Fd1Ch1 XMQs and MS

(Total: 63 marks)

1. FD1_Sample Q1 . 4 marks - FD1ch1 Algorithms
2. FD1_Specimen Q1 . 5 marks - FD1ch1 Algorithms
3. FD1_2019 Q1 . 8 marks - FD1ch1 Algorithms
4. FD1_2020 Q5 . 7 marks - FD1ch1 Algorithms
5. FD1_2021 Q5 . 10 marks - FD1ch1 Algorithms
6. FD1_2022 Q1 . 5 marks - FD1ch1 Algorithms
7. FD1(AS)_2019 Q2 . 7 marks - FD1ch1 Algorithms
8. FD1(AS)_2020 Q1 . 6 marks - FD1ch1 Algorithms
9. FD1(AS)_2021 Q1 . 11 marks - FD1ch1 Algorithms

Answer ALL questions. Write your answers in the answer book provided.

1. A list of *n* numbers needs to be sorted into descending order starting at the left-hand end of the list.

(a) Describe how to carry out the first pass of a bubble sort on the numbers in the list.

(2)

Bubble sort is a quadratic order algorithm.

A computer takes approximately 0.021 seconds to apply a bubble sort to a list of 2000 numbers.

(b) Estimate the time it would take the computer to apply a bubble sort to a list of 50 000 numbers. Make your method clear.

(2)

(Total for Question 1 is 4 marks)

Paper 3D/4D: Decision Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	In the first pass we compare the first value with the second value and we swap these values if the second is larger than the first	B1	2.4
	We then compare the value which is now second with the third value and swap if the third is larger than the second. We continue in this way until we reach the end of this list	B1	2.4
		(2)	
(b)	$t = 0.021\times\left(\frac{50000}{2000}\right)^2$	M1	1.1a
	t = 13.125(seconds)	A1	1.1b
		(2)	
			(4 marks)

Notes:

(a)

B1: Comparing first value with second value, swap if second is larger (oe) – in their reasoning it must be clear that the first value in the list is being compared with the second value in the list and swapping if the second is larger than the first

B1: Compare second with third, (third with fourth), and so on until the end of the list – must be clear in their reasoning that after the first comparison the second value in the list is compared with the third value and so on until the end of the list

(b)

M1: Correct method seen – accept 25 for 50000/2000

A1: cao

Answer ALL questions. Write your answers in the answer book provided.

1. 6 1 9 14 18 7 10 4 17 13

(a) The list of numbers shown above is to be sorted into descending order. Apply the quick sort algorithm to obtain the sorted list. You must make your pivots clear.

(3)

(b) Apply the first-fit decreasing bin packing algorithm to your ordered list to pack the numbers into bins of size 30

(2)

(Total for Question 1 is 5 marks)

1. 6 1 9 14 18 7 10 4 17 13

9FM0/3D: Decision Mathematics 01 Mark scheme

Question	Scheme	Marks	AOs
1(a)	6 1 9 14 18 $\boxed{7}$ 10 4 17 13 9 14 18 $\boxed{10}$ 17 13 $\underline{7}$ 6 $\boxed{1}$ 4	M1	1.1b
	14 18 $\boxed{17}$ 13 $\underline{10}$ 9 $\underline{7}$ 6 $\boxed{4}$ $\underline{1}$ 18 $\underline{17}$ 14 $\boxed{13}$ $\underline{10}$ 9 $\underline{7}$ 6 $\underline{4}$ $\underline{1}$	A1	1.1b
	18 $\underline{17}$ 14 $\underline{13}$ $\underline{10}$ 9 $\underline{7}$ 6 $\underline{4}$ $\underline{1}$	A1	1.1b
		(3)	
(b)	Bin 1: 18 10 1 Bin 2: 17 13 Bin 3: 14 9 7 Bin 4: 6 4	M1 A1	1.1b 1.1b
		(2)	
			(5 marks)

Notes:

(a)

M1: quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot).

A1: first two passes correct and correct pivots chosen for third pass

A1: cso (correct solution only – all previous marks in this part must have been awarded) – must include a fourth pass

(b)

M1: must be using 'sorted' list in descending order. First five items placed correctly and at least eight values placed in bins

A1: cso (so no additional/repeated values)

Answer ALL questions. Write your answers in the answer book provided.

