Cp2Ch2 XMQs and MS

(Total: 40 marks)

1. CP1_Sample Q1 . 5 marks - CP2ch2 Series
2. CP2_Specimen Q2 . 10 marks - CP2ch2 Series
3. CP1_2019 Q4 . 5 marks - CP2ch2 Series
4. CP1_2021 Q2 . 7 marks - CP2ch2 Series
5. CP2_2021 Q3 . 6 marks - CP2ch2 Series
6. CP1_2022 Q4 . 7 marks - CP2ch2 Series

Answer ALL questions. Write your answers in the spaces provided.

1. Prove that

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} = \frac{n(an+b)}{12(n+2)(n+3)}$$

where a and b are constants to be found.

(5)

Paper 1: Core Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	$\frac{1}{(r+1)(r+3)} \equiv \frac{A}{(r+1)} + \frac{B}{(r+3)} \Rightarrow A = ..., B = ...$	M1	3.1a
	$\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} =$ $\frac{1}{2\times2} - \frac{1}{2\times4} + \frac{1}{2\times3} - \frac{1}{2\times5} + ... + \frac{1}{2n} - \frac{1}{2(n+2)} + \frac{1}{2(n+1)} - \frac{1}{2(n+3)}$	M1	2.1
	$= \frac{1}{4} + \frac{1}{6} - \frac{1}{2(n+2)} - \frac{1}{2(n+3)}$	A1	2.2a
	$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{12(n+2)(n+3)}$	M1	1.1b
	$= \frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
		(5)	
	Alternative by induction: $n=1 \Rightarrow \frac{1}{8} = \frac{a+b}{12\times3\times4},\ n=2 \Rightarrow \frac{1}{8} + \frac{1}{15} = \frac{2(2a+b)}{12\times4\times5}$ $a+b=18,\ 2a+b=23 \Rightarrow a = ..., b = ...$	M1	3.1a
	Assume true for $n = k$ so $\sum_{r=1}^{k} \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)}$		
	$\sum_{r=1}^{k+1} \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)}$	M1	2.1
	$\frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)} = \frac{k(5k+13)(k+4) + 12(k+3)}{12(k+2)(k+3)(k+4)}$	A1	2.2a
	$= \frac{5k^3 + 33k^2 + 52k + 12k + 36}{12(k+2)(k+3)(k+4)} = \frac{(k+1)(k+2)(5k+18)}{12(k+2)(k+3)(k+4)}$	M1	1.1b
	$= \frac{(\underline{k+1})(5(\underline{k+1})+13)}{12(\underline{k+1}+2)(\underline{k+1}+3)}$ So true for $n = k + 1$ So $\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} = \frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
		(5)	
			(5 marks)

Question 1 notes:
Main Scheme **M1:** Valid attempt at partial fractions **M1:** Starts the process of differences to identify the relevant fractions at the start and end **A1:** Correct fractions that do not cancel **M1:** Attempt common denominator **A1:** Correct answer
Alternative by Induction: **M1:** Uses $n = 1$ and $n = 2$ to identify values for a and b **M1:** Starts the induction process by adding the $(k + 1)^{\text{th}}$ term to the sum of k terms **A1:** Correct single fraction **M1:** Attempt to factorise the numerator **A1:** Correct answer and conclusion

2. A company operating a coal mine is concerned about the mine running out of coal. It is estimated that 2.5 million tonnes of coal are left in the mine. The company wishes to mine all of this coal in 20 years.

In order to mine the coal in a regulated manner, the company models the amount of coal to be mined in the coming years by the formula

$$M_r = \frac{10}{r^2 + 8r + 15}$$

where M_r is the amount of coal, in millions of tonnes, mined in year r, with the first year being year 1

(a) Show that, according to the model, the total amount of coal, in millions of tonnes, mined in the first n years is given by

$$T_n = \frac{9n^2 + 41n}{k(n+4)(n+5)}$$

where k is a constant to be determined.

(6)

(b) Explain why, according to this model, the mine will never run out of coal.

(2)

The company decides to mine an extra fixed amount each year so that all the coal will be mined in exactly 20 years.

(c) Refine the formula for M_r so that 2.5 million tonnes of coal will be exhausted in exactly 20 years of mining.

