Cp1Ch9 XMQs and MS

(Total: 126 marks)

1. CP1_Sample Q8 . 7 marks - CP1ch9 Vectors
2. CP2_Sample Q2 . 8 marks - CP1ch9 Vectors
3. CP1_Specimen Q5 . 9 marks - CP1ch9 Vectors
4. CP1_2019 Q7 . 7 marks - CP1ch9 Vectors
5. CP1_2020 Q4 . 9 marks - CP1ch9 Vectors
6. CP1_2021 Q7 . 8 marks - CP1ch9 Vectors
7. CP2_2022 Q8 . 13 marks - CP1ch9 Vectors
8. CP(AS)_2018 Q4 . 11 marks - CP1ch9 Vectors
9. CP(AS)_2019 Q4 . 5 marks - CP1ch9 Vectors
10. CP(AS)_2019 Q8 . 12 marks - CP1ch9 Vectors
11. CP(AS)_2020 Q4 . 13 marks - CP1ch9 Vectors
12. CP(AS)_2021 Q6 . 11 marks - CP1ch9 Vectors
13. CP(AS)_2022 Q6 . 13 marks - CP1ch9 Vectors

8. The line l_1 has equation $\dfrac{x-2}{4} = \dfrac{y-4}{-2} = \dfrac{z+6}{1}$

The plane Π has equation $x - 2y + z = 6$

The line l_2 is the reflection of the line l_1 in the plane Π.

Find a vector equation of the line l_2

(7)

Question	Scheme	Marks	AOs
8	$2+4\lambda-2(4-2\lambda)-6+\lambda=6 \Rightarrow \lambda=...$	M1	1.1b
	$\lambda=2 \Rightarrow$ Required point is $(2+2(4),\ 4+2(-2),\ -6+2(1))$ $(10,\ 0,\ -4)$	A1	1.1b
	$2+t-2(4-2t)-6+t=6 \Rightarrow t=...$	M1	3.1a
	$t=3$ so reflection of $(2,4,-6)$ is $(2+6(1),4+6(-2),-6+6(1))$	M1	3.1a
	$(8,\ -8,\ 0)$	A1	1.1b
	$\begin{pmatrix}10\\0\\-4\end{pmatrix}-\begin{pmatrix}8\\-8\\0\end{pmatrix}=\begin{pmatrix}2\\8\\-4\end{pmatrix}$	M1	3.1a
	$\mathbf{r}=\begin{pmatrix}10\\0\\-4\end{pmatrix}+k\begin{pmatrix}1\\4\\-2\end{pmatrix}$ or equivalent e.g. $\left(\mathbf{r}-\begin{pmatrix}10\\0\\-4\end{pmatrix}\right)\times\begin{pmatrix}1\\4\\-2\end{pmatrix}=\mathbf{0}$	A1	2.5
		(7)	
			(7 marks)

Notes:

M1: Substitutes the parametric equation of the line into the equation of the plane and solves for λ

A1: Obtains the correct coordinates of the intersection of the line and the plane

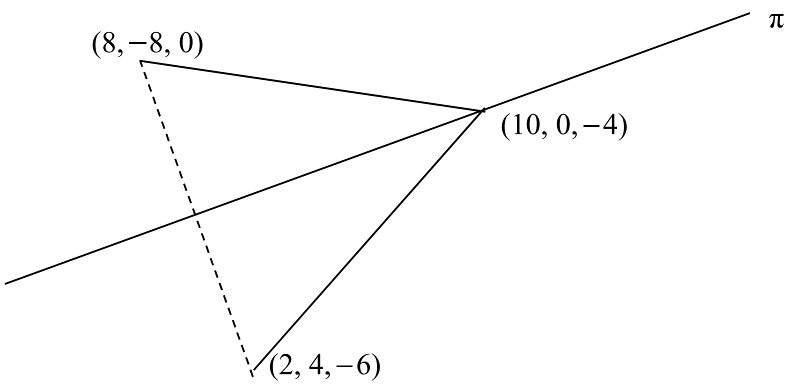
M1: Substitutes the parametric form of the line perpendicular to the plane passing through (2, 4, −6) into the equation of the plane to find t

M1: Find the reflection of (2, 4, −6) in the plane

A1: Correct coordinates

M1: Determines the direction of l by subtracting the appropriate vectors

A1: Correct vector equation using the correct notation



2. The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

where λ and μ are scalar parameters.

(b) Show that the vector $-\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ is perpendicular to Π_2

(2)

(c) Show that the acute angle between Π_1 and Π_2 is 52° to the nearest degree.

(3)

Question	Scheme	Marks	AOs
2(a)	$\begin{pmatrix}3\\-4\\2\end{pmatrix}\bullet\begin{pmatrix}6\\2\\12\end{pmatrix}=18-8+24$	M1	3.1a
	$d=\dfrac{18-8+24-5}{\sqrt{3^2+4^2+2^2}}$	M1	1.1b
	$=\sqrt{29}$	A1	1.1b
		(3)	
(b)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\5\end{pmatrix}=\ldots$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\ldots$	M1	2.1
	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\5\end{pmatrix}=0$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=0$ $\therefore\ -\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is perpendicular to Π_2	A1	2.2a
		(2)	
(c)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}3\\-4\\2\end{pmatrix}=-3+12+2$	M1	1.1b
	$\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}\cos\theta=11$ $\Rightarrow\cos\theta=\dfrac{11}{\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}}$	M1	2.1
	So angle between planes $\theta=52^\circ$ *	A1*	2.4
		(3)	
		(8 marks)	

Notes:

(a)

M1: Realises the need to and so attempts the scalar product between the normal and the position vector

M1: Correct method for the perpendicular distance

A1: Correct distance

(b)

M1: Recognises the need to calculate the scalar product between the given vector and both direction vectors

A1: Obtains zero both times and makes a conclusion

(c)

M1: Calculates the scalar product between the two normal vectors

M1: Applies the scalar product formula with their 11 to find a value for cos θ

A1*: Identifies the correct angle by linking the angle between the normal and the angle between the planes

5.

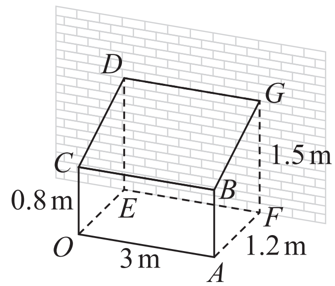


Figure 2

Figure 2 shows a sketch of a shelter against a wall. The shelter consists of two rectangular wooden boards, *OABC* and *BCDG*, which can be modelled as parts of planes. Board *OABC* is vertical and parallel to the wall and the ground may be assumed to be horizontal.
The points *E* and *F* are at the foot of the wall directly below *D* and *G* respectively.

The length *OC* is 0.8 m, the length *OA* is 3 m and the board *OABC* is 1.2 m away from the wall. The points *D* and *G* are 1.5 m above the ground.

To model the shelter, take *O* as the origin, the vector **i** to be 1 m in the direction of $\overrightarrow{OA}$, the vector **j** to be 1 m in the direction of $\overrightarrow{OE}$ and the vector **k** to be 1 m in the direction of $\overrightarrow{OC}$.

(a) Find an equation of the plane *BCDG*, giving your answer in the form $\mathbf{r} \bullet \mathbf{n} = d$

(5)

In order to support the roof of the shelter, one end of a pole is attached to the ground at the centre of the rectangle *OAFE* and the other end to a point on the roof. Modelling the pole as a rod,

(b) find, to the nearest mm, the shortest possible length for the pole.

(3)

(c) State a limitation of the assumption that the boards can be modelled as planes.

(1)

Question	Scheme	Marks	AOs
5(a)	$\overrightarrow{OC} = 0.8\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + 0.8\mathbf{k}$ and $\overrightarrow{OD} = 1.2\mathbf{j} + 1.5\mathbf{k}$, or $\overrightarrow{CB} = 3\mathbf{i}$, and $\overrightarrow{CD} = 1.2\mathbf{j} + 0.7\mathbf{k}$	B1	3.3
	So plane has equation **r** = their $\overrightarrow{OC}$ + their $\lambda\overrightarrow{CB}$ + their $\mu\overrightarrow{CD}$ (oe) OR $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k}).(3\mathbf{i}) = 0$ and $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k}).(1.2\mathbf{j}+0.7\mathbf{k}) = 0$ leading to a =..., b =... and c =... (may use vector product)	M1	1.1b
	Equation is $\mathbf{r} = 0.8\mathbf{k} + \lambda(3\mathbf{i}) + \mu(1.2\mathbf{j} + 0.7\mathbf{k})$ OR normal is $\mathbf{n} = p(7\mathbf{j} - 12\mathbf{k})$	A1	1.1b
	$x = 3\lambda$, $y = 1.2\mu$ and $z = 0.8 + 0.7\mu \Rightarrow 70y - 120z = -96$ OR $(0.8\mathbf{k}).(7\mathbf{j} - 12\mathbf{k}) = -9.6 \Rightarrow d = -9.6$	M1	1.1b
	Equation is $\mathbf{r}.(7\mathbf{j} - 12\mathbf{k}) = -9.6$ (or a multiple e.g. $\mathbf{r}.(70\mathbf{j} - 120\mathbf{k}) = -96$)	A1	2.5
		(5)	
(b)	Full attempt to find the minimum distance from the centre of the base rectangle to the plane – e.g. using the distance formula for closest point, or first finding the intersection point then finding the distance. Must have correct starting point (1.5,0.6,0).	M1	3.1b
	E.g. Minimum distance $= \dfrac{\lvert 0\times1.5 + 7\times0.6 + (-12)\times0 + 9.6\rvert}{\sqrt{0^2 + 7^2 + (-12)^2}} = \ldots$	M1	3.4
	= 0.993 m or 99.3 cm or 993 mm (to 3 s.f.) Accept awrt.	A1	1.1b
		(3)	
(c)	E.g. the boards will not have negligible thickness, which should be taken into account in the model, or wooden boards will bow and so not form planes.	B1	3.5b
		(1)	
			(9 marks)

Notes:

(a) Accept use of column vectors throughout.

