Fd1Ch2 XMQs and MS

(Total: 27 marks)

1. FD1_Sample   Q2 . 7 marks - FD1ch2 Graphs and networks
2. FD1_2021   Q1 . 4 marks - FD1ch2 Graphs and networks
3. FD1(AS)_2018 Q2 . 10 marks - FD1ch2 Graphs and networks
4. FD1(AS)_2019 Q1 . 6 marks - FD1ch2 Graphs and networks

2.

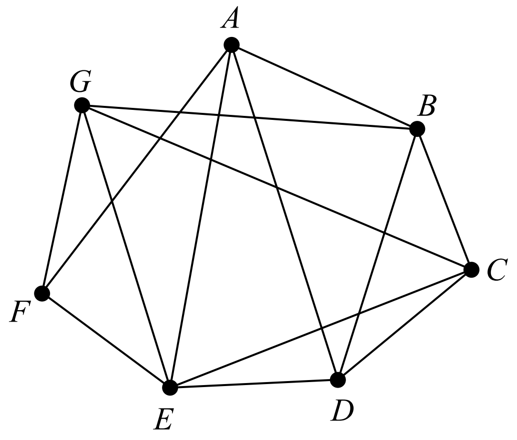


Figure 1

(a) Define what is meant by a **planar** graph.

(2)

(b) Starting at A, find a Hamiltonian cycle for the graph in Figure 1.

(1)

Arc AG is added to Figure 1 to create the graph shown in Figure 2.

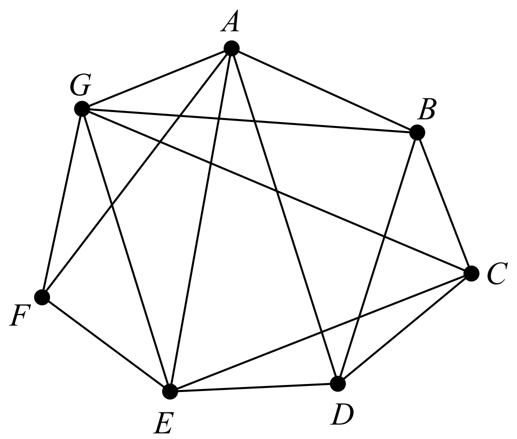


Figure 2

Taking ABCDEFGA as the Hamiltonian cycle,

(c) use the planarity algorithm to determine whether the graph shown in Figure 2 is planar. You must make your working clear and justify your answer.

(4)

(Total for Question 2 is 7 marks)

2.

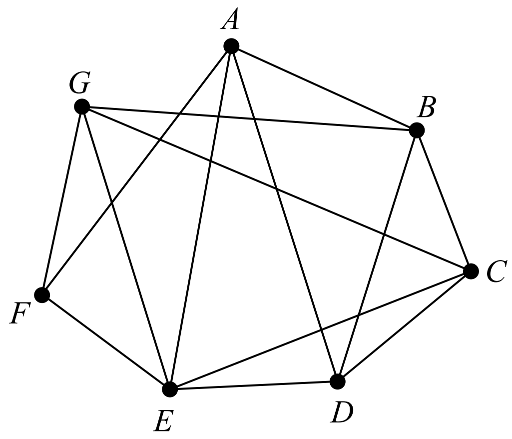


Figure 1

Question 2 continued

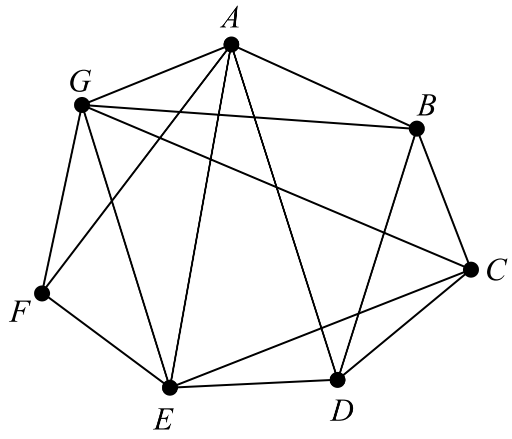


Figure 2

(Total for Question 2 is 7 marks)