(2)

Question	Scheme	Marks	AOs
2(a)	A correct method to sum the series, most likely by the method of differences. Look for $\frac{10}{r^2+8r+15}=\frac{A}{r+3}+\frac{B}{r+5} \Rightarrow A=..,\ B=..$ followed by an attempt at the sum (or with 1 instead of 10). (Induction may be attempted – see alt for (a).)	M1	3.1a
	$\frac{10}{r^2+8r+15}=\frac{5}{r+3}-\frac{5}{r+5}$ or $\frac{1}{r^2+8r+15}=\frac{1/2}{r+3}-\frac{1/2}{r+5}$	B1	1.1b
	$\sum_{r=1}^{n}\frac{10}{r^2+8r+15}=5\sum_{r=1}^{n}\left(\frac{1}{r+3}-\frac{1}{r+5}\right)$ $=5\left[\left(\frac{1}{4}-\frac{1}{6}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{6}-\frac{1}{8}\right)+\cdots+\left(\frac{1}{n+3}-\frac{1}{n+5}\right)\right]$	M1	2.1
	$=5\left(\frac{1}{4}+\frac{1}{5}-\frac{1}{n+4}-\frac{1}{n+5}\right)$	A1ft	1.1b
	$=5\left(\frac{5(n+4)(n+5)+4(n+4)(n+5)-20(n+5)-20(n+4)}{20(n+4)(n+5)}\right)=\ldots$	M1	2.1
	$=\frac{9n^2+41n}{4(n+4)(n+5)}$ (So k = 4)	A1	1.1b
		(6)	
(b)	As $n\to\infty,\ T_n\to\frac{9}{4}$ or appropriate investigation tried.	M1	3.4
	Since the sum is increasing towards $\frac{9}{4}$ which is strictly less than 2.5 T_n can never reach 2.5, so the 2.5 million remaining tonnes of coal will not all be mined no matter how long the company keeps mining.	A1	3.2b
		(2)	
(c)	In the first 20 years $T_{20}=\frac{221}{120}$ million tonnes of coal have been mined, so $2.5-\frac{221}{120}=\frac{79}{120}$ tonnes remain.	M1	2.2b
	Hence $\frac{79}{120\times 20}$ extra tonnes per year need mining, so the new model is $M_r=\frac{79}{2400}+\frac{10}{r^2+8r+15}$.	A1ft	3.5c
		(2)	
			(10 marks)

Notes:

(a)
M1: Attempts the sum using an appropriate method – ie method of differences. An attempt at partial fractions would evidence the attempt.
B1: Correct split into partial fractions.

M1: Applies method of differences showing evidence of the cancelling terms. The 5 may be missing at this stage and included later.

A1ft: Correct non-cancelling terms identified. Follow through their split into partial fractions if it leads to most terms cancelling.

M1: Puts the terms over a common denominator and simplifies. May be done in stages with the numerical fractions combined first etc, but look for appropriately adapted numerators for their method.
A1: Correct form with $k = 4$.

(b)

M1: Investigates the long term behaviour, e.g. by trying large values of n in the expression to see what happens, or by considering the long term limit.

A1: As scheme, comments that since the limit of the sum as $n \to \infty$ is 9/4 then the total amount of coal mined will never exceed 2.25 million tonnes, and so the coal will not all be mined even after a long time.

(c)

M1: Calculates the shortfall between 2.5 and the value of the sum at $n = 20$.

A1ft: Correct adaptation of the model adding (their shortfall)/20 to the original expression.

Alt (a)	Use of induction: Look for an attempt to find the value of k using $n = 1$ followed by an attempt at the inductive hypothesis.	M1	3.1a
	$n=1 \Rightarrow \dfrac{10}{1+8+15} = \dfrac{9+41}{k(5)(6)} \Rightarrow k=4$	B1	1.1b
	Assume true for $n = p$, so $\sum_{r=1}^{p+1} M_r = \dfrac{9p^2+41p}{\text{"4"}(p+4)(p+5)} + \dfrac{10}{(p+1)^2+8(p+1)+15}$ $= \dfrac{9p^2+41p}{\text{"4"}(p+4)(p+5)} + \dfrac{10}{(p+4)(p+6)}$	M1	2.1
	$= \dfrac{(9p^2+41p)(p+6)+10\times\text{"4"}(p+5)}{\text{"4"}(p+4)(p+5)(p+6)}$	M1 A1ft	1.1b 1.1b
	$= \dfrac{9p^3+95p^2+286p+200}{4(p+4)(p+5)(p+6)} = \dfrac{(p+4)(p+1)(9p+50)}{4(p+4)(p+5)(p+6)}$ $= \dfrac{(p+1)\left[9(p+1)^2+41\right]}{4((p+1)+4)((p+1)+5)}$ Hence true for $n = 1$ (with $k = 4$) and if true for $n = p$ then true for $n = p+1$ so true for all positive integers n.	A1	2.1
		(6)	