B1: Identifies three points on or two vectors in the plane that can be used to set up the model.

M1: Attempts a plane equation with their vectors OR attempts to find a normal vector using scalar (or cross) product.

A1: Correct plane equation OR correct normal vector (any multiple).

M1: Solves $x = 3\lambda$, $y = 1.2\mu$ and $z = 0.8 + 0.7\mu$ to find equation x, y and z. OR Applies **r**.**n** = d with a point on the line and their **n** to find d.

A1: Correct equation of plane in the correct form **r**.**n** = d, as shown or a multiple thereof.

(b)

M1: See scheme. Alternative methods can be used (e.g find p required for $\mathbf{r}=1.5\mathbf{i}+0.6\mathbf{j}+p(7\mathbf{j}-12\mathbf{k})$ to intersect the plane).

M1: Uses the model to attempt the minimum distance from any point to the plane, or an attempt to find the value of p for the point of intersection for the minimum distance.

A1: Correct answer awrt 993 mm or equivalent in m or cm.

(c) B1: Any reasonable limitation about the boards - e.g. those in the scheme.

5(a) **Alt**	Sets up equation of plane as $ax + by + c = d$	B1	3.3
	Identifies at least three points on the plane and substitutes in to the equation to form simultaneous equations. E.g. (3,0,0.8), (0,0,0.8), (0,1.2,1.5) and (3,1.2,1.5) give $3a + 0.8c = d$ $0.8c = d$ $1.2b + 1.5c = d$ $3a + 1.2b + 1.5c = d$ Note may use d = 1 with only 3 equations.	M1	1.1b
	Solves to find correct corresponding values. E.g. With d = 1, c = 1.25, a = 0 and b = $-\frac{35}{48}$ (so accept any appropriate multiples)	A1	1.1b
	Forms plane equation in correct form with their values. E.g. $-\frac{35}{48}y + \frac{5}{4}z = 1 (\Rightarrow 35y - 60z = -48) \Rightarrow \mathbf{r}\cdot\mathbf{n} = d$	M1	1.1b
	Equation is $\mathbf{r}\cdot(35\mathbf{i} - 60\mathbf{j}) = -48$ (or any multiple)	A1	2.5
		(5)	

(a) Alt

B1: Sets up appropriate Cartesian plane equation for the model.

M1: Identifies at least three points on the plane and forms simultaneous equations using them in the general equation.

A1: Solves the equations to find correct values for the coefficients (may be a common multiple of the ones shown).

M1: Uses their coefficients in their Cartesian equation to form an equation for the plane in the correct form.

A1: Correct equation of plane in the correct form **r**.**n** = d, as shown or a multiple thereof.

7. The line l_1 has equation

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$$

The line l_2 has equation

$$\mathbf{r} = \mathbf{i} + 3\mathbf{k} + t(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

where t is a scalar parameter.

(a) Show that l_1 and l_2 lie in the same plane.

(3)

(b) Write down a vector equation for the plane containing l_1 and l_2

(1)

(c) Find, to the nearest degree, the acute angle between l_1 and l_2

(3)

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">7(a)

Way 1</td><td colspan="2">$1+2\lambda=1+t$
$-1-\lambda=-t$
$4+3\lambda=3+2t$
$\Rightarrow t=\ldots \text{or } \lambda=\ldots$</td><td>M1</td><td>3.1a</td></tr>
<tr><td colspan="2">Checks the third equation with $t=2$ and $\lambda=1$
Or shows that the coordinate (3, -2, 7) lies on both lines</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2">As the lines intersect at a point the lines lie in the same plane.</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td rowspan="4">(a)
Way 2</td><td>$1=1+2\lambda+t$
$-1=\quad-\lambda-t$
$4=3+3\lambda+2t$
$\Rightarrow t=\ldots \text{or } \lambda=\ldots$</td><td>$1=1+2\lambda+t$
$0=-1-\lambda-t$
$3=4+3\lambda+2t$
$\Rightarrow t=\ldots \text{or } \lambda=\ldots$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Checks the third equation with
$t=2$ and $\lambda=-1$</td><td>Checks the third equation with
$t=-2$ and $\lambda=1$</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2">Second coordinates lie on the plane; therefore, the lines lie on the same plane</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td rowspan="4">(a)
Way 3</td><td colspan="2">$x=1+t,\quad y=-t,\quad z=3+2t$
$\frac{1+t-1}{2}=\frac{-t+1}{-1}=\frac{3+2t-4}{3}$
Solves a pair of equations $t=\ldots$</td><td>M1</td><td>3.1a</td></tr>
<tr><td colspan="2">Solve two pairs of equations to find $t=2$</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2">As the lines intersect at a point the lines lie in the same plane.</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td>(a)
Way 4
(Using Further Pure 2 knowledge)</td><td colspan="2">$\begin{pmatrix}2\\-1\\3\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow 2x-y+3z=0$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow x-y+2z=0$
attempts to solve the equations to find a normal vector
OR
attempts the cross product $\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\ldots$
AND
either finds the equation of one plane OR finds dot product between the normal and one coordinate</td><td>M1</td><td>3.1a</td></tr>
</table>

	$\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ or $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ **OR** $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ or $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$		
	Achieves the correct planes containing **each** line $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ or $x-y-z=-2$ o.e. **OR** Shows that $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ **and** $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ o.e.	A1	1.1b
	Both planes are the same, therefore the lines lie in the same plane.	A1	2.4
		(3)	
(b)	e.g. $\mathbf{r}=\begin{pmatrix}1\\0\\3\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}1\\-1\\4\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}0\\-1\\1\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}.k\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2k$	B1	2.5
		(1)	
(c) **Way 1**	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\2\end{pmatrix}=2+1+6$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\cos\theta=9$ $\Rightarrow\cos\theta=\dfrac{9}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	

Way 2 (Using Further Pure 2 knowledge)	$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\sin\theta = \sqrt{1^2+(-1)^2+(-1)^2}$ $\Rightarrow \sin\theta = \dfrac{\sqrt{1^2+(-1)^2+(-1)^2}}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta = 11$ cao	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)

Allow using $\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ instead of $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$ for the method mark.

Way 1

M1: Starts by attempting to find where the two lines intersect. They must set up a parametric equation for line 1 (allow sign slips and as long as the intention is clear), forms simultaneous equations by equating coordinates and attempts to solve to find a value for $t = \ldots$ or $\lambda = \ldots$.

A1: Shows that there is a unique solution by checking the third equation or shows that the coordinate (3, -2, 7) lies on both lines.

A1: Achieves the correct values $t = 2$ and $\lambda = 1$, checks the third equation and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

Way 2

M1: Finds the vector equation of the plane with the both direction vectors and one coordinate (allow a sign slip), sets equal to the other coordinate, forms simultaneous equations and attempts to solve to find a value for $t = \ldots$ or $\lambda = \ldots$.

A1: Shows that the other coordinate lies on the plane by checking the third equation.

A1: Achieves the correct values $t = -2$ and $\lambda = 1$ or $t = 2$ and $\lambda = -1$ and concludes that the second coordinate lie on the plane; therefore, the lines lie on the same plane

Way 3

M1: Substitutes line 2 into line 1 and solves a pair of equations to find a value for *t*. Allow slip with the position of 0 and sign slips as long as the intention is clear.

A1: Solve two pairs of equations to achieve $t = 2$ for each.
A1: Achieves the correct value $t = 2$ and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

Way 4 (Using Further Pure 2 knowledge)
M1: A complete method to finds a vector which is normal to both lines and attempts to finds the equation of the plane containing one line.
A1: Achieves the correct equation for the plane containing each line.
A1: Conclusion, planes are the same, therefore the lines lie in the same plane.

(b) **This may be seen in part (a)**
B1: Correct **vector** equation allow any letter for the scalers.