Question	Scheme	Marks	AOs
2(a)	A planar graph is a graph that can be drawn so that…	B1	1.2
	… no arc meets another arc except at a vertex	B1	1.2
		(2)	
(b)	e.g. ABCDEGFA	B1	1.1b
		(1)	
(c)	Creates two lists of arcs	M1	2.1
	e.g. BG AD		
	CG BD	M1	1.1b
	EG AE		
	CE AF	A1	1.1b
	Since no arc appears in both lists, the graph is planar (**or** draws a planar version)	A1	2.4
		(4)	
			(7 marks)

Notes:

(a)
B1: A clear indication that a planar graph 'can be drawn' – allow this mark even if candidate implies that arcs can cross each other
B1: cao – no arc meets another arc except at a vertex – technical language must be correct

(b)
B1: Any correct Hamiltonian cycle (must start and finish at A) – must contain 8 vertices with every vertex appearing only once (except A)

(c)
M1: Creates two list of arcs (with at least three arcs in each list) which contain no common arcs
M1: Four arcs (in each list) and within each list there are no crossing arcs
A1: cao
A1: Correct reasoning that no arc appears in both lists + so the graph is therefore planar

1.

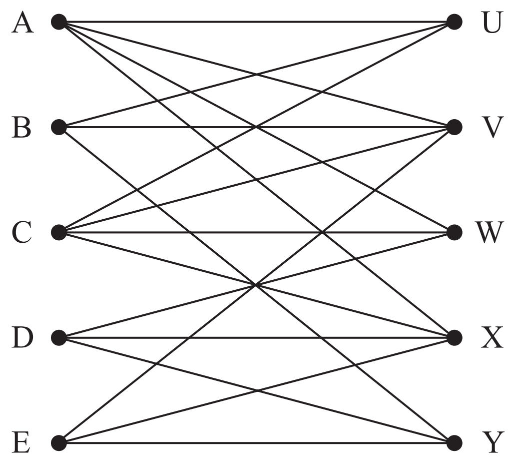


Figure 1

A Hamiltonian cycle for the graph in Figure 1 begins C, V, E, X, A, W, ….

(a) Complete the Hamiltonian cycle.

(1)

(b) Hence use the planarity algorithm to determine whether the graph shown in Figure 1 is planar. You must make your working clear and justify your answer.

(3)

(Total for Question 1 is 4 marks)

1.

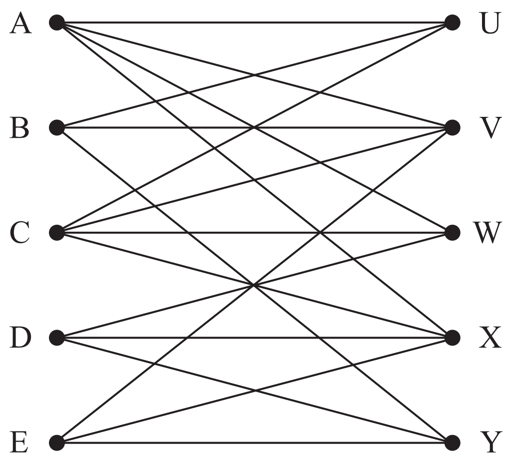
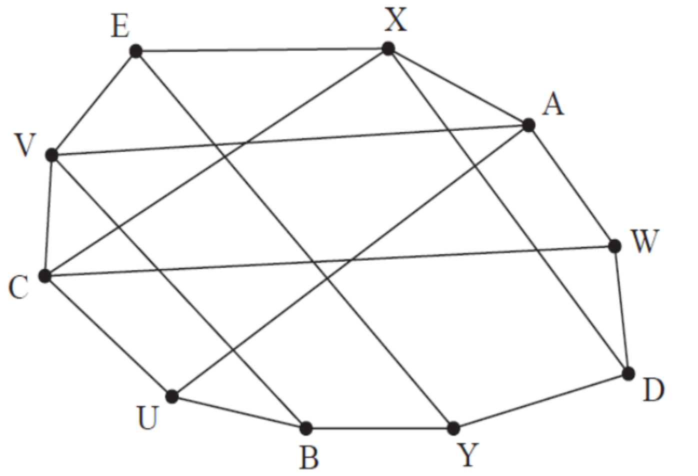