M1: For use of induction look for an attempt to find the value of k first, followed by an attempt at proving the inductive step.

B1: Deduces $k = 4$.

M1: Assumes true for some p and uses their k in the expression for T_p (may use k instead of p, which is fine if there is no confusion as they have a value in the expression).

M1: Attempts to combine over a common denominator.

A1ft: Correct single fraction expression, follow through their k.
A1: Completes the induction step and make a suitable conclusion.

4. Prove that, for $n \in \mathbb{Z}, n \geqslant 0$

$$\sum_{r=0}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{(n+a)(n+b)}{c(n+2)(n+3)}$$

where a, b and c are integers to be found.

(5)

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A1: Correct area of the glass following fully correct working. **Do not award for the correct answer following incorrect working.** M1: Subtracts their area of the glass from their area of the rectangle, as long as it does not give a negative area A1: awrt 0.155 or awrt $0.155\,\text{m}^2$ (If the units are stated they must be correct) **Note:** Using a calculator to find the area scores a maximum of B1M0M0A0M0A0M1A1

Question	Scheme	Marks	AOs
4	$\frac{1}{(r+1)(r+2)(r+3)} \equiv \frac{A}{r+1}+\frac{B}{r+2}+\frac{C}{r+3} \Rightarrow A=..., B=..., C=...$ $\left(\text{NB } A=\frac{1}{2} \quad B=-1 \quad C=\frac{1}{2}\right)$	M1	3.1a
	$r=0$ $\quad \frac{1}{2}\left[\frac{1}{1}-\frac{2}{2}+\frac{1}{3}\right]$ or $\frac{1}{2\ 1}-\frac{1}{2}+\frac{1}{2\ 3}$ or $\frac{1}{2}-\frac{1}{2}+\frac{1}{6}$ $r=1$ $\quad \frac{1}{2}\left[\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right]$ or $\frac{1}{2\ 2}-\frac{1}{3}+\frac{1}{2\ 4}$ or $\frac{1}{4}-\frac{1}{3}+\frac{1}{8}$ $r=n-1$ $\quad \frac{1}{2}\left[\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right]$ or $\frac{1}{2\ n}-\frac{1}{n+1}+\frac{1}{2\ n+2}$ or $\frac{1}{2n}-\frac{1}{n+1}+\frac{1}{2n+4}$ $r=n$ $\quad \frac{1}{2}\left[\frac{1}{n+1}-\frac{2}{n+2}+\frac{1}{n+3}\right]$ or $\frac{1}{2\ n+1}-\frac{1}{n+2}+\frac{1}{2\ n+3}$ or $\frac{1}{2n+2}-\frac{1}{n+2}+\frac{1}{2n+6}$	M1	2.1
	$\frac{1}{2}-\frac{1}{2}+\frac{1}{4}+\frac{1}{2(n+2)}-\frac{1}{n+2}+\frac{1}{2(n+3)}$ or $\frac{1}{4}-\frac{1}{2(n+2)}+\frac{1}{2(n+3)}$	A1	1.1b
	$=\frac{n^2+5n+6+2n+6-4n-12+2n+4}{4(n+2)(n+3)}$	M1	1.1b
	$=\frac{(n+1)(n+4)}{4(n+2)(n+3)}$	A1	2.2a
		(5)	
			(5 marks)