Must start with **r** = … and uses two out of the following direction vectors $\pm\begin{pmatrix}1\\-1\\2\end{pmatrix}$, $\pm\begin{pmatrix}2\\-1\\3\end{pmatrix}$ or $\pm\begin{pmatrix}0\\-1\\1\end{pmatrix}$ and one of the following position vectors $\begin{pmatrix}1\\0\\3\end{pmatrix}$, $\begin{pmatrix}1\\-1\\4\end{pmatrix}$ or $\begin{pmatrix}3\\-2\\7\end{pmatrix}$

(c)
Way 1
M1: Calculates the scalar product between the direction vectors, allow one slip, if the intention is clear
dM1: Dependent on the previous method mark. Applies the scalar product formula with their scalar product to find a value for $\cos\theta$
A1: Correct answer only
Way 2 (Using Further Pure 2 knowledge)
M1: Calculates the vector product between the direction vectors, allow one slip, if the intention is clear
dM1: Dependent on the previous method mark. Applies the vector product formula with their vector product to find a value for $\sin\theta$
A1: Correct answer only

4. The plane Π_1 has equation

$$\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Find a Cartesian equation for Π_1

(4)

The line l has equation

$$\frac{x-1}{5} = \frac{y-3}{-3} = \frac{z+2}{4}$$

(b) Find the coordinates of the point of intersection of l with Π_1

(3)

The plane Π_2 has equation

$$\mathbf{r}.(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 5$$

(c) Find, to the nearest degree, the acute angle between Π_1 and Π_2

(2)

Question	Scheme	Marks	AOs
4(a)	Attempts normal vector: E.g. let $\mathbf{n}=a\mathbf{i}+b\mathbf{j}+\mathbf{k}$ then $a+2b-3=0,\ -a+2b+1=0$ $\Rightarrow a=...,b=...$ or $\mathbf{n}=(\mathbf{i}+2\mathbf{j}-3\mathbf{k})\times(-\mathbf{i}+2\mathbf{j}+\mathbf{k})$	M1	3.1a
	$\mathbf{n}=k(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$	A1	1.1b
	$(4\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}+4\mathbf{j}-\mathbf{k})=...$	M1	1.1b
	$4x+y+2z=10$	A1	2.5
		(4)	
	Alternative:		
	$x=2+\lambda-\mu$ $y=4+2\lambda+2\mu \Rightarrow$ $2x+y=8+4\lambda$, $y-2z=6+8\lambda$ $z=-1-3\lambda+\mu$	M1 A1	3.1a 1.1b
	$2(2x+y-8)=y-2z-6$ $(4x+y+2z=10)$	M1 A1	1.1b 2.5
		(4)	
(b)	$\dfrac{x-1}{5}=\dfrac{y-3}{-3}=\dfrac{z+2}{4}\Rightarrow \mathbf{r}=\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\lambda(5\mathbf{i}-3\mathbf{j}+4\mathbf{k})$ $4(1+5\lambda)+3-3\lambda+2(4\lambda-2)=10\Rightarrow\lambda=...$	M1	3.1a
	$\lambda=\dfrac{7}{25}\Rightarrow \mathbf{r}=\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\dfrac{7}{25}(5\mathbf{i}-3\mathbf{j}+4\mathbf{k})$	dM1	1.1b
	$\left(\dfrac{12}{5},\dfrac{54}{25},-\dfrac{22}{25}\right)$	A1	1.1b
		(3)	
	Alternative: $4x+\left(-\dfrac{3}{5}(x-1)+3\right)+2\left(\dfrac{4}{5}(x-1)-2\right)=10\Rightarrow x=...$	M1	3.1a
	$\Rightarrow y=...,\ z=...$	M1	1.1b
	$\left(\dfrac{12}{5},\dfrac{54}{25},-\dfrac{22}{25}\right)$	A1	1.1b
		(3)	
(c)	$(4\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}-\mathbf{j}+3\mathbf{k})=8-1+6=13$ $13=\sqrt{14}\sqrt{21}\cos\theta\Rightarrow\theta=...$	M1	1.1b
	$\theta=41°$	A1	1.1b
		(2)	
			(9 marks)

Notes
Accept equivalent vector notation throughout. (a) M1: Starts by attempting to find a normal vector using a correct method. Allow if there are sign errors in attempts at the cross product. A1: Obtains a correct normal vector M1: Attempts scalar product between their normal and a point in the plane A1: Correct Cartesian form (accept any equivalent Cartesian equation) Alternative M1: Uses the component form to eliminate one of the scalar parameters A1: Two correct equations with one parameter eliminated OR a correct equation for each parameter in terms of *x*, *y* and *z* M1: Forms a Cartesian equation A1: Correct Cartesian equation (accept any equivalent form) (b) M1: Interprets the Cartesian form to give a parametric form (allow sign slips) and substitutes this into their Cartesian equation and proceeds to find a value for their parameter. NB: Attempts at $\begin{pmatrix} 2+\lambda-\mu \\ 4+2\lambda+2\mu \\ -1-3\lambda+\mu \end{pmatrix} = \begin{pmatrix} 1+5\lambda \\ 3-3\lambda \\ -2+4\lambda \end{pmatrix}$ will score M0 as there are only two parameters, but $\begin{pmatrix} 2+\lambda-\mu \\ 4+2\lambda+2\mu \\ -1-3\lambda+\mu \end{pmatrix} = \begin{pmatrix} 1+5\gamma \\ 3-3\gamma \\ -2+4\gamma \end{pmatrix}$ leading to a value for γ from solving three equations in three unknowns in M1. dM1: Substitutes their parameter value back into the parametric form of the line. The parameter must have come from a correct attempt to find the value at intersection. A1: Correct coordinates. Accept as *x* = …, *y*=, … *z* = … or as a vector. Alternative: M1: Eliminates two of the variables from the equation of plane using the Cartesian equation of the line and solves the linear equation. dM1: Finds the other two coordinates. A1: Correct coordinates, as above. (c) M1: Complete and correct scalar product method leading to a value for θ. Note that if sinθ is used instead of cosθ then they must also apply 90 – θ to access the method. A1: Correct angle, accept awrt 41. as their final answer (do not isw if they go on to give e.g. $(180-41)°$)

7. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that vector $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ is perpendicular to Π.

(2)

(b) Hence find a Cartesian equation of Π.

(2)

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$$

where t is a scalar parameter.

The point A lies on l.

Given that the shortest distance between A and Π is $2\sqrt{29}$

(c) determine the possible coordinates of A.

(4)

Question	Scheme	Marks	AOs
7(a)	$\begin{pmatrix}-1\\2\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=-2+6-4=0$ and $\begin{pmatrix}2\\0\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=4+0-4=0$ Alt: $\begin{pmatrix}-1\\2\\1\end{pmatrix}\times\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}2\times1-1\times0\\-(-1\times1-1\times2)\\-1\times0-2\times2\end{pmatrix}=\ldots$	M1	1.1b
	As 2**i** + 3**j** – 4**k** is perpendicular to both direction vectors (two non-parallel vectors) of *Π* then it must be perpendicular to *Π*	A1	2.2a
		(2)	
(b)	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=\begin{pmatrix}3\\3\\2\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}\Rightarrow\ldots$	M1	1.1a
	$2x+3y-4z=7$	A1	2.2a
		(2)	
(c)	$\dfrac{\lvert 2(4+t)+3(-5+6t)-4(2-3t)-7\rvert}{\sqrt{2^2+3^2+(-4)^2}}=2\sqrt{29}\Rightarrow t=\ldots$	M1	3.1a
	$t=-\dfrac{9}{8}$ and $t=\dfrac{5}{2}$	A1	1.1b
	$\boldsymbol{r}=\begin{pmatrix}4\\-5\\2\end{pmatrix}-\dfrac{9}{8}\begin{pmatrix}1\\6\\-3\end{pmatrix}=\ldots$ or $\boldsymbol{r}=\begin{pmatrix}4\\-5\\2\end{pmatrix}+\dfrac{5}{2}\begin{pmatrix}1\\6\\-3\end{pmatrix}=\ldots$	M1	1.1b
	$\left(\dfrac{23}{8},-\dfrac{47}{4},\dfrac{43}{8}\right)$ and $\left(\dfrac{13}{2},10,-\dfrac{11}{2}\right)$	A1	2.2a
		(4)	
			(8 marks)

Notes:

(a)
M1: Attempts the scalar product of each direction vector and the vector 2**i** + 3**j** – 4**k.** Some numerical calculation is required, just "= 0" is insufficient. Alternatively, attempts the cross product (allow sign slips) with the two direction vectors.
A1: Shows that both scalar products = 0 (minimum $-2+6-4=0$ and $4-4=0$) and makes a minimal conclusion with no erroneous statements. If using cross product, the calculation must be correct, and a minimal conclusion given with no erroneous statements.

(b)

M1: Applies $\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \Rightarrow ...$

A1: $2x + 3y - 4z = 7$

(c)

M1: A fully correct method for finding a value of *t*. Other methods are possible, but must be valid and lead to a value of *t*. Examples of other methods:

- $2\sqrt{29} = \pm\left(\frac{2(4+t)+3(-5+6t)-4(2-3t)}{\sqrt{2^2+3^2+(-4)^2}} - \frac{7}{\sqrt{29}}\right)$ using plane parallel to Π through origin and shortest distance from plane to origin.
- $2(4+t)+3(-5+6t)-4(2-3t)=7 \Rightarrow t = t_i$ (*t* at intersection of line and plane) and $\sin\theta = \frac{(2,3,-4)^T.(1,6,-3)^T}{\sqrt{29}\sqrt{46}}$ (sine of angle between line and plane) followed by $\sin\theta = \frac{2\sqrt{29}}{k\sqrt{46}} \Rightarrow k = ... \Rightarrow t = t_i \pm k$

A1: Correct values for *t*. Both are required.

M1: Uses a value of *t* to find a set of coordinates for *A*.

A1: Both correct sets of coordinates for *A*.

8. Two birds are flying towards their nest, which is in a tree.

Relative to a fixed origin, the flight path of each bird is modelled by a straight line.

In the model, the equation for the flight path of the first bird is

$$\mathbf{r}_1 = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ a \\ 0 \end{pmatrix}$$

and the equation for the flight path of the second bird is

$$\mathbf{r}_2 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

where λ and μ are scalar parameters and a is a constant.

In the model, the angle between the birds' flight paths is 120°

(a) Determine the value of a.

(4)

(b) Verify that, according to the model, there is a common point on the flight paths of the two birds and find the coordinates of this common point.

(5)

The position of the nest is modelled as being at this common point.

The tree containing the nest is in a park.

The ground level of the park is modelled by the plane with equation

$$2x - 3y + z = 2$$

(c) Hence determine the shortest distance from the nest to the ground level of the park.

(3)

(d) By considering the model, comment on whether your answer to part (c) is reliable, giving a reason for your answer.

