Cp2Ch6 XMQs and MS

(Total: 101 marks)

1. CP2_Sample Q5 . 10 marks - CP2ch2 Series
2. CP1_Specimen Q1 . 6 marks - CP2ch6 Hyperbolic functions
3. CP2_Specimen Q5 . 14 marks - CP2ch2 Series
4. CP2_2019 Q1 . 10 marks - CP2ch6 Hyperbolic functions
5. CP2_2019 Q3 . 6 marks - CP2ch6 Hyperbolic functions
6. CP2_2020 Q1 . 7 marks - CP2ch6 Hyperbolic functions
7. CP2_2020 Q5 . 10 marks - CP2ch6 Hyperbolic functions
8. CP1_2021 Q9 . 11 marks - CP2ch6 Hyperbolic functions
9. CP2_2021 Q7 . 9 marks - CP2ch6 Hyperbolic functions
10. CP1_2022 Q2 . 4 marks - CP2ch6 Hyperbolic functions
11. CP1_2022 Q9 . 6 marks - CP2ch3 Methods in calculus
12. CP2_2022 Q9 . 8 marks - CP2ch6 Hyperbolic functions

5.

$$y = \sin x \sinh x$$

(a) Show that $\frac{d^4y}{dx^4} = -4y$

(4)

(b) Hence find the first three non-zero terms of the Maclaurin series for y, giving each coefficient in its simplest form.

(4)

(c) Find an expression for the nth non-zero term of the Maclaurin series for y.

(2)

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Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx} = \sin x \cosh x + \cos x \sinh x$	M1	1.1a
	$\frac{d^2y}{dx^2} = \cos x \cosh x + \sin x \sinh x + \cos x \cosh x - \sin x \sinh x$ $(= 2\cos x \cosh x)$	M1	1.1b
	$\frac{d^3y}{dx^3} = 2\cos x \sinh x - 2\sin x \cosh x$	M1	1.1b
	$\frac{d^4y}{dx^4} = -4\sinh x \sin x = -4y$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{d^2y}{dx^2}\right)_0 = 2,\ \left(\frac{d^6y}{dx^6}\right)_0 = -8,\ \left(\frac{d^{10}y}{dx^{10}}\right)_0 = 32$	B1	3.1a
	Uses $y = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \ldots$ with their values	M1	1.1b
	$= \frac{x^2}{2!}(2) + \frac{x^6}{6!}(-8) + \frac{x^{10}}{10!}(32)$	A1	1.1b
	$= x^2 - \frac{x^6}{90} + \frac{x^{10}}{113400}$	A1	1.1b
		(4)	
(c)	$2(-4)^{n-1}\frac{x^{4n-2}}{(4n-2)!}$	M1 A1	3.1a 2.2a
		(2)	
			(10 marks)

Notes:

(a)

M1: Realises the need to use the product rule and attempts first derivative

M1: Realises the need to use a second application of the product rule and attempts the second derivative

M1: Correct method for the third derivative

A1*: Obtains the correct 4th derivative and links this back to y

(b)

B1: Makes the connection with part **(a)** to establish the general pattern of derivatives and finds the correct non-zero values

M1: Correct attempt at Maclaurin series with their values

A1: Correct expression un-simplified

A1: Correct expression and simplified

(c)

M1: Generalising, dealing with signs, powers and factorials

A1: Correct expression

Answer ALL questions. Write your answers in the spaces provided.

1. Solve the equation

$$6\cosh 2x + 4\sinh x = 7$$

giving your answers as exact logarithms.

(6)

9FM0/01: Core Pure Mathematics 01 Mark scheme

Question	Scheme	Marks	AOs
1	$6(1+2\sinh^2 x)+4\sinh x=7$ and rearranges to quadratic form OR substitutes correct exponential identifies and rearranges to quartic in e^x, $\cosh 2x=\frac{e^{2x}+e^{-2x}}{2}$ and $\sinh x=\frac{e^x-e^{-x}}{2}$ used.	M1	3.1a
	$12\sinh^2 x + 4\sinh x - 1 = 0$ OR $3e^{4x} + 2e^{3x} - 7e^{2x} - 2e^x + 3 = 0$	A1	1.1b
	$(6\sinh x-1)(2\sinh x+1)=0 \Rightarrow \sinh x = \ldots$ OR $(e^{2x}+e^x-1)(3e^{2x}-e^x-3)=0 \Rightarrow e^x = \ldots.$	M1	1.1b
	$\sinh x=\frac{1}{6}$ or $\sinh x=-\frac{1}{2}$ OR $e^x=\frac{-1\pm\sqrt{5}}{2}$ or $e^x=\frac{1\pm\sqrt{37}}{6}$	A1	1.1b
	$x=\ln\left(a+\sqrt{1+a^2}\right)$ where a is one of their sinh x values OR undoes exponentials using ln	M1	1.2
	$x=\ln\left(\frac{1+\sqrt{37}}{6}\right)$, $x=\ln\left(\frac{-1+\sqrt{5}}{2}\right)$	A1	1.1b
		(6)	
			(6 marks)

Notes:

M1: Identifies a correct approach to solving the problem, either through use of identity or definition of hyperbolics

A1: Reaches a correct quadratic in sinh x or a correct quartic in e^x.

M1: Solves their quadratic/quartic, may just see answers from calculator.

A1: Correct values for sinh$x$ or e^x found.

M1: Correct process of reaching x from their solutions in sinh x or $e^{x.}$

A1: Correct answers as exact simplified logarithms, and no others (in the alternative the negative exponential cases must have been rejected).

5.

$$y = \arctan(\sinh(x))$$

(a) Show that $\dfrac{d^3y}{dx^3} = \dfrac{dy}{dx} - 2\left(\dfrac{dy}{dx}\right)^3$

(7)

(b) Hence find $\dfrac{d^5y}{dx^5}$ in terms of $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\dfrac{d^3y}{dx^3}$

(4)

(c) Find the Maclaurin series for y, in ascending powers of x, up to and including the term in x^5

(3)

