Cp2Ch3 XMQs and MS

(Total: 66 marks)

1. CP1_Sample Q6 . 9 marks - CP2ch3 Methods in calculus
2. CP1_Specimen Q4 . 8 marks - CP2ch3 Methods in calculus
3. CP2_Specimen Q1 . 7 marks - CP2ch3 Methods in calculus
4. CP1_2019 Q2 . 7 marks - CP2ch3 Methods in calculus
5. CP1_2020 Q2 . 7 marks - CP2ch3 Methods in calculus
6. CP1_2021 Q5 . 7 marks - CP2ch3 Methods in calculus
7. CP2_2021 Q5 . 8 marks - CP2ch3 Methods in calculus
8. CP1_2022 Q6 . 7 marks - CP2ch3 Methods in calculus
9. CP2_2022 Q5 . 6 marks - CP2ch3 Methods in calculus

6.

$$f(x) = \frac{x + 2}{x^2 + 9}$$

(a) Show that

$$\int f(x)\,dx = A\ln(x^2 + 9) + B\arctan\left(\frac{x}{3}\right) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence show that the mean value of $f(x)$ over the interval [0, 3] is

$$\frac{1}{6}\ln 2 + \frac{1}{18}\pi$$

(3)

(c) Use the answer to part (b) to find the mean value, over the interval [0, 3], of

$$f(x) + \ln k$$

where k is a positive constant, giving your answer in the form $p + \frac{1}{6}\ln q$, where p and q are constants and q is in terms of k.

(2)

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Question	Scheme	Marks	AOs
6(a)	$\mathrm{f}(x)=\dfrac{x+2}{x^2+9}=\dfrac{x}{x^2+9}+\dfrac{2}{x^2+9}$	B1	3.1a
	$\displaystyle\int\frac{x}{x^2+9}\,\mathrm{d}x=k\ln\left(x^2+9\right)(+c)$	M1	1.1b
	$\displaystyle\int\frac{2}{x^2+9}\,\mathrm{d}x=k\arctan\left(\frac{x}{3}\right)(+c)$	M1	1.1b
	$\displaystyle\int\frac{x+2}{x^2+9}\,\mathrm{d}x=\frac{1}{2}\ln\left(x^2+9\right)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)+c$	A1	1.1b
		(4)	
(b)	$\displaystyle\int_0^3\mathrm{f}(x)\,\mathrm{d}x=\left[\frac{1}{2}\ln\left(x^2+9\right)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)\right]_0^3$ $=\dfrac{1}{2}\ln18+\dfrac{2}{3}\arctan\left(\dfrac{3}{3}\right)-\left(\dfrac{1}{2}\ln9+\dfrac{2}{3}\arctan(0)\right)$ $=\dfrac{1}{2}\ln\dfrac{18}{9}+\dfrac{2}{3}\arctan\left(\dfrac{3}{3}\right)$	M1	1.1b
	Mean value = $\dfrac{1}{3-0}\left(\dfrac{1}{2}\ln2+\dfrac{\pi}{6}\right)$	M1	2.1
	$\dfrac{1}{6}\ln2+\dfrac{1}{18}\pi$ *	A1*	2.2a
		(3)	
(c)	$\dfrac{1}{6}\ln2+\dfrac{1}{18}\pi+\ln k$	M1	2.2a
	$\dfrac{1}{6}\ln2k^6+\dfrac{1}{18}\pi$	A1	1.1b
		(2)	
		(9 marks)	

Notes:

(a)
B1: Splits the fraction into two correct separate expressions
M1: Recognises the required form for the first integration
M1: Recognises the required form for the second integration
A1: Both expressions integrated correctly and added together with constant of integration included

(b)
M1: Uses limits correctly and combines logarithmic terms
M1: Correctly applies the method for the mean value for their integration
A1*: Correct work leading to the given answer

(c)
M1: Realises that the effect of the transformation is to increase the mean value by ln k
A1: Combines ln's correctly to obtain the correct expression

4.

$$f(x) = \begin{cases} \dfrac{kx}{x^2 + 6} & \text{for } 0 \leqslant x < 3 \\ \dfrac{k}{x^2 - 4} & \text{for } 3 \leqslant x \end{cases}$$

where k is a positive constant.