1. 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

(2)

The list of numbers is now to be sorted into descending order.

(b) Perform a quick sort on the original list to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

For a list of n numbers, the quick sort algorithm has, on average, order $n \log n$.

Given that it takes 2.32 seconds to run the algorithm when $n = 450$

(c) calculate approximately how long it will take, to the nearest tenth of a second, to run the algorithm when $n = 11\,250$. You should make your method and working clear.

(2)

(Total for Question 1 is 8 marks)

1. 2.1 1.7 3.0 1.9 3.2 1.2 3.3 1.4 1.5 0.2

9FM0/3D: Decision Mathematics D1 Mark scheme

Question	Scheme	Marks	AOs
1(a)	Bin 1: $\underline{2.1}\quad\underline{1.7}\quad\underline{1.2}$ Bin 2: $\underline{3.0}\quad\underline{1.9}$ Bin 3: $\underline{3.2}\quad 1.4\quad 0.2$ Bin 4: $3.3\quad 1.5$	M1 A1	1.1b 1.1b
		(2)	
(b)	e.g. middle right		
	$2.1\ 1.7\ 3.0\ 1.9\ 3.2\ \boxed{1.2}\ 3.3\ 1.4\ 1.5\ 0.2$ Pivot: 1.2	M1	1.1b
	$2.1\ 1.7\ 3.0\ 1.9\ \boxed{3.2}\ 3.3\ 1.4\ 1.5\ \underline{1.2}\ 0.2$ Pivot(s): 3.2 (0.2)		
	$3.3\ \underline{3.2}\ 2.1\ 1.7\ 3.0\ \boxed{1.9}\ 1.4\ 1.5\ \underline{1.2}\ 0.2$ Pivot(s): (3.3) 1.9	A1	1.1b
	$3.3\ \underline{3.2}\ 2.1\ \boxed{3.0}\ \underline{1.9}\ 1.7\ \boxed{1.4}\ 1.5\ \underline{1.2}\ 0.2$ Pivots: 3.0 1.4	A1ft	1.1b
	$3.3\ \underline{3.2}\ \underline{3.0}\ 2.1\ \underline{1.9}\ 1.7\ \boxed{1.5}\ \underline{1.4}\ \underline{1.2}\ 0.2$ Pivot(s): (2.1) 1.5	A1	1.1b
	$3.3\ \underline{3.2}\ \underline{3.0}\ 2.1\ \underline{1.9}\ 1.7\ \underline{1.5}\ \underline{1.4}\ \underline{1.2}\ 0.2$		
		(4)	
(c)	$\frac{2.32(11\,250\log 11\,250)}{450\log 450}$	M1	1.1a
	=88.6 seconds	A1	1.1b
		(2)	

(8 marks)

Notes for Question 1

PLEASE NOTE NO MISREADS IN PARTS (a) and (b) – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE FOR ASCENDING ORDER IN (b)

(a) M1: First six items placed correctly and at least eight values placed in bins - condone cumulative totals for M1 only (the underlined values)

A1: CSO – all correct (so no additional/repeated values)

(b) M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then max of M1A1 only** – Bubble sort is not a MR and scores M0

A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct) – so they must be choosing (if middle right) a pivot value of 3.2 for the second pass or (if middle left) a pivot value of 1.9

A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark

A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including if middle right a fifth pass with the 1.5 used as a pivot or if middle left a fourth pass with the 1.7 used as a pivot

Sorting list into ascending order in (b)

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in **(b)**
- If the list is not reversed in **(b)** then remove the last two A marks earned in **(b)**. If the candidate says that the list needs reversing in **(b)** but does not actually show the reversed list in **(b)** then remove the last A mark earned
- **Note that if sorting into ascending order then a 'sort complete' statement is required – this could be shown by the final list being re-written or 'sorted' statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice) BEFORE list is reversed**

