 1 IS 40 MARKS

END

4.



(a) length of the shortest path from A to G: ______________________

____

(b)

Arc	Weight
AB	
AC	
AD	
BC	
BE	
CD	
CF	
CG	
DG	
EG	
FG	

____

____

____

____

Question 4 continued

(c) shortest path from A to G:

(d)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>4(a)</td><td>66</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>4(b)</td><td><table><tr><th>Arc</th><th>Weight</th></tr><tr><td>AB</td><td>25</td></tr><tr><td>AC</td><td>37</td></tr><tr><td>AD</td><td>45</td></tr><tr><td>BC</td><td>10</td></tr><tr><td>BE</td><td>31</td></tr><tr><td>CD</td><td>7</td></tr><tr><td>CF</td><td>28</td></tr><tr><td>CG</td><td>39</td></tr><tr><td>DG</td><td>28</td></tr><tr><td>EG</td><td>12</td></tr><tr><td>FG</td><td>3</td></tr></table></td><td>B1
B1
B1</td><td>3.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>4(c)</td><td>Shortest path from A to G: ABCFG</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>4(d)</td><td>Prim’s algorithm gives: 25 + 10 + 7 + CE + EF + 3 = 80
Shortest path from A to F via E which is ABEF gives: 56 + EF = 67
EF = 11 and CE = 24</td><td>M1
M1
A1</td><td>2.1
3.1b
2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
</table>

Notes:

(a)

B1: cao

(b)

B1: AB, AC and AD correct

B1: Any other four arcs correct

B1: All correct

(c)

B1: cao

(d)

M1: Use given information regarding Prim's algorithm to set up an equation for CE and EF (using the weight of the correct arcs from (b) so AB + BC + CD + CE + EF + FG = 80)

M1: Use given information regarding shortest path from A to F via E (using the path ABEF) to get an equation for EF (AB + BE + EF = 67) – this mark can be awarded for correctly stating EF as 11

A1: cao for both CE and EF

Answer ALL questions. Write your answers in the answer book provided.

1. 55 44 34 59 28 37 41 52 33 42 47

The list of eleven numbers shown above is to be sorted into ascending order.

(a) Carry out a quick sort to produce the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

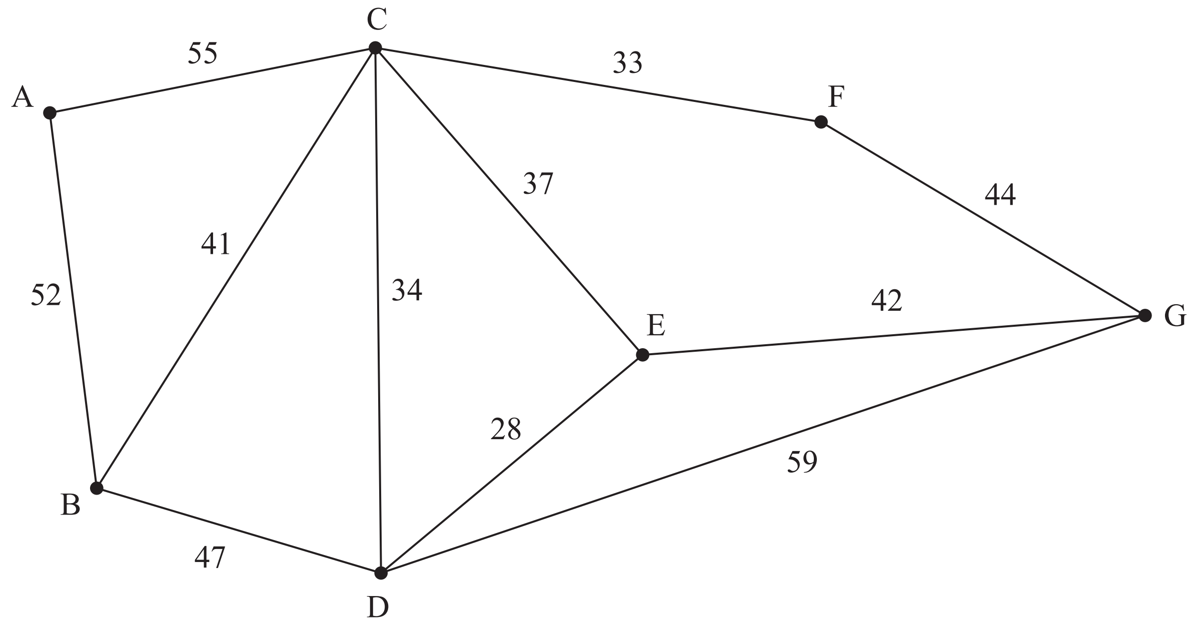


Figure 1

(b) Use Kruskal's algorithm to find the minimum spanning tree for the network in Figure 1. You should list the arcs in the order in which you consider them. For each arc, state whether or not you are adding it to your minimum spanning tree.

(3)

(c) (i) Draw the minimum spanning tree on Diagram 1 in the answer book.

(ii) State the total weight of the tree.