Notes
M1: A complete strategy to find *A*, *B* and *C* e.g. partial fractions. Allow slip when finding the constant but must be the correct form of partial fractions and correct identity. M1: Starts the process of differences to identify the relevant fractions at the start and end. Must have attempted a minimum of $r=0,\quad r=1,\quad \ldots\quad r=n-1$ and $r=n$ Follow through on their values of *A*, *B* and C. Look for $r=0\to\ \frac{A}{1}-\frac{B}{2}+\frac{C}{3}$ $\qquad r=1\to\ \frac{A}{2}-\frac{B}{3}+\frac{C}{4}$ $r=n-1\to\ \frac{A}{n}-\frac{B}{n+1}+\frac{C}{n+2}$ $\qquad r=n\to\ \frac{A}{n+1}-\frac{B}{n+2}+\frac{C}{n+3}$ A1: Correct fractions from the beginning and end that do not cancel stated. M1 Combines all 'their' fractions (at least two algebraic fractions) over their correct common denominator, does not need to be the lowest common denominator (allow a slip in the numerator). A1: Correct answer. **Note**: if they start with $r=1$ the maximum they can score is M1M0A0M1A0 **Note:** Proof by induction gains no marks

Question	Scheme	Marks	AOs
5(a)	The tank initially contains 100L. 3 L are entering every minute and 2 L are leaving every minute so overall 1 L increase in volume each minute so the tank contains 100 + *t* litres after *t* minutes	M1	3.3
	2 L leave the tank each minute and if there are *S*g of salt in the tank, the concentration will be $\frac{S}{100+t}\,g/L$ so salt leaves the tank at a rate of $2\times\frac{S}{100+t}\,g$ per minute	M1	3.3
	Salt enters the tank at a rate of $3\times 1g$ per minute	B1	2.2a
	$\therefore\ \frac{\mathrm{d}S}{\mathrm{d}t}=3-\frac{2S}{100+t}$ * cso	A1*	1.1b
		(4)	
(b)	$\frac{\mathrm{d}S}{\mathrm{d}t}+\frac{2S}{100+t}=3$		
	$I=\mathrm{e}^{\int\frac{2}{100+t}\mathrm{d}t}=(100+t)^2\Rightarrow S(100+t)^2=\int 3(100+t)^2\mathrm{d}t$	M1	3.1b
	$S(100+t)^2=(100+t)^3\,(+c)$ OR $S(100+t)^2=30\,000t+300t^2+t^3\,(+c)$	A1	1.1b
	$t=0,\ S=0\Rightarrow c=-10^6$	M1	3.4
	$t=10\Rightarrow S=100+10-\frac{10^6}{(100+10)^2}$	dM1	1.1b

2. (a) Use the Maclaurin series expansion for $\cos x$ to determine the series expansion of $\cos^2\left(\frac{x}{3}\right)$ in ascending powers of x, up to and including the term in x^4

Give each term in simplest form.

(2)

(b) Use the answer to part (a) and calculus to find an approximation, to 5 decimal places, for

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1}{x}\cos^2\left(\frac{x}{3}\right)\right) \mathrm{d}x$$

(3)

(c) Use the integration function on your calculator to evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1}{x}\cos^2\left(\frac{x}{3}\right)\right) \mathrm{d}x$$

Give your answer to 5 decimal places.

(1)

(d) Assuming that the calculator answer in part (c) is accurate to 5 decimal places, comment on the accuracy of the approximation found in part (b).

(1)

P 6 6 7 9 6 A 0 6 3 6

Question	Scheme	Marks	AOs
2 (a)	$\cos^2\frac{x}{3}=\left(1-\frac{\left(\frac{x}{3}\right)^2}{2}+\frac{\left(\frac{x}{3}\right)^4}{24}-...\right)^2=...$ or $\left(1-\frac{x^2}{18}+\frac{x^4}{1944}-...\right)^2=...$ or $\frac{1}{2}\left(1\pm\cos\frac{2x}{3}\right)=\frac{1}{2}\left(1\pm\left(1-\frac{1}{2}\left(\frac{2x}{3}\right)^2+\frac{1}{4!}\left(\frac{2x}{3}\right)^4-\right)...\right)$	M1	2.2a
	$=1-\frac{x^2}{9}+\frac{1}{243}x^4$	A1	1.1b
		(2)	
(b)	$\int\frac{1-\frac{x^2}{9}+\frac{1}{243}x^4}{x}=\int\frac{1}{x}-\frac{x}{9}+\frac{1}{243}x^3=A\ln x+Bx^2+Cx^4$ where A, B and $C\neq 0$	M1	3.1a
	$\ln x-\frac{x^2}{18}+\frac{1}{972}x^4$	A1ft	1.1b
	= awrt 0.98295	A1	2.2a
		(3)	
(c)	Calculator = awrt 0.98280	B1	1.1b
		(1)	
(d)	E.g. the approximation is correct to 3 d.p.	B1	3.2b
		(1)	
			(7 marks)