Middle left

2.1	1.7	3.0	1.9	$\boxed{3.2}$	1.2	3.3	1.4	1.5	0.2
3.3	$\underline{3.2}$	2.1	1.7	3.0	$\boxed{1.9}$	1.2	1.4	1.5	0.2
3.3	$\underline{3.2}$	$\boxed{2.1}$	3.0	$\underline{1.9}$	1.7	1.2	$\boxed{1.4}$	1.5	0.2
3.3	$\underline{3.2}$	3.0	$\underline{2.1}$	$\underline{1.9}$	$\boxed{1.7}$	1.5	$\underline{1.4}$	$\boxed{1.2}$	0.2
3.3	$\underline{3.2}$	3.0	$\underline{2.1}$	$\underline{1.9}$	$\underline{1.7}$	1.5	$\underline{1.4}$	$\underline{1.2}$	0.2

Middle right ascending (which requires a 'sort complete' statement – see above)

2.1	1.7	3.0	1.9	3.2	$\boxed{1.2}$	3.3	1.4	1.5	0.2
0.2	$\underline{1.2}$	2.1	1.7	3.0	$\boxed{1.9}$	$\boxed{3.2}$	3.3	1.4	1.5
0.2	$\underline{1.2}$	2.1	1.7	3.0	$\boxed{1.9}$	1.4	1.5	$\underline{3.2}$	3.3
0.2	$\underline{1.2}$	1.7	$\boxed{1.4}$	1.5	$\underline{1.9}$	2.1	$\boxed{3.0}$	$\underline{3.2}$	3.3
0.2	$\underline{1.2}$	$\underline{1.4}$	1.7	$\boxed{1.5}$	$\underline{1.9}$	2.1	$\underline{3.0}$	$\underline{3.2}$	3.3
0.2	$\underline{1.2}$	$\underline{1.4}$	$\underline{1.5}$	1.7	$\underline{1.9}$	2.1	$\underline{3.0}$	$\underline{3.2}$	3.3

Middle left ascending (which required a 'sort complete' statement – see above)

2.1	1.7	3.0	1.9	$\boxed{3.2}$	1.2	3.3	1.4	1.5	0.2
2.1	1.7	3.0	$\boxed{1.9}$	1.2	1.4	1.5	0.2	$\underline{3.2}$	3.3
1.7	1.2	$\boxed{1.4}$	$\boxed{1.5}$	0.2	$\underline{1.9}$	$\boxed{2.1}$	3.0	$\underline{3.2}$	3.3
$\boxed{1.2}$	0.2	$\underline{1.4}$	$\boxed{1.7}$	1.5	$\underline{1.9}$	$\underline{2.1}$	3.0	$\underline{3.2}$	3.3
0.2	$\underline{1.2}$	$\underline{1.4}$	1.5	$\underline{1.7}$	$\underline{1.9}$	$\underline{2.1}$	3.0	$\underline{3.2}$	3.3

(c) M1: Complete correct method – allow reciprocal – allow slips in values only e.g. 1250 for 11 250

A1: CAO – the exact value of 88.6 must be stated at some point (as question specifically asked for the answer to the nearest tenth of a second) – isw if 90 follows 88.6 seen. 90 with no working scores no marks. An answer of 88.6 with no working scores M1A0 – condone lack of units (but if present must be correct)

5. The nine distinct numbers in the following list are to be packed into bins of size 50

$$23 \qquad 17 \qquad 19 \qquad x \qquad 24 \qquad 8 \qquad 18 \qquad 10 \qquad 21$$

When the first-fit bin packing algorithm is applied to the numbers in the list it results in the following allocation.

Bin 1: 23 17 8

Bin 2: 19 x 10

Bin 3: 24 18

Bin 4: 21

(a) Explain why $13 < x < 21$

(3)

The same list of numbers is to be sorted into descending order. A bubble sort, starting at the left-hand end of the list, is to be used to obtain the sorted list. After the first complete pass the list is

$$23 \qquad 19 \qquad 17 \qquad 24 \qquad x \qquad 18 \qquad 10 \qquad 21 \qquad 8$$

(b) Using this information, write down the smallest interval that must contain x, giving your answer as an inequality.

(2)

When the first-fit decreasing bin packing algorithm is applied to the nine distinct numbers it results in the following allocation.

Bin 1: 24 23

Bin 2: 21 19 10

Bin 3: 18 17 x

Bin 4: 8

Given that only one of the bins is full and that x is an integer,

(c) calculate the value of x. You must give reasons for your answer.