The area between the curve $y = f(x)$ and the positive x-axis is $\frac{1}{4}$

Show that

$$k = \frac{1}{\ln a}$$

where a is a constant to be determined.

(8)

Question	Scheme	Marks	AOs
4	$\int_0^3 \frac{kx}{x^2+6}\,dx + \int_3^\infty \frac{k}{x^2-4}\,dx = \frac{1}{4} \Rightarrow k\left([\ldots]_0^3 + [\ldots]_3^\infty\right) = \frac{1}{4} \Rightarrow k = \ldots$	M1	3.1a
	$(k)\int_0^3 \frac{x}{x^2+6}\,dx = (k)\left[\frac{1}{2}\ln\left(x^2+6\right)\right]_0^3 = (k)\left(\frac{1}{2}\ln(15) - \frac{1}{2}\ln(6)\right)$	M1 A1	1.1b 1.1b
	$(k)\int \frac{1}{x^2-4}\,dx = (k)\frac{1}{4}\ln\left(\frac{x-2}{x+2}\right)$	M1	1.1b
	$\int_3^\infty \frac{1}{x^2-4}\,dx = \lim_{t\to\infty}\left[\frac{1}{4}\ln\left(\frac{x-2}{x+2}\right)\right]_3^t$ $= \frac{1}{4}\left(\lim_{t\to\infty}\ln\left(\frac{t-2}{t+2}\right) - \ln\left(\frac{1}{5}\right)\right) = \frac{1}{4}\ln 5$	M1 A1	3.1a 1.1b
	So $\frac{k}{2}\ln\left(\frac{15}{6}\right) + \frac{k}{4}\ln 5 = \frac{1}{4} \Rightarrow k\ln\left(\frac{5}{2}\right)^2 + k\ln(5) = 1$ so $k = \frac{1}{\ln\left(\frac{125}{4}\right)}$	M1 A1	2.1 1.1b
		(8)	
		(8 marks)	

Notes:

M1: For a complete overall method. Correct expression for the total area formed, a sum of two areas with an attempt made for both areas in terms of k and with area set equal to $\frac{1}{4}$.

M1: Correct form for integral of left hand part (with or without k) and attempts to apply the limits. Accept $a\ln\left(x^2+6\right)$.

A1: Integral correct, with limits applied correctly. No need to combine lns at this stage.

M1: Correct form integral for the right hand part (with or without k), quoted directly or uses partial fractions. Accept $A\ln\left(\frac{x-2}{x+2}\right)$ or $A\ln(x-2) - B\ln(x+2)$ from an attempt at partial fractions.

M1: Applies the limits on the improper integral, 3 as lower limit and t as upper with $t \to \infty$ to obtain a value for the integral. If partial fractions used, the log terms will need combining first.

A1: Fully correct integral for right hand section, as in scheme or equivalent, with or without k.

M1: Adds the results of both the integrals and equates to $\frac{1}{4}$, including k, and uses correct log laws to combine their log terms to find k.

A1: Correct answer. Accept any equivalents in the correct form, e.g. $\frac{1}{\ln(31.25)}$ or statement that $a = \frac{125}{4}$ etc.

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Find

$$\int \frac{1}{x^2 + 6x + 25}\,\mathrm{d}x$$

(3)

(b) Hence find the exact value of

$$\int_{-3}^{1} \left(1 - \frac{25}{x^2 + 6x + 25}\right)\mathrm{d}x$$

giving the answer in simplest form.

(3)

A student claims that the magnitude of the answer to part (b) gives the total area bounded by the curve $y = 1 - \dfrac{25}{x^2 + 6x + 25}$ and the x-axis between the line $x = -3$ and the line $x = 1$

(c) State, with a reason, whether or not the student is correct.