Figure 1

Question	Scheme	Marks	AOs
1(a)	…, D, Y, B, U, C	B1	1.1b
		(1)	
(b)	 Or list of arcs: AU, AV, BV, CW, CX, DX, EY	M1	2.1
	e.g., select (and label) AU (as I) and the arcs that intersect AU are BV, EY, CW and DX (so label them O so AU(I), AV, BV(O), CW(O), CX, DX(O), EY(O))	A1	1.1b
	Edges BV and CW intersect and so the graph is not planar (oe)	A1	2.2a
		(3)	
			(4 marks)

Notes:

(a)

B1: CAO (CVEXAWDYBUC) – must return to C

(b)

M1: Either draws their Hamiltonian cycle from part **(a)** as the edges of a polygon and shows the remaining arcs intersecting inside **OR** lists the arcs that are not part of the Hamiltonian cycle

A1: Selects any arc (that is not part of the Hamiltonian cycle) and lists/references the correct arcs that intersect with this selected arc – dependent on any correct Hamiltonian cycle and correct arcs that are not part of this cycle

A1: cao – based on their initial arc selection, states the two arcs that are unlabelled (or that are labelled with the same label) which intersect each other (e.g., if CX chosen as the initial selection then this arc intersects with BV, EY and AV but EY and AV intersect) **and** concludes that the graph is not planar. This mark is dependent on the correct Hamiltonian cycle stated in either **(a)** or **(b)**

2. A simply connected graph is a connected graph in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

(a) Given that a simply connected graph has exactly four vertices,

(i) write down the minimum number of arcs it can have,

(ii) write down the maximum number of arcs it can have.

(2)

(b) (i) Draw a simply connected graph that has exactly four vertices and exactly five arcs.

(ii) State, with justification, whether your graph is Eulerian, semi-Eulerian or neither.

(3)

(c) By considering the orders of the vertices, explain why there is only one simply connected graph with exactly four vertices and exactly five arcs.

(5)

(Total for Question 2 is 10 marks)

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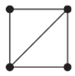
2.

Minimum number of arcs: ________________

Maximum number of arcs: ________________

(b) (i)

Question	Scheme	Marks	AOs
2(a)	Minimum number of arcs is 3	B1	2.2a
	Maximum number of arcs is 6	B1	2.2a
		(2)	
(b)	(i) e.g. 	B1	1.1b
	(ii) The graph has exactly two odd nodes and so the graph is semi-Eulerian	B1 DB1	2.4 2.2a
		(3)	
(c)	The sum of the orders of the vertices = 2(number of arcs) = 10	B1	1.2
	One possibility is that the orders are 1, 3, 3 and 3	M1	2.1
	In a simply connected graph with four vertices each of the vertices of order 3 must connect to the three other vertices therefore it is not possible to have three vertices all with order 3	A1	2.4
	The second possibility is that the orders are 2, 2, 3 and 3	M1	2.1
	There is only one way to make a graph with vertices of orders 2, 2, 3 and 3 as the two vertices of order 2 cannot be connected to each other (note that as the graph is connected no vertex can have order 0). There are no other possible graphs as the maximum order of a vertex is 3 (due to the condition that the graph must be simple)	A1	2.2a
		(5)	
			(10 marks)

Notes
(a) **B1:** Cao **B1:** Cao
(b)(i) **B1:** Cao oe (vertices must be clear) **(b)(ii)** **B1:** Explanation which consists of the graph having two odd nodes (or consistent explanation with their graph in (b)(i)) **DB1:** <u>Exactly</u> (or <u>only</u>) two odd nodes together with the deduction that therefore the graph is semi-Eulerian (from a correct graph only in (b)(i))
(c) **B1:** 10 seen – this mark can be implied if two or more lists of four numbers which sum to 10 are seen **M1:** States that the vertex orders could be 1, 3, 3, 3 or that no vertex can have an order greater than 3 **A1:** Convincing argument that 1, 3, 3, 3 is not possible **M1:** Considers the possibility of the orders being 2, 2, 3, 3 **A1:** Convincing argument that there is only one way of making a graph with vertex orders of 2, 2, 3, 3 (e.g. mention of the fact that the two vertices of order 2 cannot be connected to each other) For full marks in (c) – there must be some mention of the fact that there cannot be a vertex with order greater than 3 (withhold last A mark if this point is not considered)

1. (a) Draw the graph K_5

(1)

(b) (i) In the context of graph theory explain what is meant by 'semi-Eulerian'.