(2)

(Total for Question 5 is 7 marks)

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5. 23 17 19 x 24 8 18 10 21

Question	Scheme	Marks	AOs
5(a)	If *x* has been placed in Bin 2 then $10 < x \le 31$ - this is because Bin 1 at this stage only contains 40 and before *x* had been placed in Bin 2 it only contained 19 As the 18 has been placed in Bin 3 this implies that $x > 50-(19+18)$ so $x > 13$ As the 10 has been placed in Bin 2 after the *x* then $x \le 50-(19+10)$ so $x \le 21$ However, the number are all distinct and therefore $13 < x < 21$	B1 B1 B1	3.1a 2.4 2.2a
		(3)	
(b)	$13 < x < 17$	B1 B1	2.2a 2.2a
		(2)	
(c)	If *x* has been placed in Bin 3 then this implies that $x \le 15$ So *x* is either 14 or 15 – but as Bin 2 is full $\Rightarrow x = 14$	M1 A1	2.4 2.2a
		(2)	
			(7 marks)

Notes for Question 5

(a)

B1: Correct reasoning of why $x > 13$ accept $x > 50-(19+18)$

B1: Correct explanation of why $x \le 21$ accept $x \le 50-(19+10)$

B1: Correct deduction that $13 < x < 21$ must mention that the numbers are distinct (oe)

(b)

B1: Use first complete pass to deduce that $x < 17$

B1: Correct lower bound of $x > 13$

(c)

M1: Using first-fit decreasing in an attempt to derive new upper bound for *x* (so either for stating both *x* could equal 14 or 15, $x \le 15$ or $x < 15$)

A1: Correct deduction that $x = 14$ (must clearly state or imply that Bin 2 is full)

5. 30 12 5 2 23 18 36 10 15 24

(a) The list of ten numbers above is to be sorted into descending order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

The ten numbers are to be packed into bins of size *n*, where *n* is a positive integer.

When the first-fit bin packing algorithm is applied to the original list of ten numbers, the following allocation is obtained.

Bin 1:	30	12	2
Bin 2:	5	23	10
Bin 3:	18	15	
Bin 4:	36		
Bin 5:	24		

(b) Explain why the value of the integer *n* must be either 44 or 45

(3)

(c) Use the first-fit decreasing bin packing algorithm to determine how the numbers can be packed into bins of size 45

(3)

(Total for Question 5 is 10 marks)

5. 30 12 5 2 23 18 36 10 15 24

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>**5(a)**</td><td>e.g., middle right pivot(s)
<table>
<tr><td>30</td><td>12</td><td>5</td><td>2</td><td>23</td><td>**18**</td><td>36</td><td>10</td><td>15</td><td>24</td></tr>
<tr><td>30</td><td>23</td><td>**36**</td><td>24</td><td>$\underline{18}$</td><td>12</td><td>5</td><td>**2**</td><td>10</td><td>15</td></tr>
<tr><td>$\underline{36}$</td><td>30</td><td>**23**</td><td>24</td><td>$\underline{18}$</td><td>12</td><td>5</td><td>**10**</td><td>15</td><td>$\underline{2}$</td></tr>
<tr><td>$\underline{36}$</td><td>30</td><td>**24**</td><td>$\underline{23}$</td><td>$\underline{18}$</td><td>12</td><td>**15**</td><td>$\underline{10}$</td><td>5</td><td>$\underline{2}$</td></tr>
<tr><td>$\underline{36}$</td><td>30</td><td>$\underline{24}$</td><td>$\underline{23}$</td><td>$\underline{18}$</td><td>$\underline{15}$</td><td>12</td><td>$\underline{10}$</td><td>5</td><td>$\underline{2}$</td></tr>
</table>
Therefore, the sort is complete</td><td>M1
A1
A1ft
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>**(4)**</td><td></td></tr>
<tr><td>**(b)**</td><td>The 5 has been put in Bin 2 rather than Bin 1 which indicates that the size of the bins is less than 30 + 12 + 5 = 47 and so therefore $(42 \le n) \le 46$</td><td>B1</td><td>3.1a</td></tr>
<tr><td></td><td>The fact that there is still room for the 2 in Bin 1 indicates that $n \ge 44$</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>The 18 cannot fit in Bin 2 and so therefore $n < 5 + 23 + 18 = 46$ which implies that n is either 44 or 45</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>**(3)**</td><td></td></tr>
<tr><td>**(c)**</td><td>Bin 1: $\boxed{36}$ 5 2
Bin 2: $\boxed{30}$ $\underline{15}$
Bin 3: $\boxed{24}$ $\boxed{18}$
Bin 4: $\boxed{23}$ $\underline{12}$ $\underline{10}$</td><td>$\boxed{\text{M1}}$
$\underline{\text{A1}}$
A1</td><td>1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>**(3)**</td><td></td></tr>
<tr><td colspan="4">**(10 marks)**</td></tr>
</table>