(1)

Question	Scheme	Marks	AOs
8(a)	A complete method to use the scalar product of the direction vectors and the angle 120° to form an equation in a $$\frac{\begin{pmatrix}2\\a\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}}{\sqrt{2^2+a^2}\sqrt{1^2+(-1)^2}}=cos\,1\,20$$	M1	3.1b
	$$\frac{a}{\sqrt{4+a^2}\sqrt{2}}=-\frac{1}{2}$$	A1	1.1b
	$2a=-\sqrt{4+a^2}\sqrt{2}\Rightarrow 4a^2=8+2a^2\Rightarrow a^2=4\Rightarrow a=...$	M1	1.1b
	$a=-2$	A1	2.2a
		(4)	
(b)	Any two of **i**: $-1+2\lambda=4$ (1) **j**: $5+\text{'their}-2\text{'}\lambda=-1+\mu$ (2) **k**: $2=3-\mu$ (3)	M1	3.4
	Solves the equations to find a value of $\lambda\left\{=\frac{5}{2}\right\}$ **and** $\mu\{=1\}$	M1	1.1b
	$r_1=\begin{pmatrix}-1\\5\\2\end{pmatrix}+\frac{5}{2}\begin{pmatrix}2\\\text{'their}-2\text{'}\\0\end{pmatrix}$ or $r_2=\begin{pmatrix}4\\-1\\3\end{pmatrix}+1\begin{pmatrix}0\\1\\-1\end{pmatrix}$	dM1	1.1b
	$(4,0,2)$ or $\begin{pmatrix}4\\0\\2\end{pmatrix}$	A1	1.1b
	Checks the third equation e.g. $\lambda=\frac{5}{2}$: **L HS** $=5-2\lambda=5-5=0$ $\mu=1$: **R HS** $=-1+\mu=-1+1=0$ therefore **common point/intersect/consistent/tick** or substitutes the values of λ and μ into the relevant lines and achieves the same coordinate	B1	2.1
		(5)	
(c)	Full attempt to find the minimum distance from the point of intersection (nest) to the plane (ground) E.g. Minimum distance $=\frac{\lvert 2\times\text{'4'}+(-3)\times\text{'0'}+1\times\text{'2'}-2\rvert}{\sqrt{2^2+(-3)^2+1)^2}}=...$ **Alternatively** $\mathbf{r}=\begin{pmatrix}\text{'4'}\\\text{'0'}\\\text{'2'}\end{pmatrix}+\lambda\begin{pmatrix}2\\-3\\1\end{pmatrix}$ $2(\text{'4'}+2\lambda)-3(\text{'0'}-3\lambda)+(\text{'2'}+\lambda)=2\Rightarrow$ $\lambda=...\left\{-\frac{4}{7}\right\}$	M1 A1ft	3.1b 3.4

	$\mathbf{r} = \begin{pmatrix} '4' \\ '0' \\ '2' \end{pmatrix} + ' - \frac{4}{7}' \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{20}{7} \\ \frac{12}{7} \\ \frac{10}{7} \end{pmatrix}$ Minimum distance = $\sqrt{\left(2 \times -\frac{4}{7}'\right)^2 + \left(-3 \times -\frac{4}{7}'\right)^2 + \left(1 \times -\frac{4}{7}\right)^2} =$ … $= \sqrt{\left('4' - '\frac{20}{7}'\right)^2 + \left('0' - '\frac{12}{7}'\right)^2 + \left('2' - '\frac{10}{7}'\right)^2} =$…		
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$ or awrt 2.1	A1	2.2b
		(3)	
	Alternative Find perpendicular distance from plane to the origin $2x - 3y + z = 2$ $\lvert n \rvert = \sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{14}$ shortest distance $= \frac{2}{\sqrt{14}}$ Find perpendicular distance from the plane containing the point of intersection to the origin $2x - 3y + z = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 10$ shortest distance $= \frac{10}{\sqrt{14}}$ Minimum distance = $\frac{10}{\sqrt{14}} - \frac{2}{\sqrt{14}}$	M1 A1ft	3.1b 3.4
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$ or awrt 2.1	A1	2.2b
		(3)	
(d)	For example Not reliable as the birds will not fly in a straight line Not reliable as angle between flights paths will not always be 120° Not reliable/reliable as the ground will not be flat/smooth Not reliable as bird's nest is not a point	B1	3.2b
		(1)	
			(13 marks)

Notes:

(a)

M1: See scheme, allow a sign slip and cos 60

A1: Correct simplified equation in a, cos 120 must be evaluated to – ½ and dot product calculated

Note: If the candidate states either $\left|\frac{a⍰b}{|a||b|}\right| = cos\ \theta$ or $\left|\frac{a}{\sqrt{4+a^2}\sqrt{2}}\right| = cos\ 6\ 0$then has the equation $\frac{a}{\sqrt{4+a^2}\sqrt{2}} = \frac{1}{2}$ award this mark. If the module of the dot product is not seen then award A0 for this equation.

dM1: Solve a quadratic equation for a, by squaring and solving an equation of the form $a^2 = K$ where $K > 0$ **A1:** Deduces the correct value of a from a correct equation. Must be seen in part (a) using the angle between the lines. **Alternative cross product method** **M1:** $\begin{vmatrix} 2 & a & 0 \\ 0 & 1 & -1 \end{vmatrix} = \sqrt{2^2 + a^2}\sqrt{1^2 + (-1)^2}\, sin\, 1\,20$ **A1:** $\sqrt{a^2 + 8} = \sqrt{4 + a^2}\sqrt{2}\frac{\sqrt{3}}{2}$ Then as above **Note If they use the point of intersection to find a value for *a* this scores no marks**
(b) **M1:** Uses the model to write down any two correct equations **M1:** Solve two equations simultaneously to find a value for μ and λ **dM1**: Dependent on previous method mark. Substitutes μ and λ into a relevant equation. If no method shown two correct ordinates implies this mark. **A1:** Correct coordinates. May be seen in part (c) **B1**: Shows that the values of μ and λ give the same third coordinate or point of intersection and draws the conclusion that the **lines intersect/common point/consistent** or tick. **Note:** If an incorrect value for *a* is found in part (a) but in part (b) they find that $a = -2$ this scores **B0** but all other marks are available
(c) This is M1M1A1 on ePen marking as M1 A1ft A1 **M1:** Full attempt to find the minimum distance from a point to a plane. Condone a sign slip with the value of *d*. **A1ft:** Following through on their point of intersection. Uses the model to find a correct expression for minimum distance from the nest to the ground **A1:** Correct distance **Alternative** **M1:** Find the shortest distance from a point to plane by finding the perpendicular distance from the given plane to the origin and the perpendicular distance from the plane contacting their point of intersection to the origin and subtracts **A1ft:** Following through on their point of intersection. Uses the model to find a correct expression for minimum distance from the nest to the ground **A1:** Correct distance
(d) **B1:** Comments on one of the models • Flight path of the birds modelled as a straight line • Angle between flight paths modelled as 120° • The bird’s nest is modelled as a point • Ground modelled as a plane Then states unreliabl Any correct answer seen, ignore any other incorrect answers

4. Part of the mains water system for a housing estate consists of water pipes buried beneath the ground surface. The water pipes are modelled as straight line segments. One water pipe, W, is buried beneath a particular road. With respect to a fixed origin O, the road surface is modelled as a plane with equation $3x - 5y - 18z = 7$, and W passes through the points $A(-1, -1, -3)$ and $B(1, 2, -3)$. The units are in metres.

(a) Use the model to calculate the acute angle between W and the road surface.

(5)

A point $C(-1, -2, 0)$ lies on the road. A section of water pipe needs to be connected to W from C.

(b) Using the model, find, to the nearest cm, the shortest length of pipe needed to connect C to W.

(6)

Question	Scheme	Marks	AOs
4(a)	Attempts the scalar product between the direction of W and the normal to the road and uses trigonometry to find an angle.	M1	3.1a
	$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\left(\begin{pmatrix}-1\\-1\\-3\end{pmatrix}-\begin{pmatrix}1\\2\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$	M1 A1	1.1b 1.1b
	$\sqrt{(2)^2+(3)^3+(0)^2}\sqrt{(3)^2+(-5)^3+(-18)^2}\cos\alpha="-9"$ $\theta=90-\arccos\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ or $\theta=\arcsin\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ Angle between pipe and road $=7.58^{\circ}$ (3sf) or 0.132 radians (3sf) (Allow -7.58° or -0.132 radians)	M1 A1	1.1b 3.2a
		(5)	
(b)	$W:\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}$ or $\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$C\text{ to }W:\left\{\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$ or $\left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$	M1	3.4
	$\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow t=\ldots$ or $\begin{pmatrix}2+2\lambda\\4+3\lambda\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow\lambda=\ldots$ or $(2t)^2+(3t+1)^2+(-3)^2=\ldots$ or $(2+2t)^2+(4+3t)^2+(-3)^2=\ldots$	M1	3.1b
	$t=-\frac{3}{13}$ or $\lambda=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ or $(2t)^2+(3t+1)^2+(-3)^2=13\left(t+\frac{3}{13}\right)^2+\frac{121}{13}$ or $(2+2t)^2+(4+3t)^2+(-3)^2=13\left(\lambda+\frac{16}{13}\right)^2+\frac{121}{13}$ or $\frac{\mathrm{d}\left((2t)^2+(3t+1)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\frac{3}{13}\Rightarrow C\text{ to }W\text{ is }-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ Or $\frac{\mathrm{d}\left((2+2t)^2+(4+3t)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}\text{ is }-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$	A1	1.1b
	$d=\sqrt{\left(-\frac{6}{13}\right)^2+\left(\frac{4}{13}\right)^2+(-3)^2}$ or $d=\sqrt{\frac{121}{13}}$	**dd**M1	1.1b

	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
			(11 marks)

Notes

(a)

M1: Realises the scalar product between the direction of *W* and the normal to the road is needed and so applies it and uses trigonometry to find an angle

M1: Calculates the scalar product between $\pm\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)$ and $\pm\begin{pmatrix}3\\-5\\-18\end{pmatrix}$ (Allow sign slips as long as the intention is clear)

A1: $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$ or $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=-9$

M1: A fully complete and correct method for obtaining the acute angle

A1: Awrt 7.58° or awrt 0.132 radians **(must see units)**. Do not isw and withhold this mark if extra answers are given.