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Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx} = \frac{1}{\sinh^2 x + 1} \times ..$	M1	1.2
	$\frac{dy}{dx} = \frac{1}{\sinh^2 x + 1} \times \cosh x$	A1	1.1b
	$= \frac{\cosh x}{\cosh^2 x} = \text{sech}\, x$ or use of correct identity $\sinh^2 x + 1 = \cosh^2 x$ later in the proof.	B1	2.1
	E.g. $\frac{d^2y}{dx^2} = -\text{sech}\, x \tanh x$ or $\frac{d^2y}{dx^2} = -(\cosh x)^{-2} \times \sinh x$ or even $\frac{d^2y}{dx^2} = \frac{(\sinh x)(\sinh^2 x + 1) - (\cosh x)(2\sinh x \cosh x)}{\left(\sinh^2 x + 1\right)^2}$	M1	1.1b
	$\frac{d^3y}{dx^3} = -(-\text{sech}\, x \tanh x)(\tanh x) + (-\text{sech}\, x)(\text{sech}^2 x)$ (oe) or any valid attempt at the third derivative from their second derivative. E.g. $\frac{d^2y}{dx^2} = -\tanh x \frac{dy}{dx}$ then $\frac{d^3y}{dx^3} = -\text{sech}^2 x \frac{dy}{dx} - \tanh x \frac{d^2y}{dx^2}$	M1 A1	3.1a 1.1b
	E.g. $\frac{d^3y}{dx^3} = \text{sech}\, x \tanh^2 x - \text{sech}^3 x = \text{sech}\, x(1 - \text{sech}^2 x) - \text{sech}^3 x$ $= \text{sech}\, x - 2\text{sech}^3 x = \frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^3$ * **or** $\frac{d^3y}{dx^3} = -\text{sech}^2 x \frac{dy}{dx} - \tanh x \frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)^3 + \tanh^2 x \frac{dy}{dx}$ $= (1 - \text{sech}^2 x)\frac{dy}{dx} - \left(\frac{dy}{dx}\right)^3 = \frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^3$ *	A1*	2.1
		(7)	
(b)	$\frac{d^4y}{dx^4} = \frac{d^2y}{dx^2} - 6\left(\frac{dy}{dx}\right)^2 \times \frac{d^2y}{dx^2}$	M1 A1	1.1b 1.1b
	$\frac{d^5y}{dx^5} = \frac{d^3y}{dx^3} - 12\left(\frac{dy}{dx}\right) \times \left(\frac{d^2y}{dx^2}\right)^2 - 6\left(\frac{dy}{dx}\right)^2 \frac{d^3y}{dx^3}$	M1 A1	2.1 1.1b
		(4)	
(c)	At $x = 0, y = 0, y' = 1, y'' = 0, y^{(3)} = -1, y^{(4)} = 0$ and $y^{(5)} = -1 - 1\times0^2 - 6\times1^2\times(-1) = 5$	M1	1.1b
	So $y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y^{(3)}(0) + \frac{x^4}{4!}y^{(4)}(0) + \frac{x^5}{5!}y^{(5)}(0) + ...$ with their evaluated values.	M1	1.1b
	$y = x - \frac{x^3}{6} + \frac{x^5}{24} + ...$	A1	2.5
		(3)	
		(14 marks)	

Notes:

(a)

M1: Applies correct derivative of arctan(..)

A1: Correct derivative of *y*.

B1: Uses the identity $1+\sinh^2 x = \cosh^2 x$ to simplify the expression or anywhere later in their proof.

M1: Attempts the second derivative either using standard results, or quotient rule on unsimplified form.

M1: Simplifies and attempts the third derivative or attempts third derivative before simplifying. May even replace sech *x* with *y*' in the second derivative before using product rule. Many routes are possible at this stage (but must use product rule, chain rule, quotient rule as appropriate)

A1: A correct third derivative in any form.

A1*: Fully correct work leading to the given answer. Steps should be clear to reach the given answer.

(b)

M1: Differentiates again using the chain rule on the cube term. Constant multiple may be incorrect.

A1: Correct (unsimplified) fourth derivative.

M1: Completes the process of differentiation to reach the 5^{th} derivative.

A1: Correct answer, need not be simplified. Isw after a correct expression.

(c)

M1: Attempts the evaluation of all the derivatives at $x = 0$.

M1: Applies the Maclaurin formula with their values. Accept with 3! or 6 and with 5! or 120.

A1: Correct series, must start $y = ...$ or with $f(x) = ...$ only if this has been defined as being equal to *y* at some stage in their working.

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Prove that

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad -k < x < k$$

stating the value of the constant k.

(5)

(b) Hence, or otherwise, solve the equation

$$2x = \tanh\left(\ln\sqrt{2-3x}\right)$$

(5)

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Mark Scheme – Pre-Stand

Question	Scheme	Marks	AOs
1(a)	$y=\tanh^{-1}(x)\Rightarrow \tanh y=x\Rightarrow x=\frac{\sinh y}{\cosh y}=\frac{e^{y}-e^{-y}}{e^{y}+e^{-y}}$	M1 A1	2.1 1.1b
	Note that some candidates only have one variable and reach e.g. $x=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$ or $\tanh x=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$ Allow this to score M1A1		
	$x\left(e^{2y}+1\right)=e^{2y}-1\Rightarrow e^{2y}(1-x)=1+x\Rightarrow e^{2y}=\frac{1+x}{1-x}$	M1	1.1b
	$e^{2y}=\frac{1+x}{1-x}\Rightarrow 2y=\ln\left(\frac{1+x}{1-x}\right)\Rightarrow y=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ *	A1*	2.1
	Note that $e^{2y}(x-1)+x+1=0$ can be solved as a quadratic in e^{y}: $e^{y}=\frac{-\sqrt{0-4(x-1)(x+1)}}{2(x-1)}=\frac{-\sqrt{4(1-x)(x+1)}}{2(x-1)}=\frac{2\sqrt{(1-x)(x+1)}}{2(1-x)}$ $=\frac{\sqrt{(x+1)}}{\sqrt{(1-x)}}\Rightarrow y=\frac{1}{2}\ln\frac{(x+1)}{(1-x)}$ * Score **M1** for an attempt at the quadratic formula to make e^{y} the subject (condone ± √…) and **A1*** for a correct solution that rejects the positive root at some point and deals with the $(x-1)$ bracket correctly		
	$k = 1$ or $-1 < x < 1$	B1	1.1b
		(5)	
(a) Way 2	$\tanh^{-1}x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\Rightarrow x=\tanh\left(\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\right)=\frac{e^{\ln\frac{1+x}{1-x}}-1}{e^{\ln\frac{1+x}{1-x}}+1}$	M1 A1	2.1 1.1b
	$x=\frac{e^{\ln\frac{1+x}{1-x}}-1}{e^{\ln\frac{1+x}{1-x}}+1}=\frac{\frac{1+x}{1-x}-1}{\frac{1+x}{1-x}+1}=x$ Hence true, QED, tick etc.	M1 A1	1.1b 2.1
(b)	$2x=\tanh\left(\ln\sqrt{2-3x}\right)\Rightarrow \tanh^{-1}(2x)=\ln\sqrt{2-3x}$	M1	3.1a
	$\frac{1}{2}\ln\left(\frac{1+2x}{1-2x}\right)=\frac{1}{2}\ln(2-3x)\Rightarrow\frac{1+2x}{1-2x}=2-3x$	M1	2.1
	$6x^{2}-9x+1=0$	A1	1.1b
	$6x^{2}-9x+1=0\Rightarrow x=...$	M1	1.1b
	$x=\frac{9-\sqrt{57}}{12}$	A1	3.2a
		(5)	

	Alternative for first 2 marks of (b)		
	$2x=\tanh\left(\ln\sqrt{2-3x}\right)\Rightarrow 2x=\frac{e^{2\ln\sqrt{2-3x}}-1}{e^{2\ln\sqrt{2-3x}}+1}$	M1	3.1a
	$\Rightarrow\frac{2-3x-1}{2-3x+1}=2x$	M1	2.1
			(10 marks)

Notes

(a)

<u>If you come across any attempts to use calculus to prove the result – send to review</u>

M1: Begins the proof by expressing tanh in terms of exponentials and forms an equation in exponentials.