Notes:

(a)

M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then max of M1 only** – Bubble sort is not a MR and scores **M0**

A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct)

A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark

A1: cso (correct solution only – all previous marks in this part **must** have been awarded) including ‘sort complete’ statement

If list sorted into ascendng order then mark as a MR

(b)

B1: Correct reasoning why $n \leq 46$ or $n < 47$ or $n \leq 45$ or $n < 46$ - condone an argument which is mathematical in nature only e.g., $n < 30 + 12 + 5$ or $n < 5 + 23 + 18$

B1: Correct reasoning why $n \geq 44$ - condone 'largest bin filled is 44 so $n \geq 44$' or other similar mathematical argument. This mark can be awarded for an argument which is mathematical in nature only e.g., $n \geq 30 + 12 + 2$

B1: Completely correct reasoning for why n is either 44 or 45 only – this mark is dependent on the two previous **B** marks and must give sufficient detail (so an argument that contains no clear explanation of why certain inequalities hold cannot score this mark)

(c)

No MR or follow through in this part

M1: First five values placed correctly - with at least eight values placed (the squared values)

A1: First eight values places correctly with no additional/repeated values (the squared and underlined values)

A1: cso - no additional/repeated values

Middle left pivot(s)

30	12	5	2	**23**	18	36	10	15	24
30	**36**	24	$\underline{23}$	12	5	**2**	18	10	15
$\underline{36}$	**30**	24	$\underline{23}$	12	5	**18**	10	15	$\underline{2}$
$\underline{36}$	$\underline{30}$	24	$\underline{23}$	$\underline{18}$	12	**5**	10	15	$\underline{2}$
$\underline{36}$	$\underline{30}$	24	$\underline{23}$	$\underline{18}$	12	**10**	15	$\underline{5}$	$\underline{2}$
$\underline{36}$	$\underline{30}$	24	$\underline{23}$	$\underline{18}$	**12**	15	$\underline{10}$	$\underline{5}$	$\underline{2}$
$\underline{36}$	$\underline{30}$	24	$\underline{23}$	$\underline{18}$	15	$\underline{12}$	$\underline{10}$	$\underline{5}$	$\underline{2}$

Therefore, the sort is complete

1. A gardener needs the following lengths of string. All lengths are in metres.

4.3	6.1	5.1	4.7	2.5	5.9	3.4	1.7	2.1	0.4	1.3

She cuts the lengths from balls of string. Each ball contains 10 m of string.

(a) Calculate a lower bound for the number of balls of string the gardener needs.
You must make your method clear.

(2)

(b) Use the first-fit bin packing algorithm to determine how the lengths could be cut from the balls of string.

(3)

(Total for Question 1 is 5 marks)

1.

4.3 6.1 5.1 4.7 2.5 5.9 3.4 1.7 2.1 0.4 1.3

(Total for Question 1 is 5 marks)

Question	Scheme	Marks	AOs
1(a)	$\frac{4.3+6.1+...+1.3}{10}=\frac{37.5}{10}=3.75$ so lower bound is four (balls of string)	M1 A1	1.1b 2.2a
		(2)	
(b)	Bin 1: $\boxed{4.3}$ $\boxed{5.1}$ 0.4 Bin 2: $\boxed{6.1}$ **2.5** 1.3 Bin 3: $\boxed{4.7}$ **3.4** **1.7** Bin 4: **5.9** 2.1	$\boxed{\text{M1}}$ **A1** A1	1.1b 1.1b 1.1b
		(3)	
			(5 marks)