Notes:

(a)

M1: Deduces the required series by using the Maclaurin series for cos x, replacing x with $\frac{x}{3}$ and squares, or first applying the double angle identity (allow sign error) and then applying the series for cos x with $\frac{2x}{3}$. Attempts at finding from differentiation score M0 as the cosine series is required.

A1: Correct series

(b)

M1: Divides their series in part (a) by x and integrates to the form $A\ln x+Bx^2+Cx^4$

A1ft: Correct integration, follow through on their coefficients and need not be simplified.

A1: Deduces the definite integral awrt 0.98295

(c)

B1: Correct value.

(d)

B1: Makes a quantitative statement about the accuracy, so e.g. how many decimal places or significant figures it is correct to, or calculates a percentage accuracy to deduce it is reasonable. Do not accept just "underestimate" or similar without quantitative evidence. Allow for a reasonable comment as long as (b) is correct to at least 2 s.f. but (c) must be the correct value.

3. $$f(x) = \arcsin x \qquad -1 \leqslant x \leqslant 1$$

(a) Determine the first two non-zero terms, in ascending powers of x, of the Maclaurin series for $f(x)$, giving each coefficient in its simplest form.

(4)

(b) Substitute $x = \frac{1}{2}$ into the answer to part (a) and hence find an approximate value for π

Give your answer in the form $\frac{p}{q}$ where p and q are integers to be determined.

(2)

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Question	Scheme	Marks	AOs
3(a)	$f'(x) = A(1-x^2)^{-\frac{1}{2}}$ $f''(x) = Bx(1-x^2)^{-\frac{3}{2}}$ and $f'''(x) = C(1-x^2)^{-\frac{3}{2}} + Dx^2(1-x^2)^{-\frac{5}{2}}$ or $\dfrac{C(1-x^2)^{\frac{3}{2}} + Dx^2(1-x^2)^{\frac{1}{2}}}{(1-x^2)^3}$	M1	2.1
	$f'(x) = (1-x^2)^{-\frac{1}{2}}$ or $\dfrac{1}{\sqrt{1-x^2}}$ $f''(x) = x(1-x^2)^{-\frac{3}{2}}$ or $\dfrac{x}{(1-x^2)^{\frac{3}{2}}}$ and $f'''(x) = (1-x^2)^{-\frac{3}{2}} + 3x^2(1-x^2)^{-\frac{5}{2}}$ or $\dfrac{1}{(1-x^2)^{\frac{3}{2}}} + \dfrac{3x^2}{(1-x^2)^{\frac{5}{2}}}$ from quotient rule $\dfrac{(1-x^2)^{\frac{3}{2}} + 3x^2(1-x^2)^{\frac{1}{2}}}{(1-x^2)^3}$	A1	1.1b
	Finds $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$ and applies the formula $f(x) = f(0) + f'(0)x + f''(0)\dfrac{x^2}{2} + f'''(0)\dfrac{x^3}{6}$ $\{f(0) = 0,\ f'(0) = 1,\ f''(0) = 0,\ f'''(0) = 1\}$	M1	1.1b
	$f(x) = x + \dfrac{x^3}{6}$ cso	A1	1.1b
		(4)	
(b)	$\arcsin\left(\dfrac{1}{2}\right) = \dfrac{1}{2} + \dfrac{(1/2)^3}{6} = \dfrac{\pi}{6} \Rightarrow \pi = ...$	M1	1.1b
	$\pi = \dfrac{25}{8}$ o.e.	A1ft	2.2b
		(2)	
			(6 marks)

Notes:

(a)

M1: Finds the correct form of the first three derivatives, may be unsimplified – the third may come later.

A1: Correct first three derivatives, may be unsimplified – the third may come later.

M1: Finds $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$ and applies to the correct formula, needs to go up to x^3.