(1)

9FM0/02: Core Pure Mathematics 02 Mark scheme

Question	Scheme	Marks	AOs
1(a)	$\int \frac{1}{x^2+6x+25}\,dx = \int \frac{1}{(x+3)^2-9+25}\,dx = \int \frac{1}{(x+3)^2+16}\,dx$ or reaches integral in θ if using substitution.	M1	3.1a
	$= k\arctan\left(\frac{x+b}{a}\right)(+c)$ (or $k\theta$ where $4\tan\theta = x+3$)	M1	1.1b
	$= \frac{1}{4}\arctan\left(\frac{x+3}{4}\right)+c$	A1	1.1b
		(3)	
(b)	$\int_{-3}^{1}\left(1-\frac{25}{x^2+6x+25}\right)dx = \left[x-\frac{25}{4}\arctan\left(\frac{x+3}{4}\right)\right]_{-3}^{1} = (1-...)-(-3-...)$	M1	1.1b
	$= \left(1-\frac{25}{4}\arctan\left(\frac{4}{4}\right)\right)-\left(-3-\frac{25}{4}\arctan 0\right)$	A1ft	1.1b
	$= 4-\frac{25\pi}{16}$	A1	2.1
		(3)	
(c)	Since the graph crosses the x-axis at $x = 0$ the area lies partially below the x-axis, hence the expression does not give the total area as the part below the axis counts as negative which cancels the positive area, so the student is not correct.	B1	2.3
		(1)	
			(7 marks)

Notes:

(a)

M1: Identifies the need to and completes the square in the numerator to achieve a standard form, or selects the appropriate substitution $x + 3 = 4\tan\theta$. If using substitution, the integrand and dx must be dealt with and an integral in θ reached (or their chosen variable).

M1: Carries out the integration to a form $k\arctan\left(\frac{x+b}{a}\right)$

A1: Correct integral with or without c

(b)

M1: Applies limits to $x - 25\times$"their answer to (a)" and subtracts correct way.

A1ft: A correct unsimplified answer following through their answer to (a).

A1: Correct simplified exact answer.

(c)

B1: As scheme. Must refer to graph crossing the x-axis and signs of areas being different.

2. Show that

$$\int_0^{\infty} \frac{8x - 12}{(2x^2 + 3)(x + 1)}\,dx = \ln k$$

where k is a rational number to be found.

(7)

M1: Solve their four equation (using calculator) to find at least one value. This will need checking if incorrect equations used. A1: Correct quartic in terms of *z* or correct values for *a*, *b*, *c* and *d* stated. **Note:** Correct answer only will score 5/5

Question	Scheme	Marks	AOs
2	$\dfrac{8x-12}{(2x^2+3)(x+1)}=\dfrac{Ax+B}{2x^2+3}+\dfrac{C}{x+1}$	M1	3.1a
	$8x-12=(Ax+B)(x+1)+C(2x^2+3)$ E.g. $x=-1\Rightarrow C=-4, x=0\Rightarrow B=0, x=1\Rightarrow A=8$ Or Compares coefficients and solves $(A+2C=0 \quad A+B=8 \quad B+3C=-12)$ $\Rightarrow A=..., B=..., C=...$	dM1	1.1b
	$A=8 \quad B=0 \quad C=-4$	A1	1.1b
	$\int\left(\dfrac{8x}{2x^2+3}-\dfrac{4}{x+1}\right)\mathrm{d}x=2\ln(2x^2+3)-4\ln(x+1)$	A1ft	1.1b
	$2\ln(2x^2+3)-4\ln(x+1)=\ln\left(\dfrac{(2x^2+3)^2}{(x+1)^4}\right)$ or $2\ln(2x^2+3)-4\ln(x+1)=2\ln\left(\dfrac{(2x^2+3)}{(x+1)^2}\right)$	M1	2.1
	$\lim\limits_{x\to\infty}\left\{\ln\dfrac{(2x^2+3)^2}{(x+1)^4}\right\}=\ln 4$ or $\lim\limits_{x\to\infty}\left\{2\ln\dfrac{(2x^2+3)}{(x+1)^2}\right\}=2\ln 2$	B1	2.2a
	$\Rightarrow\int_0^{\infty}\dfrac{8x-12}{(2x^2+3)(x+1)}\mathrm{d}x=\ln\dfrac{4}{9}$ cao	A1	1.1b
		(7)	
			(7 marks)

Notes

M1: Selects the correct form for partial fractions.
dM1: Full method for finding values for all three constants. Dependent on having the correct form for the partial fractions. Allow slips as long as the intention is clear.
A1: Correct constants or partial fractions.