Notes for Question 1

a1M1: Attempt to find the lower bound $(37.5 \pm 6.1)/10$ (a value of 3.75 seen with no working can imply this mark)
a1A1: cso – a lower bound of 4 with either a correct calculation seen or 3.75 or 'total is 37.5 and if each ball contains 10 this gives a lower bound of 4'. An answer of 4 with no working (or from part **(b)**) scores M0A0. Any incorrect working loses this mark e.g. a correct calculation followed by an incorrect value followed by 4 is A0

b1M1: First four items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the boxed values)
b1A1: First eight items placed correctly (the boxed **and** bold values), and all eleven correct values only placed in bins (so no additional/repeated values)
b2A1: cso (no additional/repeated values)

Condone working in cm provided consistent
No MR in this question – mark according to the scheme

1. (a) Draw the graph K_5

(1)

(b) (i) In the context of graph theory explain what is meant by 'semi-Eulerian'.

(ii) Draw two semi-Eulerian subgraphs of K_5, each having five vertices but with a different number of edges.

(3)

(c) Explain why a graph with exactly five vertices with vertex orders 1, 2, 2, 3 and 4 cannot be a tree.

(2)

(Total for Question 1 is 6 marks)

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2. The following algorithm produces a numerical approximation for the integral

$$I = \int_{A}^{B} x^4 \, dx$$

```
Step 1     Start
Step 2     Input the values of A, B and N
Step 3     Let H = (B – A) / N
Step 4     Let C = H / 2
Step 5     Let D = 0
Step 6     Let D = D + $A^4$ + $B^4$
Step 7     Let E = A
Step 8     Let E = E + H
Step 9     If E = B go to Step 12
Step 10    Let D = D + 2 × $E^4$
Step 11    Go to Step 8
Step 12    Let F = C × D
Step 13    Output F
Step 14    Stop
```

For the case when A = 1, B = 3 and N = 4,

(a) (i) complete the table in the answer book to show the results obtained at each step of the algorithm.

(ii) State the final output.

(4)

(b) Calculate, to 3 significant figures, the percentage error between the exact value of I and the value obtained from using the approximation to I in this case.

(3)

(Total for Question 2 is 7 marks)

2.

You may not need to use all the rows in this table.

It may not be necessary to complete all boxes in each row.

A	B	N	H	C	D	E	F

Final output: ______________________________

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>2(a)</td><td>(i)
<table>
<tr><th>A</th><th>B</th><th>N</th><th>H</th><th>C</th><th>D</th><th>E</th><th>F</th></tr>
<tr><td>1</td><td>3</td><td>4</td><td>0.5</td><td>0.25</td><td>0</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>82</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td>1.5</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>92.125</td><td>2</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>124.125</td><td>2.5</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>202.25</td><td>3</td><td>50.5625</td></tr>
</table>
(ii) Final output = 50.5625</td><td>M1
A1
A1
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>$\int_1^3 x^4 \mathrm{d}x = 48.4$
$\left(\frac{50.5625-48.4}{48.4}\right)\times 100$
4.47%</td><td>B1
M1
A1ft</td><td>1.1b
1.1a
3.2b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(7 marks)</td></tr>
<tr><td colspan="4" align="center">Notes</td></tr>
</table>

(a)(i)

M1: At least three rows of cells completed (so at least two values of D and E given) with either a correct first row or 82 found for D – condone repeated values in all columns or a single value in each row

A1: CAO – the values in the second, third and fourth rows correct (so up to the 92.125 in column D and the 2 in column E) – accept exact equivalent fractions

A1: CAO – all values correct in columns A to E – accept exact equivalent fractions

(ii)

A1: CAO (output = 50.5625) (or equivalent e.g. $50\frac{9}{16}$) – allow if stated only in column F

(b)

B1: CAO (48.4)

M1: Correct method (including multiplying by 100) using candidate’s final output from (a)(ii) and their value for *I*

A1ft: Follow through their final output from (a)(ii) (for reference: 4.4679752…) must be using 48.4 - dependent on M mark in (a) and percentage error being < 10% (answer must be given to 3 significant figures)

1. 3.7 2.5 5.4 1.9 2.7 3.2 3.1 2.7 4.2 2.0

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 8.5

(3)

The first-fit bin packing algorithm is to be used to pack *n* numbers into bins. The number of comparisons is used to measure the order of the first-fit bin packing algorithm.