A1: $x + \dfrac{x^3}{6}$ cso ignore any higher terms whether correct or not

Special case: If they think that their $f''(0) \neq 0$ then maximum score M1 A0 M1 A0

M1 for correct form of the first two derivatives

M1 Correctly uses their $f(0)$, $f'(0)$, $f''(0)$ and applies to the correct formula

Note: If candidates do not find the first three derivatives but use $f(0)=0,\ f'(0)=1,\ f''(0)=0,\ f'''(0)=1$ and use these correctly in the formula this can score M0 A0 M1 A0
(b) **M1:** Substitutes $x=\frac{1}{2}$ into both sides and rearranges to find $\pi = \ldots$ **A1ft:** Infers that $\pi=\frac{25}{8}$ o.e. Follow through their $6f\left(\frac{1}{2}\right)$

4. (a) Use the method of differences to prove that for $n > 2$

$$\sum_{r=2}^{n} \ln\left(\frac{r+1}{r-1}\right) \equiv \ln\left(\frac{n(n+1)}{2}\right)$$

(4)

(b) Hence find the exact value of

$$\sum_{r=51}^{100} \ln\left(\frac{r+1}{r-1}\right)^{35}$$

Give your answer in the form $a\ln\left(\frac{b}{c}\right)$ where a, b and c are integers to be determined.

(3)

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Question	Scheme	Marks	AOs
4(a)	Applies $ln\left(\frac{r+1}{r-1}\right) = ln(r+1) - ln(r-1)$ to the problem in order to apply differences.	M1	3.1a
	$\sum_{r=2}^{n}(ln(r+1) - ln(r-1))$ $= (ln(3) - ln(1)) + (ln(4) - ln(2)) + (ln(5) - ln(3)) + \ldots$ $+(ln(n) - ln(n-2)) + (ln(n+1) - ln(n-1))$	dM1	1.1b
	$ln(n) + ln(n+1) - ln\,2$	A1	1.1b
	$ln\left(\frac{n(n+1)}{2}\right)$ * cso	A1 *	2.1
		(4)	
(b)	$\sum_{r=51}^{100} ln\left(\frac{r+1}{r-1}\right) = \sum_{r=2}^{100} ln\left(\frac{r+1}{r-1}\right) - \sum_{r=2}^{50} ln\left(\frac{r+1}{r-1}\right)$ $= ln\left(\frac{100 \times 101}{2}\right) - ln\left(\frac{50 \times 51}{2}\right)$	M1	1.1b
	$\sum_{r=51}^{100} ln\left(\frac{r+1}{r-1}\right)^{35} = 35\,ln\left(\frac{100\times101}{2} \div \frac{50\times51}{2}\right)$	M1	3.1a
	$= 35\,ln\left(\frac{202}{51}\right)$	A1	1.1b
		(3)	
		(7 marks)	

Notes:

(a)

M1: Uses the subtraction laws of logs to start the method of differences process.

dM1: Demonstrates the method of differences process, should have a minimum of e.g $r = 2$, $r = 3$, $r = 4$, $r = n - 1$ and $r = n$ shown -- enough to establish *at least one cancelling term* and *all non-disappearing terms* though the latter may be implied by correct extraction if only the first few cases are shown. Allow this mark if an extra term for $r = 1$ has been included.

A1: Correct terms that do not cancel - must not contradict their list of terms so e.g. if $r = 1$ was included, then A0A0 follows. The ln 1 may be included for this mark.

A1*: Achieves the printed answer, with no errors or omissions **and** must have had a complete list (as per dM1) before extraction (but condone missing brackets on ln terms). If working with r throughout, they must replace by n to gain the last A, but all other marks are available.

NB For attempts at combining log terms instead of using differences, full marks may be awarded for the equivalent steps, but attempts that do not make progress in combining terms will score no marks.

(b) Condone a bottom limit of 0 or 1 being used throughout part (b).

M1: Attempts to split into (the sum up to 100) – (the sum up to k) where k is 49, 50 or 51 **and** apply the result of (a) in some way. Condone slips with the power.

M1: Having attempted to apply (a), uses difference and power log laws correctly to reach an expression of the required form.

A1: Correct answer**.** Accept equivalents in required form, such as $35\,ln\frac{5050}{1275}$