(ii) Draw two semi-Eulerian subgraphs of K_5, each having five vertices but with a different number of edges.

(3)

(c) Explain why a graph with exactly five vertices with vertex orders 1, 2, 2, 3 and 4 cannot be a tree.

(2)

(Total for Question 1 is 6 marks)

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2. The following algorithm produces a numerical approximation for the integral

$$I = \int_A^B x^4 \, dx$$

Step 1	Start
Step 2	Input the values of A, B and N
Step 3	Let H = (B – A) / N
Step 4	Let C = H / 2
Step 5	Let D = 0
Step 6	Let $D = D + A^4 + B^4$
Step 7	Let E = A
Step 8	Let E = E + H
Step 9	If E = B go to Step 12
Step 10	Let $D = D + 2 \times E^4$
Step 11	Go to Step 8
Step 12	Let $F = C \times D$
Step 13	Output F
Step 14	Stop

For the case when A = 1, B = 3 and N = 4,

(a) (i) complete the table in the answer book to show the results obtained at each step of the algorithm.

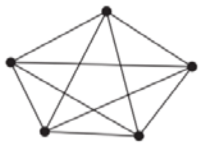
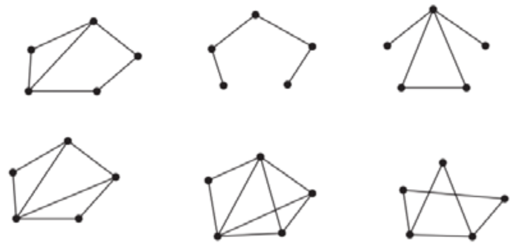
(ii) State the final output.

(4)

(b) Calculate, to 3 significant figures, the percentage error between the exact value of I and the value obtained from using the approximation to I in this case.

(3)

(Total for Question 2 is 7 marks)

Question	Scheme	Marks	AOs
1(a)		B1	1.2
		(1)	
(b)(i)	A semi-Eulerian graph contains <u>exactly</u> <u>two</u> <u>nodes</u> of <u>odd</u> <u>order</u> (and any number of nodes of even order)	B1	2.5
(b)(ii)	e.g. (two semi-Eulerian subgraphs of K_5 with a different number of edges) 	B1 B1	1.1b 1.1b
		(3)	
(c)	e.g. The graph with five vertices has $\frac{1+2+2+3+4}{2}=6$ arcs but a tree on five nodes would contain only 4 arcs	B1 B1dep	2.2a 2.4
		(2)	
			(6 marks)

Notes

(a)

B1: CAO (give bod for position of nodes)

(b)(i)

B1: CAO (accept ‘there are exactly two odd nodes’ but must contain exact oe (e.g. ‘only two odd nodes’ or ‘all but 2 nodes have an even order’ but not ‘the graph has two odd nodes’))

(b)(ii)

B1: One correct semi-Eulerian subgraph of K_5 with five nodes

B1: Two correct semi-Eulerian subgraphs of K_5 with five nodes – note that the graphs must have a different number of edges

(c)

B1: Deducing that the graph has 6 arcs **or** a tree on five nodes has 4 arcs **or** the node of order 4 must be connected to the other 4 nodes **or** an argument based on the sum of the orders of both the graph and the tree (but must relate the orders to the number of arcs and not the number of nodes) **or** the node with order 4 and one of the nodes of orders 2 or 3 would create a cycle **or** a tree must have two nodes of order 1

B1dep: Complete argument – graph has 6 arcs and the tree would only have 4 arcs **or** the sum of the orders is 12 compared to 8 for the tree **or** the node of order 4 must be connected to the other 4 nodes therefore all the other vertices would have to have order 1 **or** the graph has 6 arcs and therefore with 5 vertices there would have to be cycles **or** the node of order 4 is connected to the other 4 nodes and so together with the node of order 3 (or 2) a cycle would be formed **or** a tree must have at least two nodes of order 1 as otherwise a cycle would be formed

Note: no marks in **(c)** for attempts based only on examples of graphs drawn with the vertex orders as stated