(b) By considering the worst case, determine the order of the first-fit bin packing algorithm in terms of *n*. You must make your method and working clear.

(3)

(Total for Question 1 is 6 marks)

1.

3.7 2.5 5.4 1.9 2.7 3.2 3.1 2.7 4.2 2.0

Question	Scheme	Marks	AOs
1(a)	Bin 1: $\boxed{3.7}$ $\boxed{2.5}$ $\boxed{1.9}$ Bin 2: $\boxed{5.4}$ $\underline{2.7}$ Bin 3: $\underline{3.2}$ $\underline{3.1}$ 2.0 Bin 4: $\underline{2.7}$ 4.2	$\boxed{\text{M1}}$ $\underline{\text{A1}}$ A1	1.1b 1.1b 1.1b
		(3)	
1(b)	In the worst case the second number must be compared with the first number so 1 comparison, then the third number must be compared with the first and second numbers so 2 comparisons… so, in total there are 1 + 2 + 3 + … + (n – 1) comparisons in total	M1	2.1
	$1+2+...+(n-1)=\frac{1}{2}(n-1)n$	A1	2.2a
	$\frac{1}{2}(n-1)n$ so quadratic order	B1	1.1b
		(3)	
			(6 marks)

Notes

(a)

M1: First four items placed correctly (the values in boxes) with at least eight values placed – allow cumulative totals for M1 only

A1: First eight items placed correctly (the values in boxes and underlined) – no repeated values

A1: CSO (so no repeated values)

(b)

M1: Considers the correct worst case and attempts to sum the total number of comparisons in the first (n – 1) comparisons – this mark can be implied by the correct summation

A1: Correct sum evaluation seen or implied from a correct simplified formula <u>together</u> with the correct method for determining the total number of comparisons in the worst case

For those candidates who simply state that the total number of comparisons is $\frac{1}{2}n(n-1)$ then M1A0

As a minimum for M1A1 accept (total comparisons =) $\sum_{r=1}^{n-1} r=\frac{1}{2}n(n-1)$ (or considers 1 + 2 + … + (n – 1) together with the correct expression for this sum)

B1: Or equivalent e.g. order n^2, O(n^2), etc. (this mark is independent of the previous M and A mark)

Write your answers in the answer book provided.

1. 3.5 6.3 2.9 5.4 3.1 2.8 3.7 1.7 4.1 3.3 2.2

The numbers listed above are to be sorted into descending order.

(a) (i) Perform **one** pass of a bubble sort, starting at the left-hand end of the list. You must write down the list that results at the end of this first pass.

(ii) Write down the number of comparisons and the number of swaps performed during this first pass.

(3)

After a second pass using this bubble sort, the updated list is

6.3 5.4 3.5 3.1 3.7 2.9 4.1 3.3 2.8 2.2 1.7

(b) Use a quick sort on this updated list to obtain the fully sorted list. You should show the result of each pass and identify your pivots clearly.

(3)

(c) Apply the first-fit decreasing bin packing algorithm to the fully sorted list to pack the numbers into bins of size 11.5

(3)

(d) Determine whether your answer to part (c) uses the minimum number of bins. You must justify your answer.

(2)

(Total for Question 1 is 11 marks)