A1ft: Integrates $\int \frac{px}{2x^2+3} - \frac{q}{x+1}\,dx = \frac{p}{4}\ln(2x^2+3) - q\ln(x+1)$ and no extra terms

M1: Combines two algebraic log terms correctly.
B1: Correct upper limit for $x \to \infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0). This can be implied.

A1: Deduces the correct value for the improper integral in the correct form, cao A0 for $2\ln\frac{2}{3}$

Correct answer with no working seen is no marks.
Note: Incorrect partial fraction form,
$\frac{A}{2x^2+3} + \frac{B}{x+1}$ or $\frac{Ax}{2x^2+3} + \frac{B}{x+1}$ the maximum it can score is M0M0A0A0M1B1A0

Question	Scheme	Marks	AOs
3(a)(i)	$2(0.4+a)=1.2$ or $0.4+a=0.6$ or $0.4+a\cos 0=0.6$ $\Rightarrow a = ...$	M1	3.4
	$a = 0.2$ * cso	A1*	1.1b
		(2)	
(b)	Area of rectangle is $1.2\times 0.6(=0.72)$	B1	1.1b
	Area enclosed by curve = $\frac{1}{2}\int (0.4+0.2\cos 2\theta)^2 (\mathrm{d}\theta)$	M1	3.1a
	$(0.4+0.2\cos 2\theta)^2 = 0.16+0.16\cos 2\theta+0.04\cos^2 2\theta$ $= 0.16+0.16\cos 2\theta+0.04\left(\frac{\cos 4\theta+1}{2}\right)$	M1	2.1
	$\frac{1}{2}\int (0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta = \frac{1}{2}\left[0.18\theta+0.08\sin 2\theta+0.005\sin 4\theta(+c)\right]$ $= 0.09\theta+0.04\sin 2\theta+0.0025\sin 4\theta(+c)$ o.e.	A1ft	1.1b
	Area enclosed by curve = $\left[0.09\theta+0.04\sin 2\theta+0.0025\sin 4\theta\right]_0^{2\pi}$ or Area enclosed by curve = $2\left[0.09\theta+0.04\sin 2\theta+0.0025\sin 4\theta\right]_0^{\pi}$ or Area enclosed by curve = $4\left[0.09\theta+0.04\sin 2\theta+0.0025\sin 4\theta\right]_0^{\pi/2}$	dM1	3.1a
	$= \frac{9}{50}\pi$ or $0.18\pi(=0.5654...)$	A1	1.1b

2. (a) Explain why $\int_1^{\infty} \frac{1}{x(2x+5)}\,\mathrm{d}x$ is an improper integral.

(1)

(b) Prove that

$$\int_1^{\infty} \frac{1}{x(2x+5)}\,\mathrm{d}x = a\ln b$$

where a and b are rational numbers to be determined.

(6)

Question	Scheme	Marks	AOs
2(a)	E.g. • Because the interval being integrated over is unbounded • Accept because the upper limit is infinity • Accept because a limit is required to evaluate it	B1	2.4
		(1)	
(b)	$\frac{1}{x(2x+5)} = \frac{A}{x} + \frac{B}{2x+5} \Rightarrow A = ..., B = ...$	M1	3.1a
	$\frac{1}{x(2x+5)} = \frac{1}{5x} - \frac{2}{5(2x+5)}$	A1	1.1b
	$\int \frac{1}{5x} - \frac{2}{5(2x+5)} \mathrm{d}x = \frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5)$	A1ft	1.1b
	$\frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5) = \frac{1}{5}\ln\frac{x}{(2x+5)}$	M1	2.1
	$\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{x}{2x+5}\right\} = \frac{1}{5}\ln\frac{1}{2}$	B1	2.2a
	$\Rightarrow \int_1^\infty \frac{1}{x(2x+5)} \mathrm{d}x = \frac{1}{5}\ln\frac{1}{2} - \frac{1}{5}\ln\frac{1}{7} = \frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	
		(7 marks)	

Notes

(a)

B1: For a suitable explanation with no contrary reasoning. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept "Because the upper limit is infinity". Do not award if there are erroneous statements e.g. referring to as $x = 0$ the integrand is not defined. Do not accept "because one of the limits is undefined" unless they state they mean ∞. Do not accept "it is undefined when $x = \infty$" without reference to "it" being the upper limit.