The exponential form can be any of $\frac{\left(e^{y}-e^{-y}\right)/2}{\left(e^{y}+e^{-y}\right)/2}, \frac{e^{y}-e^{-y}}{e^{y}+e^{-y}}, \frac{e^{2y}-1}{e^{2y}+1}$

Allow any variables to be used **but the final answer must be in terms of *x***. Allow alternative notation for $\tanh^{-1}x$ e.g. artanh, arctanh.

A1: Correct expression for "x" in terms of exponentials

M1: Full method to make $e^{2"y"}$ the subject of the formula. This must be correct algebra so allow sign errors only.

A1*: Completes the proof by using logs correctly and reaches the printed answer with no errors.

Allow e.g. $\frac{1}{2}\ln\left(\frac{x+1}{1-x}\right), \frac{1}{2}\ln\frac{x+1}{1-x}, \frac{1}{2}\ln\left|\frac{x+1}{1-x}\right|$. Need to see $\tanh^{-1}x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ as a conclusion but allow if the proof concludes that $y=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ with y defined as $\tanh^{-1}x$ earlier.

B1: Correct value for k or writes $-1 < x < 1$

Way 2

M1: Starts with result, takes tanh of both sides and expresses in terms of exponentials

A1: Correct expression

M1: Eliminates exponentials and logs and simplifies

A1: Correct result (i.e. $x = x$) with conclusion

B1: Correct value for k or writes $-1 < x < 1$

(b)

M1: Adopts a correct strategy by taking $\tanh^{-1}$ of both sides

M1: Makes the link with part (a) by replacing artanh($2x$) with $\frac{1}{2}\ln\left(\frac{1+2x}{1-2x}\right)$ and demonstrates the use of the power law of logs to obtain an equation with logs removed **correctly**.

A1: Obtains the correct 3TQ

M1: Solves their 3TQ using a correct method (see General Guidance – if no working is shown (calculator) and the roots are correct for their quadratic, allow M1)

A1: Correct value with the other solution rejected (accept rejection by omission) so $x=\frac{9\pm\sqrt{57}}{12}$ scores A0 unless the positive root is rejected

Alternative for first 2 marks of (b)

M1: Adopts a correct strategy by expressing tanh in terms of exponentials

M1: Demonstrates the use of the power law of logs to obtain an equation with logs removed correctly

3.

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)\,dx = A\sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence find, in exact form in terms of natural logarithms, the mean value of $f(x)$ over the interval [0, 3].

(2)

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Question	Scheme	Marks	AOs
3(a) Way 1	$x = \frac{3}{2}\sinh u$	B1	2.1
	$\int \frac{dx}{\sqrt{4x^2+9}} = \int \frac{1}{\sqrt{4\left(\frac{9}{4}\right)\sinh^2 u + 9}} \times \frac{3}{2}\cosh u \, du$	M1	3.1a
	$= \int \frac{1}{2} \, du$	A1	1.1b
	$= \int \frac{1}{2} \, du = \frac{1}{2}u = \frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right) + c$	A1	1.1b
		(4)	
(a) Way 2	$x = \frac{3}{2}\tan u$	B1	2.1
	$\int \frac{dx}{\sqrt{4x^2+9}} = \int \frac{1}{\sqrt{4\left(\frac{9}{4}\right)\tan^2 u + 9}} \times \frac{3}{2}\sec^2 u \, du$	M1	3.1a
	$= \int \frac{1}{2}\sec u \, du$	A1	1.1b
	$= \frac{1}{2}\ln\left(\sec u + \tan u\right) = \frac{1}{2}\ln\left(\frac{2x}{3} + \sqrt{1 + \left(\frac{2x}{3}\right)^2}\right)$ $u = \frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right) + c$	A1	1.1b
(a) Way 3	$x = \frac{1}{2}u$ or $x = ku$ where $k > 0$ $k \neq 1$	B1	2.1
	$\int \frac{dx}{\sqrt{4x^2+9}} = \int \frac{1}{\sqrt{4\left(\frac{1}{4}\right)u^2 + 9}} \times \frac{1}{2} du$	M1	3.1a
	$= \frac{1}{2}\int \frac{1}{\sqrt{u^2+9}} du$ $\left(\text{or } \frac{1}{2}\int \frac{1}{\sqrt{u^2 + \frac{9}{4k^2}}} du \text{ for } x = ku\right)$	A1	1.1b
	$= \frac{1}{2}\sinh^{-1}\frac{u}{3} = \frac{1}{2}\sinh^{-1}\frac{2x}{3} + c$	A1	1.1b
(b)	Mean value = $\frac{1}{3(-0)}\left[\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)\right]_0^3 = \frac{1}{3} \times \frac{1}{2}\sinh^{-1}\left(\frac{2\times 3}{3}\right)(-0)$	M1	2.1
	$= \frac{1}{6}\ln\left(2+\sqrt{5}\right)$ **(Brackets are required)**	A1ft	1.1b
		(2)	
			(6 marks)

Notes
(a) B1: Selects an appropriate substitution leading to an integrable form M1: Demonstrates a fully correct method for the substitution that includes substituting into the function and dealing with the "dx". The substitution being substituted does not need to be "correct" for this mark but the substitution must be an attempt at $\int \frac{1}{\sqrt{4\left[\mathrm{f}(u)\right]^2+9}} \times \mathrm{f}'(u)\ \mathrm{d}u$ with the $\mathrm{f}'(u)$ correct for their substitution. E.g. if $x = \frac{1}{2}u$ is used, must see $\mathrm{d}x = \frac{1}{2}\mathrm{d}u$ not 2du. A1: Correct simplified integral in terms of u from correct work and from a correct substitution A1: Correct answer including "+ c". Allow arcsinh or arsinh for $\sinh^{-1}$ from correct work and from a correct substitution (b) M1: Correctly applies the method for the mean value for their integration which must be of the form specified in part (a) and substitutes the limits 0 and 3 but condone omission of 0 A1: Correct exact answer (follow through their A and B). **Brackets are required if appropriate.**

1. The curve C has equation

$$y = 31\sinh x - 2\sinh 2x \qquad x \in \mathbb{R}$$

Determine, in terms of natural logarithms, the exact x coordinates of the stationary points of C.