(b)

B1ft: Forms the correct parametric form for the pipe *W*. Follow through their direction vector for *W* from part (a).

M1: Identifies the need to and forms a vector connecting *C* to *W* using a parametric form for *W*

M1: Uses the model to form the scalar product of *C* to *W* and the direction of *W* to find the value of their parameter **or** finds the distance *C* to *W* or (*C* to *W*)^2 in terms of their parameter

A1: Correct vector or correct completion of the square

ddM1: Correct use of Pythagoras on their vector *CW* or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.**

A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m

Alternatives for part (b):

4(b) Way 2	$\mathbf{AC}=\begin{pmatrix}0\\1\\-3\end{pmatrix},\quad \mathbf{AB}=\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC.AB}=\begin{pmatrix}0\\1\\-3\end{pmatrix}.\begin{pmatrix}2\\3\\0\end{pmatrix}=3$	M1	3.4
	$\Rightarrow \cos CAB=\frac{3}{\sqrt{10}\sqrt{13}}\Rightarrow CAB=...$	M1	3.1b
	$CAB=74.74....°$	A1	1.1b
	$d=\sqrt{10}\sin 74.74....°$	**dd**M1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	

	Notes		
	(b) B1ft: Forms the correct vectors. Follow through their direction vector for *W* from part (a). M1: Identifies the need to and forms the scalar product between **AC** and **AB** M1: Uses the model to form the scalar product and uses this to find the angle *CAB* A1: Correct angle ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.** A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m		

4(b) Way 3	$\mathbf{AC} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{AB} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC} \times \mathbf{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 3 \\ 2 & 3 & 0 \end{vmatrix} = \begin{pmatrix} -9 \\ 6 \\ 2 \end{pmatrix}$	M1	3.4
	$\lvert \mathbf{AC} \times \mathbf{AB} \rvert = \sqrt{9^2 + 6^2 + 2^2} = \ldots$	M1	3.1b
	$= 11$	A1	1.1b
	$d = \dfrac{11}{\lvert \mathbf{AB} \rvert} = \dfrac{11}{\sqrt{2^2 + 3^2}} = \ldots$	**dd**M1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
	Notes		
	(b) B1ft: Forms the correct vectors. Follow through their direction vector for *W* from part (a). M1: Identifies the need to and forms the vector product between **AC** and **AB** M1: Uses the model to find the magnitude of their vector product A1: Correct value ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.** A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m		

4. The line l has equation

$$\frac{x+2}{1} = \frac{y-5}{-1} = \frac{z-4}{-3}$$

The plane Π has equation

$$\mathbf{r}.(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = -7$$

Determine whether the line l intersects Π at a single point, or lies in Π, or is parallel to Π without intersecting it.

(5)

Question	Scheme	Marks	AOs
4.	$(\mathbf{r}=)\begin{pmatrix}-2+\lambda\\5-\lambda\\4-3\lambda\end{pmatrix}$ **or** $\begin{pmatrix}-2\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\-3\end{pmatrix}$ (oe)	M1	1.1b
	So meet if $\begin{pmatrix}-2+\lambda\\5-\lambda\\4-3\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\1\end{pmatrix}=-7\Rightarrow(-2+\lambda)\times1+(5-\lambda)\times-2+(4-3\lambda)\times1=-7$	M1 A1	3.1a 1.1b
	$\Rightarrow 0\lambda-8=-7\Rightarrow-8=-7$ a contradiction so no intersection	A1ft	2.3
	Hence *l* is parallel to Π but not in it.	A1cso	3.2a
		(5)	
	(5 marks)		

Notes

M1 Forms a parametric form for the line. Allow one slip.

M1 Substitutes into the equation of the plane to an equation in λ. May use Cartesian form of plane to substitute into.

A1 Correct equation in λ

A1ft Simplifies and derives a contradiction and deduces line and plane do not meet. Follow through in their initial equation in λ so
- contradiction so no intersection if λ disappears and constants unequal
- line lies in plane if a tautology is arrived at
- meet in a point if a solution for λ is found.

But do not allow for incorrect simplification from a correct initial equation in λ

Note that a miscopy/misread of 7 instead of −7 can therefore score a maximum of M1M1A0A1A0.

A1cso Correct deduction from correct working. This may be seen two separate statements in their working. You may see attempts at showing the line is parallel before/after deducing there is no intersection.

Question	Scheme	Marks	AOs
Alt 1	Note that some may a attempt a mix of the main scheme and Alt 1. Mark under main scheme unless Alt 1 would score higher.		
	$\begin{pmatrix}1\\-1\\-3\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\1\end{pmatrix}=1\times1+(-1)\times(-2)+(-3)\times1=0$	M1	3.1a
	Hence *l* is parallel to Π	A1	1.1b
	$(-2,5,4)$ on *l*, but $(1)(-2)+(-2)(5)+1(4)=-8$	M1	1.1b
	$-8\neq-7$ so $(-2,5,4)$ is not on the plane.	A1ft	2.3
	Hence *l* is (parallel to Π but) not in the plane.	A1cso	3.2a
		(5)	
	(5 marks)		

Alt 1 Notes

M1 Attempts the dot product between the two direction vectors.

A1 Shows dot product is zero and makes the correct deduction that line is parallel to plane.

M1 Finds a point on *l* and substitutes into the equation of Π (vector or Cartesian)

A1ft Simplifies and derives a contradiction – follow through their equation, so if arrive at a tautology, they should deduce the line is in the plane.

A1cso Correct deduction from correct working but may be split across working.

Question	Scheme	Marks	AOs
Alt 2	Attempts to solve $\frac{x+2}{1}=\frac{y-5}{-1}=\frac{z-4}{-3}$ and $x-2y+z=-7$ simultaneously – eliminates one variable for M mark.	M1	3.1a
	e.g. $y=-(x+2)+5=-x+3 \Rightarrow x-2(-x+3)+z=-7 \Rightarrow 3x+z=-1$ (oe)	A1	1.1b
	Solves reduced equations, e.g. $-3(x+2)=z-4 \Rightarrow 3x+z=-2$ and $3x+z=-1 \Rightarrow (3x+z)-(3x+z)=-2-(-1)$	M1	1.1b
	$\Rightarrow 0=-1$ a contradiction so no intersection	A1ft	2.3
	Hence l is parallel to Π but not in it.	A1cso	3.2a
		(5)	
	(5 marks)		

Alt 2 notes

M1	Attempts to solve the Cartesian equation of the line and plane, using the plane equation to eliminate one variable for the M.
A1	Correct elimination of their chosen variable. (E.g may see $3-3y+z=-7$ or $-2x-2y-2=-7$ etc)
M1	Solves the reduced equations in two variables…
A1ft	… and derives a contradiction/line and plane do not meet. Follow through their result, so may reach a tautology and deduce lies in plane, or find single solution and deduce meet in a point.
A1cso	Correct deduction from correct working.

8. A gas company maintains a straight pipeline that passes under a mountain.

The pipeline is modelled as a straight line and one side of the mountain is modelled as a plane.

There are accessways from a control centre to two access points on the pipeline.

Modelling the control centre as the origin O, the two access points on the pipeline have coordinates $P(-300, 400, -150)$ and $Q(300, 300, -50)$, where the units are metres.

(a) Find a vector equation for the line PQ, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where λ is a scalar parameter.

(2)

The equation of the plane modelling the side of the mountain is $2x + 3y - 5z = 300$

The company wants to create a new accessway from this side of the mountain to the pipeline.

The accessway will consist of a tunnel of shortest possible length between the pipeline and the point $M(100, k, 100)$ on this side of the mountain, where k is a constant.

(b) Using the model, find

(i) the coordinates of the point at which this tunnel will meet the pipeline,

(ii) the length of this tunnel.

(7)

It is only practical to construct the new accessway if it will be significantly shorter than both of the existing accessways, OP and OQ.

(c) Determine whether the company should build the new accessway.

(2)

(d) Suggest one limitation of the model.