(b)

M1: Selects the correct form for partial fractions and proceeds to find values for *A* and *B*

A1: Correct constants or partial fractions

A1ft: $\int \frac{p}{x} + \frac{q}{2x+5} \mathrm{d}x = p\ln x + \frac{q}{2}\ln(2x+5)$ Note that $\frac{1}{5}\ln 5x - \frac{1}{5}\ln(10x+25)$ is correct.

M1: Combines logs correctly. May see $-\frac{1}{5}\ln\left(\frac{2x+5}{x}\right) = -\frac{1}{5}\ln\left(2+\frac{5}{x}\right)$

B1: Correct upper limit for $x \to \infty$ by recognising the dominant terms. (Simply replacing *x* with ∞ scores B0)

A1: Deduces the correct value for the improper integral in the correct form

Question	Scheme	Marks	AOs
(b) **Way 2**	$\dfrac{1}{x(2x+5)}=\dfrac{1}{2\left(x^2+\frac{5}{2}x\right)}=\dfrac{1}{2}\times\dfrac{1}{\left(x+\frac{5}{4}\right)^2-\frac{25}{16}}$	M1 A1	3.1a 1.1b
	$\displaystyle\int\frac{1}{x(2x+5)}\,dx=\frac{1}{2}\times\frac{2}{5}\ln\left\|\frac{x+\frac{5}{4}-\frac{5}{4}}{x+\frac{5}{4}+\frac{5}{4}}\right\|=\frac{1}{5}\ln\left\|\frac{2x}{2x+5}\right\|$	M1 A1ft	2.1 1.1b
	$\displaystyle\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{2x}{2x+5}\right\}=\frac{1}{5}\ln\frac{2}{2}=0$	B1	2.2a
	$\displaystyle\Rightarrow\int_1^{\infty}\frac{1}{x(2x+5)}\,dx=0-\frac{1}{5}\ln\frac{2}{7}=\frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	

Notes

Note the method marks as MAMABA, and should be entered in this order on ePEN.

M1: Expands the denominator and completes the square.

A1: Correct expression

M1: For $\dfrac{1}{(x+p)^2-a^2}\to k\ln\left|\dfrac{x+p-a}{x+p+a}\right|$

A1ft: $\dfrac{1}{2}\dfrac{1}{(x+a)^2-a^2}\to\dfrac{1}{2a}\ln\left|\dfrac{x}{x+2a}\right|$ with their a (may be simplified as in scheme).

B1: Correct upper limit for $x \to \infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0) Note in this method the upper limit evaluates to zero.

A1: Deduces the correct value for the improper integral in the correct form. Accept $-\dfrac{1}{5}\ln\dfrac{2}{7}$

5. (i) Evaluate the improper integral

$$\int_1^{\infty} 2e^{-\frac{1}{2}x}\,dx$$

(3)

(ii) The air temperature, θ °C, on a particular day in London is modelled by the equation

$$\theta = 8 - 5\sin\left(\frac{\pi}{12}t\right) - \cos\left(\frac{\pi}{6}t\right) \qquad 0 \leqslant t \leqslant 24$$

where t is the number of hours after midnight.

(a) Use calculus to show that the mean air temperature on this day is 8 °C, according to the model.

(3)

Given that the actual mean air temperature recorded on this day was higher than 8 °C,

(b) explain how the model could be refined.