(2)

(Total for Question 1 is 9 marks)

1. 55 44 34 59 28 37 41 52 33 42 47

P 6 8 7 9 2 A 0 2 1 2

Question 1 continued

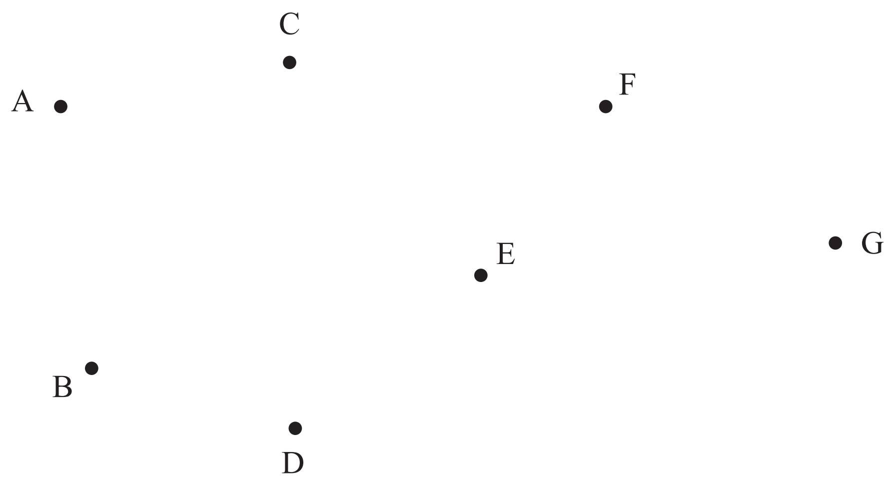
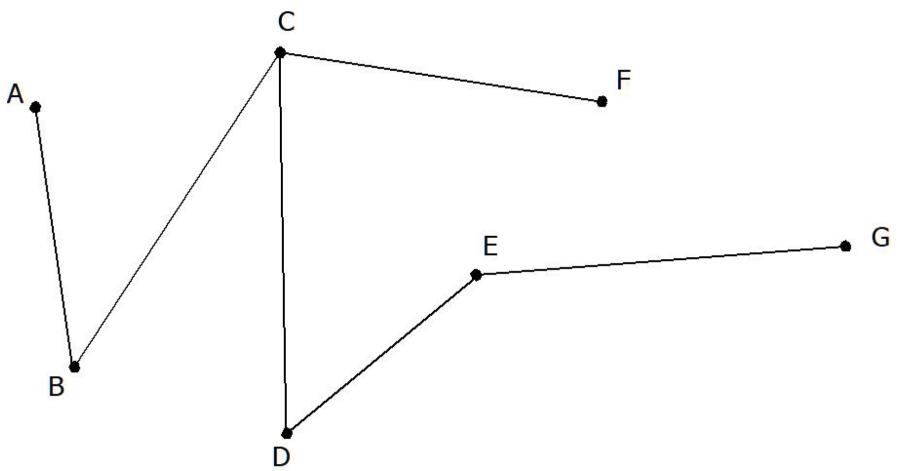


Diagram 1

(Total for Question 1 is 9 marks)

Qu	Scheme	Marks	AOs
1(a)	middle right Pivot(s) 55 44 34 59 28 37 41 52 33 42 47 37 34 28 33 **37** 55 44 59 41 52 42 47 28 41 **28** 34 33 **37** **41** 55 44 59 52 42 47 33 52 **28** **33** 34 **37** **41** 44 42 47 **52** 55 59 (34) 42 59 **28** **33** 34 **37** **41** **42** 44 47 **52** 55 **59** 47 (55) **28** **33** 34 **37** **41** **42** 44 **47** **52** 55 **59** (Sort complete)	M1 A1 A1ft A1	1.1b 1.1b 1.1b 1.1b
	middle left Pivot(s) 55 44 34 59 28 37 41 52 33 42 47 37 34 28 33 **37** 55 44 59 41 52 42 47 28 41 **28** 34 33 **37** **41** 55 44 59 52 42 47 34 59 **28** 33 **34** **37** **41** 55 44 52 42 47 **59** (33) 52 **28** 33 **34** **37** **41** 44 42 47 **52** 55 **59** 42 (55) **28** 33 **34** **37** **41** **42** 44 47 **52** 55 **59** 44 **28** 33 **34** **37** **41** **42** **44** 47 **52** 55 **59** (Sort Complete)		
		(4)	
(b)	DE, CF, CD, reject CE; BC, EG, reject FG; reject BD, AB, (reject AC, reject DG)	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(c)(i)		B1	2.2a
(ii)	(Total weight =) 230	B1	2.2a
		(2)	
			(9 marks)

Notes for Question 1
a1M1: Quick Sort, pivot, p, chosen (must be choosing middle right or middle left). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). **If choosing one pivot per iteration then M1 only.** This mark can be scored if one number only is either missing or incorrect or an additional number is added to the list. **a1A1:** First two passes correct (pivots for third pass need not be chosen) **a2A1ft:** Third and fourth passes correct (follow through from their second pass and choice of pivots) (pivot(s) for the fifth pass need not be chosen). After their second pass their list must contain either 10, 11 or 12 numbers (so allow one additional/missing number) **a3A1:** cso – if choosing middle right pivots then they must include a fifth pass and if choosing middle left then they must include a sixth pass **SC: If list is sorted into descending order, then award a maximum of M1A1A0A0 (so 2 marks) as in the scheme above even if the list is re-ordered after the sort is complete**
b1M1: Kruskal: first three arcs correctly chosen (DE, CF, CD), and arc CE rejected at the correct time. No follow through from an incorrect list. Condone list of weights for this mark only (28, 33, 34 and reject 37) **b1A1:** First five arcs correctly chosen (DE, CF, CD, BC, EG), and arc FG rejected at the correct time. Must state the arcs and not the corresponding weights for this mark **b2A1:** cso including all rejections correct and at the correct time. We do not need to see the explicit rejection of arcs AC and DG but if these are explicitly rejected then they must be in the correct order. Note that a list of all the arcs in the correct order followed by a list of the arcs in the MST can score full marks
ci1B1: Correct MST drawn **cii1B1:** Correct weight