(1)

Question	Scheme	Marks	AOs
8. (a)	**Note:** Allow alternative vector forms throughout, e.g row vectors, **i**, **j**, **k** notation $\mathbf{b} = \pm\left[\begin{pmatrix}300\\300\\-50\end{pmatrix} - \begin{pmatrix}-300\\400\\-150\end{pmatrix}\right] = \pm\begin{pmatrix}600\\-100\\100\end{pmatrix}$	M1	1.1b
	So $\mathbf{r} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \lambda\begin{pmatrix}600\\-100\\100\end{pmatrix}$ oe $\left(\text{e.g. } \mathbf{r} = \begin{pmatrix}300\\300\\-50\end{pmatrix} + \lambda\begin{pmatrix}6\\-1\\1\end{pmatrix}\right)$	A1	2.5
		(2)	
(b)(i)	$k = 200$	B1	2.2a
	If M is the point on mountain, and X a general point on the line then eg. $\overrightarrow{MX} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \lambda\begin{pmatrix}600\\-100\\100\end{pmatrix} - \begin{pmatrix}100\\k\\100\end{pmatrix} = \begin{pmatrix}-400+600\lambda\\400-k-100\lambda\\-250+100\lambda\end{pmatrix} = \begin{pmatrix}-400+600\lambda\\200-100\lambda\\-250+100\lambda\end{pmatrix}$ May be in terms of k or with $k = 200$ used.	M1	3.1b
	e.g. $\begin{pmatrix}-400+600\lambda\\200-100\lambda\\-250+100\lambda\end{pmatrix} \bullet \begin{pmatrix}600\\-100\\100\end{pmatrix} = 0 \Rightarrow \lambda = \ldots$	dM1	1.1b
	So e.g. $\overrightarrow{OX} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \frac{3}{4}\begin{pmatrix}600\\-100\\100\end{pmatrix} = \ldots$	M1	3.4
	So coordinates of X are (150, 325, −75) Accept as $\begin{pmatrix}150\\325\\-75\end{pmatrix}$	A1	1.1b
		(5)	
(ii)	Length of tunnel is $\sqrt{(150-100)^2 + (325-200)^2 + (-75-100)^2} = \ldots$	M1	1.1b
	Awrt 221m from correct working, so λ must have been correct. (Must include units)	A1	1.1b
		(2)	
(c)	$\left\|\overrightarrow{OP}\right\| = \sqrt{(-300)^2 + 400^2 + (-150)^2} \approx 522$ $\left\|\overrightarrow{OQ}\right\| = \sqrt{300^2 + 300^2 + 50^2} \approx 427$	M1	1.1b
	New tunnel length is signficantly shorter than these values so it is likely that the company will decide to build the accessway. Reason and conclusion needed.	A1ft	2.2b
		(2)	
(d)	E.g. The mountainside is not likely to be flat so a plane may not be a good model. The tunnel and/or pipeline will not have negligible thickness so modelling as lines may not be appropriate. A shortest length tunnel may not be possible, or most practical, as the strata of the rock in the mountain have not been considered by the model.	B1	3.5b
		(1)	
	(12 marks)		

Notes		
(a)	**M1**	Attempts the direction between positions P and Q. If no method shown, two correct entries imply the method.
	A1	A correct equation in the correct form. Any point on the line may used, and any non-zero multiple of the direction. Must begin **r** = …
(b)		**Note:** mark part (b) as a whole.
(i)	**B1**	Correct value of k deduced.
	M1	Realises the need to find the distance from the point on the mountain to a general point on the line.
	dM1	Takes the dot product with the direction vector of line and sets to zero and proceeds to find a value of λ. If working with k as well, allow for finding either λ in terms of k or k in terms of λ.
	M1	Substitutes their λ into their line equation. (This may not have come from correct work, but the method is for using the line equation here.) May be implied by two out of three correct coordinates for their λ **Note:** May omit this step and substitute λ into $\overrightarrow{MX}$. This gains M0 here, but can gain M1A1 in (ii) for finding the length of $\overrightarrow{MX}$.
	A1	Correct point.
(b)(ii)	**M1**	Uses the distance formula with their point and M, or with their $\overrightarrow{MX}$ from (i). (May be implied by two out of three correct coordinates for their λ)
	A1	Correct distance, including units. Accept awrt 221 m or $25\sqrt{78}$ m
(c)	**M1**	Calculates the two distances OP and OQ.
	A1ft	Makes an appropriate conclusion for their tunnel length, but distances OP and OQ must be correct. A reason and a conclusion is needed. Accept for reason e.g "significantly shorter" or "tunnel is more than 100m less than either existing accessway", as these act as a comparative judgement. But do not accept just "shorter" or just inequalities given with no comparative evidence.
(d)	**B1**	Any appropriate criticism of the model given. The model must be referred to in some way – e.g. criticise the straightness/thickness of line, flatness of plane or lack of taking strata etc of mountain into account (as e.g this means line may not be straight). Note: reference to measurements not being correct is **NOT** a limitation of the model.

<u>For reference</u> Some of the other common equations/values of λ in (b)(i) are:

$$\overrightarrow{MX} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} -400+6\lambda \\ 200-\lambda \\ -250+\lambda \end{pmatrix} \Rightarrow \lambda = 75$$

$$\overrightarrow{MX} = \begin{pmatrix} 300 \\ 300 \\ -50 \end{pmatrix} + \lambda \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} 200+600\lambda \\ 100-100\lambda \\ -150+100\lambda \end{pmatrix} \Rightarrow \lambda = -\frac{1}{4}$$

$$\overrightarrow{MX} = \begin{pmatrix} 300 \\ 300 \\ -50 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} 200+6\lambda \\ 100-\lambda \\ -150+\lambda \end{pmatrix} \Rightarrow \lambda = -25$$

(If the negative direction vectors are used in any case, the value of λ is just the negative of the above.)

See Appendix for some alternatives to part (b)

Appendix: Alternatives to 8(b)

Note that variations may occur with the line equation chosen in part (a), but mark as follows:

Question	Scheme	Marks	AOs
Alt 1 (b)(i)	As per main scheme.	B1 M1	2.2a 3.1b
	$d^2 = (-400+600\lambda)^2 + (200-100\lambda)^2 + (-250+100\lambda)^2$ $= 380000\lambda^2 - 570000\lambda + 262500$ $= 380000\left(\lambda - \frac{3}{4}\right)^2 + 48750 \Rightarrow \lambda = \ldots$	dM1	1.1b
	As per main scheme.	M1 A1	3.4 1.1b
		(5)	
(ii)	Length of tunnel is $\sqrt{"48750"} = \ldots$	M1	1.1b
	Awrt 221m from correct working, so completion of square must have been correct. (Must include units)	A1	1.1b
		(2)	

	Notes	
(i)	**B1M1**	As per main scheme.
	M1	Realises the need to find the distance from the point on the mountain to a general point on the line.
	dM1	Attempts the distance or distance squared of $\overrightarrow{MX}$, expands and completes the square to find the value of λ for which distance is minimum. May obtain other forms for the completed square. Look for $A(B\lambda - C)^2 - D + "262500"$ where $A, B, C, D \neq 0$ but B may be 1.
	M1A1	As per main scheme.
(ii)	**M1**	Correct method for the distance. May be as per main scheme, or via extracting from the completed square constant term.
	A1	Correct distance, including units. Accept awrt 221 m or $25\sqrt{78}$ m

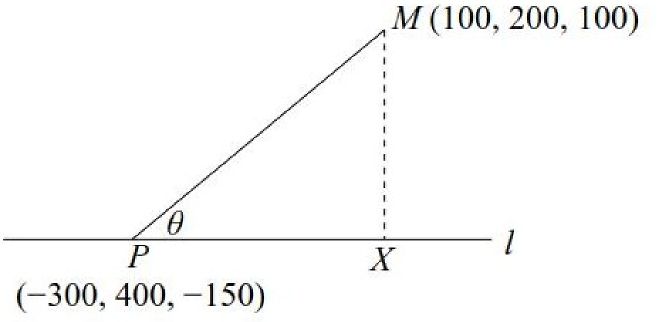
Question	Scheme	Marks	AOs
Alt 2 (b)(i)	As per main scheme.	B1 M1	2.2a 3.1b
	$d^2 = (-400+600\lambda)^2 + (200-100\lambda)^2 + (-250+100\lambda)^2$ $= 380000\lambda^2 - 570000\lambda + 262500$ $\frac{\mathrm{d}}{\mathrm{d}x}(d^2) = 0 \Rightarrow 760000\lambda - 570000 = 0 \Rightarrow \lambda = \ldots$	dM1	1.1b
	As per main scheme.	M1 A1	3.4 1.1b
		(5)	
(ii)	Length of tunnel is $\sqrt{(150-100)^2 + (325-200)^2 + (-75-100)^2} = \ldots$	M1	1.1b
	Awrt 221m from correct working, differentiation etc must have been correct. (Must include units)	A1	1.1b
		(2)	

Notes		
		As per main scheme except for:
(i)	**dM1**	Attempts the distance or distance squared of $\overrightarrow{MX}$, differentiates and set to zero to find λ for minimum distance.
(ii)	**M1**	May substitute λ into the distance squared formula to find distance.