(7)

Question	Scheme	Marks	AOs
1	$\dfrac{dy}{dx}=31\cosh x-4\cosh 2x$	B1	1.1b
	$\dfrac{dy}{dx}=31\cosh x-4\left(2\cosh^2 x-1\right)$	M1	3.1a
	$8\cosh^2 x-31\cosh x-4=0$	A1	1.1b
	$\left(8\cosh x+1\right)\left(\cosh x-4\right)=0\Rightarrow \cosh=...$	M1	1.1b
	$\cosh x=4,\left(-\dfrac{1}{8}\right)$	A1	1.1b
	$\cosh x=\alpha\Rightarrow x=\ln\left(\alpha+\sqrt{\alpha^2-1}\right)$ or $\ln\left(\alpha+\sqrt{\alpha^2-1}\right)$ or $-\ln\left(\alpha+\sqrt{\alpha^2-1}\right)$ or $\ln\left(\alpha-\sqrt{\alpha^2-1}\right)$ or $\dfrac{e^x+e^{-x}}{2}=4\Rightarrow e^{2x}-8e^x+7=0\Rightarrow e^x=...\Rightarrow x=\ln(...)$	M1	1.2
	$\pm\ln\left(4+\sqrt{15}\right)$ or $\ln\left(4\pm\sqrt{15}\right)$	A1	2.2a
		(7)	
	Alternative $\dfrac{dy}{dx}=31\cosh x-4\cosh 2x$ or $31\left(\dfrac{e^x+e^{-x}}{2}\right)-4\left(\dfrac{e^{2x}+e^{-2x}}{2}\right)$	B1	1.1b
	Using $\cosh x=\left(\dfrac{e^x+e^{-x}}{2}\right)$ and $\sinh x=\left(\dfrac{e^x-e^{-x}}{2}\right)$ as required $\Rightarrow 31\left(\dfrac{e^x+e^{-x}}{2}\right)-4\left(\dfrac{e^{2x}+e^{-2x}}{2}\right)=0$ leading to $4e^{4x}-31e^{3x}-31e^x+4=0$ o.e.	M1 A1	3.1a 1.1b
	Solves $4e^{4x}-31e^{3x}-31e^x+4=0$ $\Rightarrow e^x=...$	M1	1.1b
	$e^x=4\pm\sqrt{15}$ or awrt 7.87, 0.13	A1	1.1b
	$x=\ln(b)$ where b is a real exact value	M1	1.2
	$\ln\left(4\pm\sqrt{15}\right)$	A1	2.2a
		(7)	
			(7 marks)

Notes

B1: Correct differentiation
M1: Identifies a correct approach by using a correct identity to make progress to obtain a quadratic in cosh *x*
A1: Correct 3 term quadratic obtained
M1: Solves their 3TQ
A1: Correct values (may only see 4 here)
M1: Correct process to reach at least one value for *x* from their cosh *x*

A1: Deduces the correct 2 values with no incorrect values or work involving $\cosh x = -\frac{1}{8}$
Alternative B1: Correct differentiation M1: Using the exponential form for cosh *x*, and sinh *x* if required, and forms a quartic equation for e^x with all terms simplified and all on one side A1: Correct quartic equation for e^x M1: Solves their quartic equation in e^x A1: Correct values to two decimal places or exact values M1: $x = \ln(b)$ where b is a real exact value A1: Deduces the correct 2 values only

5. (a)

$$y = \tan^{-1} x$$

Assuming the derivative of $\tan x$, prove that

$$\frac{dy}{dx} = \frac{1}{1 + x^2}$$

(3)

$$f(x) = x \tan^{-1} 4x$$

(b) Show that

$$\int f(x)\,dx = Ax^2 \tan^{-1} 4x + Bx + C \tan^{-1} 4x + k$$

where k is an arbitrary constant and A, B and C are constants to be determined.

(5)

(c) Hence find, in exact form, the mean value of $f(x)$ over the interval $\left[0, \frac{\sqrt{3}}{4}\right]$

(2)

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Question	Scheme	Marks	AOs
5(a)	$y=\tan^{-1}x \Rightarrow \tan y = x \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\sec^2 y$ $y=\tan^{-1}x \Rightarrow \tan y = x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}\sec^2 y = 1$	M1	3.1a
	$\frac{\mathrm{d}x}{\mathrm{d}y}=1+\tan^2 y$ or $\frac{\mathrm{d}y}{\mathrm{d}x}\left(1+\tan^2 y\right)=1$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+\tan^2 y}=\frac{1}{1+x^2}$ *	A1*	2.1
		(3)	
(b)	$\frac{\mathrm{d}\left(\tan^{-1}4x\right)}{\mathrm{d}x}=\frac{4}{1+16x^2}$	B1	1.1b
	$\int x\tan^{-1}4x\,\mathrm{d}x=\alpha x^2\tan^{-1}4x-\int \alpha x^2\times\left(\frac{4}{1+16x^2}\right)\mathrm{d}x$	M1	2.1
	$\int x\tan^{-1}4x\,\mathrm{d}x=\frac{x^2}{2}\tan^{-1}4x-\int\frac{x^2}{2}\times\frac{4}{1+16x^2}\,\mathrm{d}x$	A1	1.1b
	$=\ldots-\frac{1}{8}\int\frac{16x^2+1-1}{1+16x^2}\,\mathrm{d}x=\ldots-\frac{1}{8}\int\left(1-\frac{1}{1+16x^2}\right)\mathrm{d}x$ or let $4x=\tan u \Rightarrow \frac{1}{8}\int\frac{\tan^2 u}{1+\tan^2 u}\times\frac{1}{4}\sec^2 u\,\mathrm{d}u$ $\Rightarrow \frac{1}{32}\int\tan^2 u\,\mathrm{d}u=\frac{1}{32}\int\sec^2 u-u\,\mathrm{d}u$	M1	3.1a
	$=\frac{x^2}{2}\tan^{-1}4x-\frac{1}{8}x+\frac{1}{32}\tan^{-1}4x+k$	A1	2.1
		(5)	
(c)	Mean value $=\left(\frac{1}{\frac{\sqrt{3}}{4}-0}\right)\left[\frac{x^2}{2}\tan^{-1}4x-\frac{1}{8}x+\frac{1}{32}\tan^{-1}4x\right]_0^{\frac{\sqrt{3}}{4}}$ $=\frac{4}{\sqrt{3}}\left(\left(\frac{3}{32}\times\frac{\pi}{3}-\frac{1}{8}\times\frac{\sqrt{3}}{4}+\frac{1}{32}\times\frac{\pi}{3}\right)-0\right)$	M1	2.1
	$=\frac{\sqrt{3}}{72}\left(4\pi-3\sqrt{3}\right)$ or $\frac{\sqrt{3}}{18}\pi-\frac{1}{8}$ oe	A1	1.1b
		(2)	
			(10 marks)