(1)

Question	Scheme	Marks	AOs
5(i)	$\int 2\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x = -4\mathrm{e}^{-\frac{1}{2}x}$	B1	1.1b
	$\int_1^{\infty} 2\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x = \lim_{t\to\infty}\left[\left(-4\mathrm{e}^{-\frac{1}{2}t}\right)-\left(-4\mathrm{e}^{-\frac{1}{2}}\right)\right]$	M1	2.1
	$= 4\mathrm{e}^{-\frac{1}{2}}$	A1	1.1b
		(3)	
(ii)(a)	Mean temperature $= \frac{1}{24}\int_0^{24}\left(8-5\sin\left(\frac{\pi}{12}t\right)-\cos\left(\frac{\pi}{6}t\right)\right)\mathrm{d}t$	B1	1.2
	$= \frac{1}{24}\left[\left(8t+\frac{60}{\pi}\cos\left(\frac{\pi}{12}t\right)-\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right)\right]_0^{24} = \frac{1}{24}[...]$	M1	1.1b
	$= \frac{1}{24}\left[\left(8(24)+\frac{60}{\pi}-\frac{6}{\pi}\times 0\right)-\left(\frac{60}{\pi}\right)\right] = 8$ * cso	A1*	2.1
		(3)	
(ii)(b)	E.g. increase the value of the constant 8 / adapt the constant 8 to a function which takes values greater than 8.	B1	3.5c
		(1)	
			(7 marks)

Notes:

(i)
B1: Correct integration.

M1: Attempt to integrate to a form $\lambda \mathrm{e}^{-\frac{1}{2}x}$ where $\lambda \neq 2$, and applies correct limits with some consideration of the infinite limit given (e.g. with the limit statement). Only allow with ∞ used as the limit if subsequent work shows the term is zero.
A1: Correct value

(ii)(a)
B1: Recalls the correct formula for finding the mean value of a function. You may see the division by "24" only at the end. No integration is necessary, just a correct statement with an integral.

M1: Integrates to a form $\alpha t+\beta\cos\left(\frac{\pi}{12}t\right)+\delta\sin\left(\frac{\pi}{6}t\right)$ and uses the limits of 0 and 24 (the correct way around). If no explicit substitution is seen, accept any value following the integral as an attempt. Answers from a calculator with no correct integral seen score M0 as the question requires calculus to be used.
A1*cso: Achieves 8 with no errors seen following a full attempt at the substitution. Must have seen some evidence of the limits used, minimum required for substitution is $\left[\left(8(24)+\frac{60}{\pi}\right)-\left(\frac{60}{\pi}\right)\right]$.

(ii)(b)
B1: Accept any reasonable adaptation to the equation that will increase the mean value. E.g. as in scheme, or introduce another positive term, or decrease the constant 5 etc. It must be clear which constant they are referring to in their reason, not just "increase the constant".

5. The curve C has equation

$$y = \arccos\left(\frac{1}{2}x\right) \qquad -2 \leqslant x \leqslant 2$$

(a) Show that C has no stationary points.

(3)

The normal to C, at the point where $x = 1$, crosses the x-axis at the point A and crosses the y-axis at the point B.

Given that O is the origin,

(b) show that the area of the triangle OAB is $\frac{1}{54}\left(p\sqrt{3} + q\pi + r\sqrt{3}\,\pi^2\right)$ where p, q and r are integers to be determined.

(5)

Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx}=\frac{-\lambda}{\sqrt{1-\beta x^2}}$ where $\lambda>0$ and $\beta>0$ and $\beta\neq 1$ Alternatively $2\cos y = x \Rightarrow \frac{dx}{dy}=\alpha\sin y \Rightarrow \frac{dy}{dx}=\frac{1}{\alpha\sin y}$	M1	1.1b
	$\frac{dy}{dx}=\frac{-\frac{1}{2}}{\sqrt{1-\frac{1}{4}x^2}}$ or $\frac{dy}{dx}=\frac{-1}{2\sqrt{1-\frac{1}{4}x^2}}$ o.e. or $\frac{dy}{dx}=-\frac{1}{2\sin y}$ or	A1	1.1b
	States that $\frac{dy}{dx}\neq 0$ therefore C has no stationary points. Tries to solve $\frac{dy}{dx}=0$ and ends up with a contradiction e.g. -1 = 0 therefore C has no stationary points. As cosec y > 1 therefore C has no stationary points.	A1	2.4
		(3)	
(b)	$\frac{dy}{dx}=\frac{-1}{2\sqrt{1-\frac{1}{4}\times 1^2}}=\left\{-\frac{1}{\sqrt{3}}\right\}$	M1	1.1b
	Normal gradient = $-\frac{1}{m}$ and $y-\frac{\pi}{3}=m_n(x-1)$ Alternatively $\frac{\pi}{3}=m_n(1)+c \Rightarrow c=...\left\{\frac{\pi}{3}-\sqrt{3}\right\}$ and then $y=m_n x+c$	M1	1.1b
	$y=0\Rightarrow 0-\frac{\pi}{3}=\sqrt{3}(x_A-1)\Rightarrow x_A=...\left\{1-\frac{\pi}{3\sqrt{3}} \text{ or } 1-\frac{\pi\sqrt{3}}{9}\right\}$ and $x=0\Rightarrow y_B-\frac{\pi}{3}=\sqrt{3}(0-1)\Rightarrow y_B=...\left\{\frac{\pi}{3}-\sqrt{3}\right\}$	M1	3.1a
	Area $=\frac{1}{2}\times x_A\times -y_B=\frac{1}{2}\left(1-\frac{\pi}{3\sqrt{3}}\right)\left(\sqrt{3}-\frac{\pi}{3}\right)$	M1	1.1b
	Area $\frac{1}{54}\left(27\sqrt{3}-18\pi+\sqrt{3}\pi^2\right)$ $(p=27, q=-18, r=1)$	A1	2.1
		(5)	
			(8 marks)

Notes:

(a)

M1: Finds the correct form for $\frac{dy}{dx}$

A1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: States or shows that $\frac{\mathrm{d}y}{\mathrm{d}x} \neq 0$ and draws the required conclusion. This mark can be scored as long as the M mark has been awarded.

(b)

M1: Substitutes $x = 1$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Finds the normal gradient and finds the equation of the normal using $y - \frac{\pi}{3} = m_n(x-1)$

M1: Finds where their normal cuts the *x*-axis and the *y*-axis.

M1: Finds the area of the triangle *OAB* $= \frac{1}{2} \times x_A \times -y_B$.

A1: Correct area

Special case: If finds the tangent to the curve, the *x* and *y* intercepts and the area of the triangle max score M1 M0 M1 M0 A0

Note common error

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-1}{\sqrt{1-\frac{1}{4}x^2}}$ In part (b) this leads to $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{\sqrt{3}}$ leading to normal gradient $\frac{\sqrt{3}}{2}$ and

$y = \frac{\sqrt{3}}{2}x - \frac{\sqrt{3}}{2} + \frac{\pi}{3}$ and $\left(0, \frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$ and $\left(1 - \frac{2\pi}{3\sqrt{3}}, 0\right)$ therefore area = $\frac{1}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\left(\frac{2\pi}{3\sqrt{3}} - 1\right)$

This can score M1 M1 M1 M1 A0

6. (a) Express as partial fractions

$$\frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}$$

(3)

(b) Hence, show that

$$\int_0^2 \frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}\,dx = \ln\left(a\sqrt{2}\right) + b\pi$$

where a and b are constants to be determined.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