Question	Scheme	Marks	AOs
1(a)	**(i)** 6.3 3.5 5.4 3.1 2.9 3.7 2.8 4.1 3.3 2.2 1.7	B1	1.1b
	(ii) Number of comparisons: 10 Number of swaps: 7	B1 B1	1.1b 1.1b
		(3)	
1(b)	Middle right pivot(s) $6.3\;\; 5.4\;\; 3.5\;\; 3.1\;\; 3.7\;\; \boxed{2.9}\;\; 4.1\;\; 3.3\;\; 2.8\;\; 2.2\;\; 1.7$ $6.3\;\; 5.4\;\; 3.5\;\; \boxed{3.1}\;\; 3.7\;\; 4.1\;\; 3.3\;\; \underline{2.9}\;\; 2.8\;\; \boxed{2.2}\;\; 1.7$ $6.3\;\; 5.4\;\; 3.5\;\; \boxed{3.7}\;\; 4.1\;\; 3.3\;\; \underline{3.1}\;\; \underline{2.9}\;\; 2.8\;\; \underline{2.2}\;\; 1.7$ $6.3\;\; \boxed{5.4}\;\; 4.1\;\; \underline{3.7}\;\; 3.5\;\; \boxed{3.3}\;\; \underline{3.1}\;\; \underline{2.9}\;\; 2.8\;\; \underline{2.2}\;\; 1.7$ $6.3\;\; \underline{5.4}\;\; 4.1\;\; \underline{3.7}\;\; 3.5\;\; \underline{3.3}\;\; \underline{3.1}\;\; \underline{2.9}\;\; 2.8\;\; \underline{2.2}\;\; 1.7$ Middle left pivot(s) $6.3\;\; 5.4\;\; 3.5\;\; 3.1\;\; \boxed{3.7}\;\; 2.9\;\; 4.1\;\; 3.3\;\; 2.8\;\; 2.2\;\; 1.7$ $6.3\;\; \boxed{5.4}\;\; 4.1\;\; \underline{3.7}\;\; 3.5\;\; 3.1\;\; 2.9\;\; \boxed{3.3}\;\; 2.8\;\; 2.2\;\; 1.7$ $6.3\;\; \underline{5.4}\;\; 4.1\;\; \underline{3.7}\;\; 3.5\;\; \underline{3.3}\;\; 3.1\;\; 2.9\;\; \boxed{2.8}\;\; 2.2\;\; 1.7$ $6.3\;\; \underline{5.4}\;\; 4.1\;\; \underline{3.7}\;\; 3.5\;\; \underline{3.3}\;\; \boxed{3.1}\;\; 2.9\;\; \underline{2.8}\;\; \boxed{2.2}\;\; 1.7$ $6.3\;\; \underline{5.4}\;\; 4.1\;\; \underline{3.7}\;\; 3.5\;\; \underline{3.3}\;\; \underline{3.1}\;\; 2.9\;\; \underline{2.8}\;\; \underline{2.2}\;\; 1.7$	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
1(c)	Bin 1: $\boxed{6.3}\;\; \boxed{4.1}$ Bin 2: $\boxed{5.4}\;\; \boxed{3.7}\;\; 2.2$ Bin 3: $\underline{3.5}\;\; \underline{3.3}\;\; \underline{3.1}$ Bin 4: $\underline{2.9}\;\; 2.8\;\; 1.7$	$\boxed{\text{M1}}$ $\underline{\text{A1}}$ A1	1.1b 1.1b 1.1b
		(3)	
1(d)	Lower bound for the number of bins required is given by $\frac{6.3+5.4+4.1+3.7+3.5+3.3+3.1+2.9+2.8+2.2+1.7}{11.5}$ $=\frac{39}{11.5}=3.391...$ therefore the minimum number of bins is 4 So yes, the answer to (c) does use the minimum number of bins.	M1 A1	1.1b 2.4
		(2)	
(11 marks)			

Notes:

(a)

(i) B1: cao for first pass of bubble sort

(ii) B1: correct deduction of the number of comparisons

B1: correct deduction of the number of swaps

(b)

M1: Quick sort – pivots, p, selected and first pass gives >p, p, <p. If only choosing 1 pivot per iteration M1 only. Using bubble sort in this part is M0. If sorting into ascending order M1 only.

A1: Second and third passes correct

A1: cso – including fourth pass

(c)

M1: First four items placed correctly (the values in boxes) with at least eight values placed – allow cumulative totals for M1 only

A1: First eight items placed correctly (the values in boxes and underlined) – all correct eleven values must have been placed in bins

A1: cso (therefore no repeated values)

(d)

M1: Lower bound calculation attempted ([32.7,45.3] / 11.5)

A1: cso correct calculation (rounded or truncated to at least one decimal place) and correct mathematical argument for why the answer to (c) does use the minimum number of bins. **This mark is dependent on the correct bin packing in (c)**