Notes

(a)
M1: Makes progress in establishing the derivative by taking the tan of both sides and differentiating with respect to *y* or implicitly with respect to *x*
M1: Use of the correct identity
A1*: Fully correct proof

(b)
B1: Correct derivative
M1: Uses integration by parts in the correct direction
A1: Correct expression
M1: Adopts a correct strategy for the integration by splitting into two fractions or using a substitution of $4x = \tan u$ to get to an integrable form
A1: Correct answer
(c)
M1: Correctly applies the method for the mean value for their integration. The limit of zero can be implied if it comes to 0.
A1: Correct exact answer. Allow exact equivalents e.g. $\frac{4\pi\sqrt{3}-9}{72}$, $\frac{\pi\sqrt{3}}{18}-\frac{1}{8}$

9. (a) Use a hyperbolic substitution and calculus to show that

$$\int \frac{x^2}{\sqrt{x^2 - 1}}\, dx = \frac{1}{2}\left[x\sqrt{x^2 - 1} + \operatorname{arcosh} x\right] + k$$

where k is an arbitrary constant.

(6)

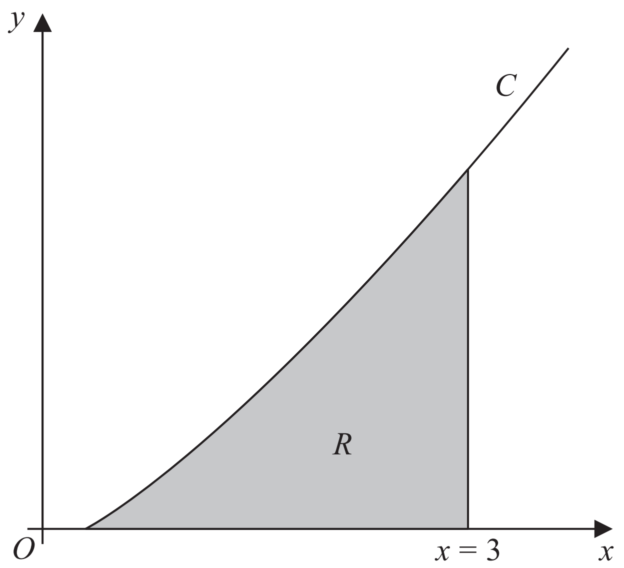

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = \frac{4}{15} x \operatorname{arcosh} x \qquad x \geqslant 1$$

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Using algebraic integration and the result from part (a), show that the area of R is given by

$$\frac{1}{15}\left[17\ln\left(3 + 2\sqrt{2}\right) - 6\sqrt{2}\right]$$

(5)

Question	Scheme	Marks	AOs
9(a)	$\int \frac{x^2}{\sqrt{x^2-1}}\,dx \to \int f(u)\,du$ Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of u only, including replacing the dx	M1	3.1a
	$\int \frac{\cosh^2 u}{\sqrt{\cosh^2 u - 1}} \sinh u\,(du)$	A1	1.1b
	Uses correct identities $\cosh^2 u - 1 = \sinh^2 u$ and $\cosh 2u = 2\cosh^2 u - 1$ to achieve an integral of the form $A\int (\cosh 2u \pm 1)\,du \qquad A > 0$	M1	3.1a
	Integrates to achieve $A\left(\pm\frac{1}{2}\sinh 2u \pm u\right)(+c) \qquad A > 0$	M1	1.1b
	Uses the identity $\sinh 2u = 2\sinh u\cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ $\to \sinh 2u = 2x\sqrt{x^2-1}$	M1	2.1
	$\frac{1}{2}\left[x\sqrt{x^2-1} + \text{ar}\cosh x\right] + k$ * cso	A1*	1.1b
		(6)	
(b)	Uses integration by parts the correct way around to achieve $\int \frac{4}{15} x \text{ arcosh}x\text{d}x = Px^2\text{arcosh}x - Q\int \frac{x^2}{\sqrt{x^2-1}}\,dx$	M1	2.1
	$= \frac{4}{15}\left(\frac{1}{2}x^2\text{arcosh}x - \frac{1}{2}\int \frac{x^2}{\sqrt{x^2-1}}\,dx\right)$	A1	1.1b
	$= \frac{4}{15}\left(\frac{1}{2}x^2\text{arcosh}x - \frac{1}{2}\left(\frac{1}{2}\left[x\sqrt{x^2-1} + \text{arcosh}x\right]\right)\right)$	B1ft	2.2a
	Uses the limits x =1 and x = 3 the correct way around and subtracts $= \frac{4}{15}\left(\frac{1}{2}(3)^2\text{ arcosh}3 - \frac{1}{2}\left(\frac{1}{2}\left[3\sqrt{(3)^2-1} + \text{arcosh}3\right]\right)\right) - \frac{4}{15}(0)$	dM1	1.1b
	$= \frac{4}{15}\left(\frac{9}{2}\ln\left(3+\sqrt{8}\right) - \frac{3\sqrt{8}}{4} - \frac{1}{4}\ln\left(3+\sqrt{8}\right)\right)$ $= \frac{1}{15}\left[17\ln\left(3+2\sqrt{2}\right) - 6\sqrt{2}\right]$*	A1*	1.1b
		(5)	
		(11 marks)	

Notes:

(a)

M1: Uses the substitution $x = \cosh u$ fully to achieve an integral in terms of *u* only. Must have replaced the d*x* but allow if the d*u* is missing.

A1: Correct integral in terms of *u*. (Allow if the d*u* is missing.)

M1: Uses correct identities $\cosh^2 u - 1 = \sinh^2 u$ and $\cosh 2u = 2\cosh^2 u - 1$ to achieve an integrand of the required form

M1: Integrates to achieve the correct form, may be sign errors.

M1: Uses the identities $\sinh 2u = 2\sinh u \cosh u$ and $\cosh^2 u - 1 = \sinh^2 u$ to attempt to find $\sinh 2u$ in terms of *x*. If using exponentials there must be a full and complete method to attempt the correct form.

A1*: Achieves the printed answer with no errors seen, cso

NB attempts at integration by parts are not likely to make progress – to do so would need to split the integrand as $x\frac{x}{\sqrt{x^2-1}}$. If you see any attempts that you feel merit credit, use review.

(b)

M1: Uses integration by parts the correct way around to achieve the required form.

A1: Correct integration by parts

B1ft: Deduces the integral by using the result from part (a). Follow through on their '*uv*'

dM1: Dependent on previous method mark. Uses the limits *x* = 1 and *x* = 3 the correct way around and subtracts

A1*cso: Achieves the printed answer with at least one intermediate step showing the evaluation of the arcosh 3, and no errors seen.