Alt 3 (b)(i)	$k = 200$	B1	2.2a
	If M is the point on mountain, then e.g (may use Q rather than P) $\overrightarrow{MP} = \begin{pmatrix} -400 \\ 200 \\ -250 \end{pmatrix} \Rightarrow \cos\theta = \dfrac{\begin{pmatrix} -400 \\ 200 \\ -250 \end{pmatrix} . \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix}}{\sqrt{(-400)^2 + 200^2 + (-250)^2}\sqrt{600^2 + (-100)^2 + 100^2}}$ $\Rightarrow \cos\theta = \ldots$ or $\theta = \ldots$ (where θ is the angle between the line and $\overrightarrow{MP}$)	M1	3.1b
	$\Rightarrow \left\lvert\overrightarrow{PX}\right\rvert = \left\lvert\overrightarrow{MP}\right\rvert \cos\theta = \ldots$	dM1	1.1b
	So e.g. $\overrightarrow{OX} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \dfrac{\left\lvert\overrightarrow{PX}\right\rvert}{\left\lvert\begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix}\right\rvert}\begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \dfrac{"75\sqrt{8}"}{100\sqrt{38}}\begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} = \ldots$	M1	3.4
	So coordinates of X are (150, 325, −75) Accept as $\begin{pmatrix} 150 \\ 325 \\ -75 \end{pmatrix}$	A1	1.1b
		(5)	
(ii)	Length of tunnel is $\left\lvert\overrightarrow{MP}\right\rvert \sin\theta = \ldots$ (oe)	M1	1.1b
	Awrt 221m from correct working. (Must include units)	A1	1.1b
		(2)	

Notes		
(i)	**B1**	Correct value of k deduced.
	M1	Finds $\overrightarrow{MP}$ (or $\overrightarrow{MQ}$) and attempts scalar product formula with this and the direction of the line to find the angle or cosine of the angle between line and $\overrightarrow{MP}$ (or $\overrightarrow{MQ}$)
	dM1	Uses their angle with the cosine to find the length of $\overrightarrow{PX}$ (or $\overrightarrow{QX}$). Accept equivalent trigonometric methods (e.g. finding opposite side first and using tangent or Pythagoras.
	M1	Uses the length of and $\overrightarrow{PX}$ (or $\overrightarrow{QX}$) to find the coordinates of the point on the line at shortest distance from M.
	A1	Correct point.
(ii)	**M1**	Correct method for the distance. May be as per main scheme, or use of sine ratio with their angle between the line and and $\overrightarrow{MP}$ (or $\overrightarrow{MQ}$). Accept equivalent trigonometric methods.
	A1	Correct distance, including units. Accept awrt 221 m or $25\sqrt{78}$ m

Useful diagram:



Note for P, $\cos\theta = \pm\dfrac{57}{\sqrt{38}\sqrt{105}}$,

$\theta = 25.5\ldots°$ and $\left\lvert\overrightarrow{PX}\right\rvert = 75\sqrt{38}$

For Q $\cos\theta = \pm\dfrac{19}{\sqrt{38}\sqrt{29}}$,

$\theta = 55.08\ldots°$, $\left\lvert\overrightarrow{QX}\right\rvert = 25\sqrt{38}$

4. **All units in this question are in metres.**

A lawn is modelled as a plane that contains the points $L(-2, -3, -1)$, $M(6, -2, 0)$ and $N(2, 0, 0)$, relative to a fixed origin O.

(a) Determine a vector equation of the plane that models the lawn, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

(3)

(b) (i) Show that, according to the model, the lawn is perpendicular to the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

(ii) Hence determine a Cartesian equation of the plane that models the lawn.

(4)

There are two posts set in the lawn.
There is a washing line between the two posts.
The washing line is modelled as a straight line through points at the top of each post with coordinates $P(-10, 8, 2)$ and $Q(6, 4, 3)$.

(c) Determine a vector equation of the line that models the washing line.

(2)

(d) State a limitation of one of the models.

(1)

The point $R(2, 5, 2.75)$ lies on the washing line.

(e) Determine, according to the model, the shortest distance from the point R to the lawn, giving your answer to the nearest cm.

(2)

Given that the shortest distance from the point R to the lawn is actually 1.5 m,

(f) use your answer to part (e) to evaluate the model, explaining your reasoning.

(1)

P 6 2 6 8 5 A 0 8 2 8

<table>
<tr><th>Question</th><th colspan="4">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">4(a)</td><td colspan="4">Finds any two vectors $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$
$\pm\begin{pmatrix}8\\1\\1\end{pmatrix}$ or $\pm\begin{pmatrix}4\\3\\1\end{pmatrix}$ or $\pm\begin{pmatrix}-4\\2\\0\end{pmatrix}$ two out of three values correct is sufficient
to imply the correct method</td><td>M1</td><td>3.3</td></tr>
<tr><td colspan="4">Applies the vector equation of the plane formula $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$
Where a is any coordinate from L, M & N and vectors b and c come from an attempt at finding any two vectors that lie on the plane.</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="4">A correct equation for the plane $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$
$\mathbf{a} = \begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-2\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$
b and c are any two vectors from $\pm\begin{pmatrix}8\\1\\1\end{pmatrix}$ or $\pm\begin{pmatrix}4\\3\\1\end{pmatrix}$ or $\pm\begin{pmatrix}-4\\2\\0\end{pmatrix}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="4"></td><td>(3)</td><td></td></tr>
<tr><td rowspan="3">(b)(i)

(ii)</td><td>Applies ‘their’ $\mathbf{b}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$
AND
‘their’ $\mathbf{c}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$</td><td>Alternative 1
Finds ‘their b’ – ‘their c’ or vice versa and applies the dot product with $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ AND one of their b or c</td><td>Alternative 2
Applies ‘their’ $\mathbf{b}.\begin{pmatrix}x\\y\\z\end{pmatrix}$ AND
‘their’ $\mathbf{c}.\begin{pmatrix}x\\y\\z\end{pmatrix}$ and
solves to find values of x, y and z</td><td>Alternative 3
Applies the dot product between their answer to part (a) and the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">Show that both dot product(s) = 0 therefore the lawn is perpendicular</td><td>Alternative 1
Shows results is parallel to $\begin{pmatrix}1\\2\\-10\end{pmatrix}$
therefore the lawn is perpendicular</td><td>Alternative 2
Achieves the value 2 and concludes as a constant therefore the lawn is perpendicular</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="4">Outside Specification for this paper – using the cross product
Finds the cross product between ‘their b’ and ‘their c’ and either</td><td>M1</td><td>1.1b</td></tr>
</table>

	compares with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ to show parallel or applies the dot product formula with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ to show parallel		
	Concludes **parallel** therefore the lawn is **perpendicular**	A1	2.4
	Attempts $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}1\\2\\-10\end{pmatrix}=\mathbf{a}\cdot\begin{pmatrix}1\\2\\-10\end{pmatrix}$ where $\mathbf{a}=\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-3\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$ Allow $\mathbf{r}\cdot\begin{pmatrix}1\\2\\-10\end{pmatrix}=\mathbf{a}\cdot\begin{pmatrix}1\\2\\-10\end{pmatrix}$ for this mark	M1	1.1b
	$x+2y-10z=2$ or $x+2y-10z-2=0$	A1	1.1b
		(4)	
(c)	Finds the vector $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ and uses it as the direction vector in the formula $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$ Two out three values correct is sufficient to imply the correct method	M1	3.3
	$\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$ where $\mathbf{a}=\begin{pmatrix}-10\\8\\2\end{pmatrix}$ or $\begin{pmatrix}6\\4\\3\end{pmatrix}$ and $\mathbf{d}=\pm\begin{pmatrix}16\\-4\\1\end{pmatrix}$	A1	1.1b
		(2)	
(d)	For example: The lawn will not be flat The washing line will not be straight	B1	3.5b
		(1)	
(e)	Applies the distance formula $\dfrac{\lvert(2\times1)+5\times2+(2.75\times-10)-2\rvert}{\sqrt{1^2+2^2+(-10)^2}}$	M1	3.4
	= 1.71 m or 171 cm	A1	2.2b
		(2)	
(f)	Must have an answer to part (e). Compares their answer to part (e) with 1.5 m and makes an appropriate comment about the model that is consistent with their answer to part (e). If their answer to part (e) is close to 1.5 (e.g. 1.4 to 1.6) they must compare and conclude that the model therefore is good If their answer to part (e) is significantly different to 1.5 they must compare and conclude that the model therefore it is not a good model.	B1ft	3.5a

		(1)	
(13 marks)			

Notes:

(a)

M1: Finds any two vectors $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$ by subtracting relevant vectors. Two out three values correct is sufficient to imply the correct method

M1: Applies the vector equation of the plane formula $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ where **a** is any point on the plane and the vectors **b** and **c** are any two from their $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$

A1: Any correct equation for the plane. Must start with **r** =…

(b)(i)

M1: Applies the dot product between their vectors **b AND c** with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

A1: Shows both dot products = 0 and concludes that the lawn is **perpendicular** to the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

(b)(i) Alternative 1

M1: Applies the dot product between their vector **b – c AND** one of their vectors **b** or **c** with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

A1: Shows both dot products = 0 and concludes that the lawn is **perpendicular** to the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

(b)(i) Alternative 2

M1: Applies the dot product between their vectors **b** and **c** $\begin{pmatrix}x\\y\\z\end{pmatrix}$ and attempts to find values of *x*, *y* and *z*

A1: Shows results is **parallel** to $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ therefore the lawn is **perpendicular**

(b)(i) Alternative 3

M1: Applies the dot product between their answer to part (a) and the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

A1: Achieves the value 2 and concludes as a constant therefore the lawn is **perpendicular**

(b)(i) Outside Specification for this paper – using the cross product

M1: Finds the cross product between 'their **b**' and 'their **c**' and shows parallel to $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

A1: Concludes **parallel** therefore the **lawn** is **perpendicular**

(b)(ii)

M1: Applies the formula $\mathbf{r.n}=\mathbf{a.n}$ where $\mathbf{n}=\begin{pmatrix}1\\2\\-10\end{pmatrix}$ and $\mathbf{a}=\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-2\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$

A1: Correct Cartesian equation of the plane

Note: If no method is shown then it must be correct to score **M1 A1**, if incorrect scores **M0 A0.** Look at part (i) to see if there is any method as long as it if used in (ii)

(c)

M1: Finds the vector $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ and uses it as the direction vector in the formula. $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$. Two out three values correct is sufficient to imply the correct method

A1: A correct equation including **r** =…

(d)

B1: States an acceptable limitation of the model for the lawn or washing line

(e)

M1: Applies the distance formula using the point (2, 5, 2.75) and the normal vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

A1: 1.71 m or 171 cm

(f)

B1ft: Compares their answer to part (e) with 1.5 and makes an assessment of the model with a reason with no contradictory statements.

6. A mining company has identified a mineral layer below ground.

The mining company wishes to drill down to reach the mineral layer and models the situation as follows.