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Question	Scheme	Marks	AOs
6(a)	$\frac{2x^2+3x+6}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \Rightarrow 2x^2+3x+6$ $= A(x^2+4) + (Bx+C)(x+1)$	M1	1.1b
	e.g. $x=-1 \Rightarrow A = \ldots,\ x=0 \Rightarrow C = \ldots,$ coeff $x^2 \Rightarrow B = \ldots$ or Compares coefficients and solves to find values for A, B and C $2 = A+B,\ 3 = B+C,\ 6 = 4A+C$	dM1	1.1b
	$A = 1,\quad B = 1,\quad C = 2$	A1	1.1b
		(3)	
(b)	$\int_0^2 \frac{1}{x+1} + \frac{x+2}{x^2+4}\,dx = \int_0^2 \frac{1}{x+1} + \frac{x}{x^2+4} + \frac{2}{x^2+4}\,dx$ $= \left[\alpha \ln(x+1) + \beta \ln(x^2+4) + \lambda \arctan\left(\frac{x}{2}\right)\right]_0^2$	M1	3.1a
	$= \left[ln(x+1) + \frac{1}{2}ln(x^2+4) + arctan\left(\frac{x}{2}\right)\right]_0^2$	A1	2.1
	$= \left[ln(3) + \frac{1}{2}ln(8) + arctan\ 1\right] - \left[ln(1) + \frac{1}{2}ln(4) + arctan(0)\right]$ = $= \left[ln(3) + \frac{1}{2}ln(8) + arctan(1)\right] - \left[\frac{1}{2}ln\ 4\right] = \underline{ln\left(\frac{3\sqrt{8}}{2}\right)} + \frac{\pi}{4}$	dM1	2.1
	$ln(3\sqrt{2}) + \frac{\pi}{4}$	A1	2.2a
		(4)	
		(7 marks)	

Notes:

(a)
M1: Selects the correct form for partial fractions and multiplies through to form suitable identity or uses a method to find at least one value (e.g. cover up rule).
dM1: Full method for finding values for all three constants. Dependent on first M. Allow slips as long as the intention is clear.
A1: Correct constants or partial fractions.

(b)
M1: Splits the integral into an integrable form and integrates at least two terms to the correct form. They may use a substitution on the arctan term
A1: Fully correct Integration.
dM1: Uses the limits of 0 and 2 (or appropriate for a substitution), subtracts the correct way round and combines the ln terms from separate integrals to a single term with evidence of correct ln laws at least once.
A1: Correct answer

5. (a) Given that

$$y = \arcsin x \qquad -1 \leqslant x \leqslant 1$$

show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

(3)

(b)

$$f(x) = \arcsin(e^x) \qquad x \leqslant 0$$

Prove that f(x) has no stationary points.

(3)

Question	Scheme		Marks	AOs
5(a)	$\sin y = x \Rightarrow \cos y \dfrac{dy}{dx} = 1$	$\sin y = x \Rightarrow \dfrac{dx}{dy} = \cos y$	M1	1.1b
	Uses $\sin^2 y + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1 - \sin^2 y} \Rightarrow \sqrt{1 - x^2}$		M1	2.1
	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ * cso		A1*	1.1b
			(3)	
(b)	Using the answer to (a) $f'(x) = \dfrac{1}{\sqrt{1-e^{2x}}} \times \ldots$	Restart $\sin y = e^x \Rightarrow \cos y \dfrac{dy}{dx} = e^x$	M1	3.1a
	$f'(x) = \dfrac{1}{\sqrt{1-e^{2x}}} \times e^x$	$f'(x) = \dfrac{e^x}{\cos y}$	A1	1.1b
	$e^x \neq 0$ (or $e^x > 0$) therefore, there are no stationary points Alternatively, $e^x = 0$ leading to $x = \ln 0$ which is impossible/undefined therefore there are no stationary points.		A1	2.4
			(3)	
			(6 marks)	

Notes:

(a)

M1: Finds *x* in terms of *y* and differentiates

M1: Uses the trig identity $\sin^2 y + \cos^2 y = 1$ to express $\cos y$ in terms of *x*. This may be seen in their derivative or stated on the side

A1*: Correctly achieves the printed answer $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$. cso

(b)

M1: Differentiates using the chain rule to achieve the correct form, condone $f'(x) = \frac{1}{\sqrt{1-e^{2x}}}$

Note $f'(x) = \frac{1}{\sqrt{1-e^x}}$ is B0 for incorrect form

Alternatively restart, finds *x* in terms of *y* and differentiates

A1: Correct differentiation

A1: Follows correct differentiation. States that as $e^x \neq 0$ (or $e^x > 0$) or no solutions to $e^x = 0$ therefore there are no stationary points.

Alternatively, $e^x = 0$ leading to $x = \ln 0$ which is impossible/undefined/error therefore there are no stationary points. Ignore any reference to the denominator = 0