7. Solutions based entirely on graphical or numerical methods are not acceptable.

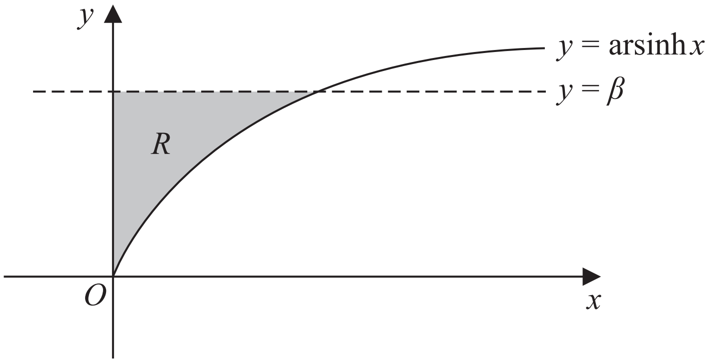

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \operatorname{arsinh} x \qquad x \geqslant 0$$

and the straight line with equation $y = \beta$

The line and the curve intersect at the point with coordinates (α, β)

Given that $\beta = \frac{1}{2}\ln 3$

(a) show that $\alpha = \frac{1}{\sqrt{3}}$

(3)

The finite region R, shown shaded in Figure 1, is bounded by the curve with equation $y = \operatorname{arsinh} x$, the y-axis and the line with equation $y = \beta$

The region R is rotated through 2π radians about the y-axis.

(b) Use calculus to find the exact value of the volume of the solid generated.

(6)

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Question	Scheme		Marks	AOs
7(a)	Using $\text{arsinh}\,\alpha = \frac{1}{2}\ln 3$ $\alpha = \dfrac{e^{\frac{1}{2}\ln 3} - e^{-\frac{1}{2}\ln 3}}{2}$	$\ln\left(\alpha + \sqrt{\alpha^2+1}\right) = \frac{1}{2}\ln 3$	B1	1.2
	$\alpha = \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{2} \Rightarrow \alpha = \ldots$	$\alpha + \sqrt{\alpha^2+1} = \sqrt{3}$ $\sqrt{\alpha^2+1} = \sqrt{3} - \alpha$ $\alpha^2 + 1 = 3 - 2\sqrt{3}\alpha + \alpha^2 \Rightarrow \alpha = \ldots$	M1	1.1b
	$\alpha = \dfrac{\sqrt{3}}{3}$ or $\dfrac{1}{\sqrt{3}}$		A1	2.2a
			(3)	
(b)	Volume = $\pi\int_0^{\frac{1}{2}\ln 3} \sinh^2 y\,dy$		B1	2.5
	$\{\pi\}\int\left(\dfrac{e^y - e^{-y}}{2}\right)^2 dy = \{\pi\}\int\left(\dfrac{e^{2y} - 2 + e^{-2y}}{4}\right)dy$ or $\{\pi\}\int \frac{1}{2}\cosh 2y - \frac{1}{2}\,dy$		M1	3.1a
	$\frac{1}{4}\left(\frac{1}{2}e^{2y} - 2y - \frac{1}{2}e^{-2y}\right)$ or $\frac{1}{4}\sinh 2y - \frac{1}{2}y$		dM1 A1	1.1b 1.1b
	Use limits $y = 0$ and $y = \frac{1}{2}\ln 3$ and subtracts the correct way round		M1	1.1b
	$\frac{\pi}{4}\left(\frac{4}{3} - \ln 3\right)$ or exact equivalent		A1	1.1b
			(6)	
			(9 marks)	

Notes:

(a)

B1: Recalls the definition for $\sinh\left(\frac{1}{2}\ln 3\right)$ or forms an equation for arcsinh x

M1: Uses logarithms to find a value for α or forms and solves a correct equation without log

A1: Deduces the correct exact value for α

Note using the result

$$\ln\left(\frac{1}{\sqrt{3}} + \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}\right) = \ln\left(\frac{1}{\sqrt{3}} + \sqrt{\frac{4}{3}}\right) = \ln\sqrt{3} = \frac{1}{2}\ln 3 \text{ therefore } \text{arsinh}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{2}\ln 3$$

B1 for substituting in α into arcsinh*x*, M1 for rearranging to show $\frac{1}{2}\ln 3$, A1 for conclusion
(b) **B1:** Correct expression for the volume $\pi\int_0^{\frac{1}{2}\ln 3}\sinh^2 y\,\mathrm{d}y$ requires integration signs, d*y* and correct limits. **M1:** Uses the exponential formula for sinh *y* or the identity $\cosh 2y = \pm 1 \pm 2\sinh^2 y$ to write in a form which can be integrated at least one term **dM1:** Dependent of previous method mark, integrates. **A1:** Correct integration. **M1:** Correct use of the limits $y=0$ and $y=\frac{1}{2}\ln 3$ **A1**: Correct exact volume.

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Determine the values of x for which

$$64\cosh^4 x - 64\cosh^2 x - 9 = 0$$

Give your answers in the form $q\ln 2$ where q is rational and in simplest form.

(4)

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Question	Scheme	Marks	AOs
2	Solves the quadratic equation for $cosh^2 x$ e.g. $\left(8\,cosh^2 x - 9\right)\left(8\,cosh^2 + 1\right) = 0 \Rightarrow cosh^2 x = \ldots$	M1	3.1a
	$cosh^2 x = \frac{9}{8}\ \left\{-\frac{1}{8}\right\}$	A1	1.1b
	$cosh\,x = \frac{3}{4}\sqrt{2} \Rightarrow x = ln\left[\frac{3}{4}\sqrt{2} + \sqrt{\left(\frac{3}{4}\sqrt{2}\right)^2 - 1}\right]$ **Alternatively** $cosh\,x = \frac{3}{4}\sqrt{2} \Rightarrow \frac{1}{2}(e^x + e^{-x}) \Rightarrow e^{2x} - \frac{3}{2}\sqrt{2}e^x + 1 = 0$ $\Rightarrow e^x = \sqrt{2}$ or $\frac{\sqrt{2}}{2} \Rightarrow x = \ldots$	M1	1.1b
	$x = \pm\frac{1}{2}ln\,2$	A1	2.2a
		(4)	
			(4 marks)

Notes:

M1: Solves the quadratic equation for $cosh^2 x$by any valid means. If by calculator accept for reaching the positive value for $cosh^2 x$ (negative may be omitted or incorrect) but do not allow for going directly to a value for $cosh\,x$ Alternatively score a correct process leading to a value for sinh $2x$ or its square (Alt 1) or use of correct exponential form for cosh x to form and expand to an equation in e^{4x} and e^{2x} (Alt 2)

A1: Correct value for $cosh^2 x$ (ignore negative or incorrect extra roots.). In Alt 1 score for a correct value for $\sinh^2 2x$ or sinh $2x$. In Alt 2 score for a correct simplified equation in e^{4x}.

M1: For a correct method to achieve at least one value for x (from cosh $^2 x$). In the main scheme or Alt 1, takes positive square root (if appropriate) and uses the correct formula for arcosh x or arsinh x to find a value for x. (No need to see negative square root rejected.) In Alt 2 it is for solving the quadratic in e^{4x} and proceeding to find a value for x.

Alternatively uses the exponential definition for cosh x, forms and solves a quadratic for e^x leading to a value for x

A1: Deduces (both) the correct values for x and no others. Must be in the form specified.

SC Allow M0A0M1A1 for cases where a calculator was used to get the value for cosh x with no evidence if a correct method for find both values is shown.

2 Alt 1	$64\cosh^2 x\left(\cosh^2 x-1\right)-9=0 \Rightarrow 64\cosh^2 x\sinh^2 x-9=0$ $\Rightarrow 16\sinh^2 2x=9 \Rightarrow \sinh^2 2x=\frac{9}{16}$ Or $(8\,sinh\,x\,cosh\,x-3)(8\,sinh\,x\,cosh\,x+3)=0 \Rightarrow sinh\,2\,x=\pm\frac{3}{4}$	M1 A1	3.1a 1.1b
	$sinh\,2\,x=\pm\frac{3}{4} \Rightarrow x=\frac{1}{2}ln\left[\pm\frac{3}{4}+\sqrt{\frac{9}{16}+1}\right]$ (or use exponentials, or proceed via cosh4x)	M1	1.1b
	$x=\pm\frac{1}{2}ln\,2$	A1	2.2a
		(4)	
2 Alt 2	$64\left(\frac{e^x+e^{-x}}{2}\right)^4-64\left(\frac{e^x+e^{-x}}{2}\right)^2-9=0 \Rightarrow$ $4(e^{4x}+4e^{2x}+6+4e^{-2x}+e^{-4x})-16(e^{2x}+2+e^{-2x})-9=0$	M1	3.1a
	$4e^{4x}-17+4e^{-4x}=0$	A1	1.1b
	$(4e^{4x}-1)(1-4e^{-4x})=0 \Rightarrow e^{4x}=\ldots \Rightarrow x=\ldots$	M1	1.1b
	$x=\pm\frac{1}{2}ln\,2$	A1	2.2a
		(4)	

9. (i) (a) Explain why $\int_0^{\infty} \cosh x \, dx$ is an improper integral.

(1)

(b) Show that $\int_0^{\infty} \cosh x \, dx$ is divergent.

(3)

(ii) $$4\sinh x = p\cosh x \qquad \text{where } p \text{ is a real constant}$$

Given that this equation has real solutions, determine the range of possible values for p

(2)

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Question	Scheme	Marks	AOs
9(i) (a)	E. g. • Because the interval being integrated over is unbounded. • cosh x is undefined at the limit of ∞ • the upper limit is infinite	B1	1.2
		(1)	
(i) (b)	$\int_0^{\infty} \cosh x \, dx = \lim_{t \to \infty} \int_0^t \cosh x \, dx$ or $\lim_{t\to\infty} \int_0^t \frac{1}{2}(e^x + e^{-x})dx$	B1	2.5
	$\int_0^t \cosh x \, dx = [\sinh x]_0^t = \sinh t \, (-0)$ or $\frac{1}{2}\int_0^t e^x + e^{-x} \, dx = \frac{1}{2}[e^x - e^{-x}]_0^t = \frac{1}{2}[e^t - e^{-t}] \left(-\frac{1}{2}[e^0 - e^0]\right)$	M1	1.1b
	When $t \to \infty$ $e^t \to \infty$and $e^{-t} \to 0$ therefore the integral is divergent	A1	2.4
		(3)	
(ii)	$4\,sinh\,x = p\,cosh\,x \Rightarrow tanh\,x = \frac{p}{4}$ or $4\,tanh\,x = p$ **Alternative** $\frac{4}{2}(e^x - e^{-x}) = \frac{p}{2}(e^x + e^{-x}) \Rightarrow 4e^x - 4e^{-x} = pe^x + pe^{-x}$ $e^{2x}(4-p) = p + 4 \Rightarrow e^{2x} = \frac{p+4}{4-p}$	M1	3.1a
	$\left\{-1 < \frac{p}{4} < 1 \Rightarrow\right\} -4 < p < 4$	A1	2.2a
		(2)	
		(6 marks)	

(i)(a)

B1: For a suitable explanation. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept "Because the upper limit is infinity", but **not** "because it is infinity" without reference to what "it" is. Do not accept "the upper limit tends to infinity" or "the integral is unbounded".

(i)(b)

B1: Writes the integral in terms of a limit as $t \to \infty$ (or other variable) with limits 0 and "t", or implies the integral is a limit by subsequent working by correct language.

M1: Integrates cosh$x$ correctly either as sinh x or in terms of exponentials and applies correctly the limits of 0 and "t". The bottom limit zero may be implied. No need for the $\lim_{t\to\infty}$ for this mark but substitution of ∞ is M0.

A1: cso States that (as $t \to \infty$) $sinh\,t \to \infty$ or $e^t \to \infty$and $e^{-t} \to 0$ therefore divergent (or not convergent), or equivalent working. Accept sinh t is undefined as $t \to \infty$

(ii)

M1: Divides through by cosh$x$ to find an expression involving tanh x

Alternative: uses the correct exponential definitions and finds an expression for e^{2x} or solves a quadratic in e^{2x}

A1: Deduces the correct inequality for p. Note $|p| < 4$ is a correct inequality for p.

9. $$y = \cosh^n x \qquad n \geqslant 5$$

(a) (i) Show that

$$\frac{d^2 y}{dx^2} = n^2 \cosh^n x - n(n-1)\cosh^{n-2} x$$

(4)

(ii) Determine an expression for $\frac{d^4 y}{dx^4}$

(2)

(b) Hence determine the first three non-zero terms of the Maclaurin series for y, giving each coefficient in simplest form.

(2)

Question	Scheme	Marks	AOs
9(a)(i)	$\frac{dy}{dx} = \ldots cosh^{n-1}x\, sinh\, x$ $\frac{d^2y}{dx^2} = \ldots cosh^{n-2}x\, sinh^2 x + \ldots cosh^{n-1}\, x\, cosh\, x$ **Alternatively** $y = \left(\frac{e^x+e^{-x}}{2}\right)^n$ leading to $\frac{dy}{dx} = \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + \ldots\left(\frac{e^x+e^{-x}}{2}\right)^n$	M1	1.1b
	$\frac{dy}{dx} = n\, cosh^{n-1}x\, sinh\, x$ $\frac{d^2y}{dx^2} = n(n-1)\, cosh^{n-2}x\, sinh^2 x + n\, cosh^n x$ **Alternatively** $\frac{dy}{dx} = n\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = n(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + n\left(\frac{e^x+e^{-x}}{2}\right)^n$	A1	2.1
	$\frac{d^2y}{dx^2} = n(n-1)\, cosh^{n-2}x\,(cosh^2 x - 1) + n\, cosh^n\, x$	M1	2.1
	$\frac{d^2y}{dx^2} = n^2\, cosh^n x - n(n-1)\, cosh^{n-2}x$* cso	A1*	1.1b
		(4)	
(a)(ii)	$\frac{d^3y}{dx^3} = \ldots cosh^{n-1}x\, sinh\, x - \ldots cosh^{n-3}x\, sinh\, x$ $\frac{d^4y}{dx^4}$ $= \ldots cosh^{n-2}x\, sinh^2 x + \ldots cosh^n\, x - \ldots cosh^{n-4}x\, sinh^2 x - \ldots cos$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3\, cosh^{n-1}x\, sinh\, x - n(n-1)(n-2)\, cosh^{n-3}x\, sinh\, x$ $\frac{d^4y}{dx^4} = n^3(n-1)\, cosh^{n-2}x\, sinh^2 x + n^3\, cosh^n\, x$ $-n(n-1)(n-2)(n-3)\, cosh^{n-4}x\, sinh^2 x - n(n-1)(n$ $-2)\, cosh^{n-2}x$	A1	1.1b
		(2)	
	Alternative 1 using $\frac{d^2y}{dx^2} = n^2y - n(n-1)\, cosh^{n-2}x$ leading to $\frac{d^3y}{dx^3} = n^2\frac{dy}{dx} - \ldots cosh^{n-3}x\, sinh\, x$ $\frac{d^4y}{dx^4} = n^2\frac{d^2y}{dx^2} - \ldots cosh^{n-4}x\, sinh^2 x - \ldots cosh^{n-2}x$	M1	1.1b

	$\frac{d^3y}{dx^3} = n^2 \frac{dy}{dx} - n(n-1)(n-2) \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = n^2 \frac{d^2y}{dx^2} - n(n-1)(n-2)(n-3) \cosh^{n-4} x \sinh^2 x - n(n-1)(n-2) \cosh^{n-2} x$	A1	1.1b
		(2)	
	Alternative 2 $y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x$ $y = \cosh^{n-2} x \Rightarrow \frac{d^2y}{dx^2} = \ldots \cosh^{n-2} x - \ldots \cosh^{n-4} x$ $\frac{d^4y}{dx^4} = n^2\left[n^2 \cosh^n x - n(n-1) \cosh^{n-2} x\right] - n(n-1)\left[\ldots \cosh^{n-2} x - \ldots \cosh^{n-4} x\right]$	M1	1.1b
	$y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x$ $y = \cosh^{n-2} x \Rightarrow \frac{d^2y}{dx^2} = (n-2)^2 \cosh^{n-2} x - (n-2)(n-3) \cosh^{n-4} x$ $\frac{d^4y}{dx^4} = n^2\left[n^2 \cosh^n x - n(n-1) \cosh^{n-2} x\right] - n(n-1)\left[(n-2)^2 \cosh^{n-2} x - (n-2)(n-3) \cosh^{n-4} x\right]$	A1	1.1b
		(2)	
	Alternative 3 Using $\frac{d^2y}{dx^2} = n^2 \left(\frac{e^x+e^{-x}}{2}\right)^n - n(n-1) \left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ leading to $\frac{d^3y}{dx^3} = \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-1} \left(\frac{e^x-e^{-x}}{2}\right) - \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-3} \left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^4y}{dx^4} = \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-2} \left(\frac{e^x-e^{-x}}{2}\right)^2 + \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-2} - \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-4} \left(\frac{e^x-e^{-x}}{2}\right)^2 - \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3 \left(\frac{e^x+e^{-x}}{2}\right)^{n-1} \left(\frac{e^x-e^{-x}}{2}\right) - n(n-1)(n-2) \left(\frac{e^x+e^{-x}}{2}\right)^{n-3} \left(\frac{e^x-e^{-x}}{2}\right)$	A1	1.1b

	$\frac{d^4y}{dx^4} = n^3(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + n^3\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ $-n(n-1)(n-2)(n-3)\left(\frac{e^x+e^{-x}}{2}\right)^{n-4}\left(\frac{e^x-e^{-x}}{2}\right)^2 - n(n$ $-1)(n-2)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$		
		(2)	
(b)	When $x = 0$ $y = 1,\quad y' = 0,\quad y'' = n^2 - n(n-1),\quad y^{(3)} = 0,$ $y^{(4)} = n^3 - n(n-1)(n-2)$ Uses their values in the expansion $y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y^{(3)}(0) + \frac{x^4}{4!}y^{(4)}(0) + \ldots$	M1	1.1b
	$y = 1 + \frac{nx^2}{2} + \frac{(3n^2-2n)x^4}{24} + \ldots$ cso	A1	2.5
		(2)	
			(8 marks)

Notes:

(a)(i)

M1: Uses the chain rule and product rule to find the first and second derivatives which must be of the required form, condone sign slips

Alternatively uses the exponential definition and uses the chain rule and product rule to find the first and second derivatives which must be of the required form.

A1: Correct unsimplified first and second derivatives, may be in exponential form.

M1: Uses the identity $\pm cosh^2 x \pm sinh^2 x = 1$

A1*: Achieves the printed answer with no errors or omissions e.g. missing x's

(a)(ii)

M1: Uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 1

M1: Using $\frac{d^2y}{dx^2} = n^2y - n(n-1)\,cosh^{n-2}x$ to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 2

M1: Using $y = cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2 cosh^n x - n(n-1)\,cosh^{n-2}x$

$y = cosh^{n-2}x \Rightarrow \frac{d^2y}{dx^2} = \ldots cosh^{n-2}x - \ldots cosh^{n-4}x$ leading to

$$\frac{d^4y}{dx^4} = n^2\left[n^2\,cosh^n\,x - n(n-1)\,cosh^{n\,-\,2}\,x\right] - n(n-1)\left[\text{their}\,\frac{d(cosh^{n-2}\,x)}{dx}\right]$$

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 3

M1: Uses the exponential definition and uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form.

A1: Correct fourth derivative, does not need to be simplified ISW

(b)

M1: Attempts the evaluation of all four of their derivatives at $x = 0$ and applies the Maclaurin formula with their values. Note that $y^{(1)}(0) = 0$ and $y^{(3)}(0) = 0$ may be implied as they will have a multiple of sinh0. If their $y^{(3)}(0) \neq 0$ they allow this mark for their first 3 non-zero terms

A1: Correct simplified expansion from correct derivatives cso