With respect to a fixed origin O,

- the ground is modelled as a horizontal plane with equation $z = 0$
- the mineral layer is modelled as part of the plane containing the points $A(10, 5, -50)$, $B(15, 30, -45)$ and $C(-5, 20, -60)$, where the units are in metres

(a) Determine an equation for the plane containing A, B and C, giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = d$

(5)

(b) Determine, according to the model, the acute angle between the ground and the plane containing the mineral layer. Give your answer to the nearest degree.

(3)

The mining company plans to drill vertically downwards from the point (5, 12, 0) on the ground to reach the mineral layer.

(c) Using the model, determine, in metres to 1 decimal place, the distance the mining company will need to drill in order to reach the mineral layer.

(2)

(d) State a limitation of the assumption that the mineral layer can be modelled as a plane.

(1)

Question	Scheme	Marks	AOs
6(a)	Any two of: $\begin{cases} \pm k\overrightarrow{AB} = \pm k(5\mathbf{i}+25\mathbf{j}+5\mathbf{k}), \\ \pm k\overrightarrow{AC} = \pm k(-15\mathbf{i}+15\mathbf{j}-10\mathbf{k}), \\ \pm k\overrightarrow{BC} = \pm k(-20\mathbf{i}-10\mathbf{j}-15\mathbf{k}) \end{cases}$	B1	3.3
	Let normal vector be $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(\mathbf{i}+5\mathbf{j}+\mathbf{k})=0,\ (a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(-3\mathbf{i}+3\mathbf{j}-2\mathbf{k})=0$ $\Rightarrow a+5b+c=0,\ -3a+3b-2c=0 \Rightarrow a=...,\ b=...,\ c=...$ Alternative: cross product $\begin{vmatrix} 1 & 5 & 1 \\ -3 & 3 & -2 \end{vmatrix} = (-10-3)\mathbf{i}-(-2+3)\mathbf{j}+(3+15)\mathbf{k}$	M1	1.1b
	$\mathbf{n}=k(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})$	A1	1.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet(10\mathbf{i}+5\mathbf{j}-50\mathbf{k})=...$	M1	1.1b
	$\mathbf{r}\bullet(13\mathbf{i}+\mathbf{j}-18\mathbf{k})=1035$ o.e. $\mathbf{r}\bullet(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})=-1035$ $\mathbf{r}\bullet(325\mathbf{i}+25\mathbf{j}-450\mathbf{k})=25875$	A1	2.5
		(5)	
(b)	Attempts the scalar product between their normal vector and the vector **k** and uses trigonometry to find an angle	M1	3.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet\mathbf{k}=-18=\sqrt{13^2+1^2+18^2}\cos\alpha$	M1	1.1b
	$\cos\alpha=\frac{-18}{\sqrt{494}} \Rightarrow \alpha=144.08... \Rightarrow \theta=36°$	A1	3.2a
		(3)	
(c)	Distance required is $\lvert\lambda\rvert$ where $\begin{pmatrix} 13 \\ 1 \\ -18 \end{pmatrix}\bullet\begin{pmatrix} 5 \\ 12 \\ \lambda \end{pmatrix}=1035$	M1	3.4
	$\lvert\lambda\rvert=53.2\text{m}$	A1	1.1b
		(2)	
(d)	E.g. • The mineral layer will not be perfectly flat and will not form a plane • The mineral layer will have a depth and this should be taken into account	B1	3.5b
		(1)	
			(11 marks)

Notes

(a)
B1: Identifies 2 correct vectors in the plane that can be used to set up the model
M1: Attempts a normal vector using an appropriate method. E.g. as in main scheme or may use vector product or parametric form
A1: A correct normal vector

M1: Applies **r.n** = *d* with their normal vector and a point in the plane to find a value for *d*
A1: Correct equation (allow any multiple)
(b)
M1: Realises the scalar product between their from part (a) and a vector parallel to **k** and so applies it and uses trigonometry to find an angle
M1: Forms the scalar product between their from part (a) and a vector parallel to **k**
A1: Correct angle
(c)
M1: Uses the model and a correct strategy to establish the distance from (5, 12, 0) to the plane vertically downwards
A1: Correct distance
(d)
B1: Any reasonable limitation – see scheme

6. The surface of a horizontal tennis court is modelled as part of a horizontal plane, with the origin on the ground at the centre of the court, and

- **i** and **j** are unit vectors directed across the width and length of the court respectively
- **k** is a unit vector directed vertically upwards
- units are metres

After being hit, a tennis ball, modelled as a particle, moves along the path with equation

$$\mathbf{r} = (-4.1 + 9\lambda - 2.3\lambda^2)\mathbf{i} + (-10.25 + 15\lambda)\mathbf{j} + (0.84 + 0.8\lambda - \lambda^2)\mathbf{k}$$

where λ is a scalar parameter with $\lambda \geqslant 0$

Assuming that the tennis ball continues on this path until it hits the ground,

(a) find the value of λ at the point where the ball hits the ground.

(2)

The direction in which the tennis ball is moving at a general point on its path is given by

$$(9 - 4.6\lambda)\mathbf{i} + 15\mathbf{j} + (0.8 - 2\lambda)\mathbf{k}$$

(b) Write down the direction in which the tennis ball is moving as it hits the ground.

(1)

(c) Hence find the acute angle at which the tennis ball hits the ground, giving your answer in degrees to one decimal place.

(4)

The net of the tennis court lies in the plane $\mathbf{r}.\mathbf{j} = 0$

(d) Find the position of the tennis ball at the point where it is in the same plane as the net.

(3)

The maximum height above the court of the top of the net is 0.9 m.

Modelling the top of the net as a horizontal straight line,

(e) state whether the tennis ball will pass over the net according to the model, giving a reason for your answer.

(1)

With reference to the model,

(f) decide whether the tennis ball will actually pass over the net, giving a reason for your answer.

(2)

Question	Scheme	Marks	AOs
6(a)	Need **k** component to be zero at ground, so $0.84 + 0.8\lambda - \lambda^2 = 0 \Rightarrow \lambda = ...$	M1	1.1b
	$\lambda = -\frac{3}{5}, \frac{7}{5}$, but $\lambda \geqslant 0$ so $\lambda = \frac{7}{5}$	A1	1.1b
		(2)	
(b)	Direction is $(9 - 4.6 \times 1.4)\mathbf{i} + 15\mathbf{j} + (0.8 - 2 \times 1.4)$ $= 2.56\mathbf{i} + 15\mathbf{j} - 2\mathbf{k}$ or $\frac{64}{25}\mathbf{i} + 15\mathbf{j} - 2\mathbf{k}$	B1ft	2.2a
		(1)	
(c)	Direction perpendicular to ground is $a\mathbf{k}$, so angle to perpendicular is given by $(\cos\theta) = \frac{a\mathbf{k}.(2.56\mathbf{i} + 15\mathbf{j} - 2\mathbf{k})}{a \times \lvert 2.56\mathbf{i} + 15\mathbf{j} - 2\mathbf{k} \rvert}$ or $\frac{\begin{pmatrix} 2.56 \\ 15 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}}{\begin{vmatrix} 2.56 \\ 15 \\ -2 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ a \end{vmatrix}}$ or angle between $\begin{pmatrix} 2.56 \\ 15 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 2.56 \\ 15 \\ 0 \end{pmatrix}$ is given by $(\cos\theta) = \frac{\begin{pmatrix} 2.56 \\ 15 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2.56 \\ 15 \\ 0 \end{pmatrix}}{\begin{vmatrix} 2.56 \\ 15 \\ -2 \end{vmatrix} \begin{vmatrix} 2.56 \\ 15 \\ 0 \end{vmatrix}}$	M1	1.1b
	$= \frac{-2}{\sqrt{2.56^2 + 15^2 + (-2)^2}} (= -0.130\ldots)$ Or $= \frac{231.5536}{\sqrt{2.56^2 + 15^2 + (-2)^2}\sqrt{2.56^2 + 15^2 + (0)^2}} = 0.991...$	M1	1.1b
	$90° - \arccos('-0.130...') = -7.48...$ or $\arccos(0.991...)$	ddM1	3.1b
	So the tennis ball hits ground at angle of $7.5°$ (1d.p.) cao	A1	3.2a
	Alternative Finds the length of the vector in the **ij** plane $= \sqrt{2.56^2 + 15^2}$	M1	1.1b
	$\tan\theta = \frac{2}{\sqrt{2.56^2 + 15^2}}$	M1	1.1b
	$\theta = \arctan\left(\frac{2}{\sqrt{2.56^2 + 15^2}}\right)$ or $\theta = 90 - \arctan\left(\frac{\sqrt{2.56^2 + 15^2}}{2}\right)$	ddM1	3.1b

	So the tennis ball hits ground at angle of $7.5°$ (1d.p.)	A1	3.2a
		(4)	
(d)	In same plane as net when $\mathbf{r}.\mathbf{j}=0$, $\begin{pmatrix}-4.1+9\lambda-2.3\lambda^2\\-10.25+15\lambda\\0.84+0.8\lambda-\lambda^2\end{pmatrix}\bullet\begin{pmatrix}0\\1\\0\end{pmatrix}$ leading to $-10.25+15\lambda=0\Rightarrow\lambda=...$ $\left(=\frac{41}{60}=0.683333...\right)$	M1	3.1b
	So is at position $\left(-4.1+9\times\frac{41}{60}-2.3\left(\frac{41}{60}\right)^2\right)\mathbf{i}+0\mathbf{j}+\left(0.84+0.8\times\frac{41}{60}-\left(\frac{41}{60}\right)^2\right)\mathbf{k}$	M1	1.1b
	$=$ awrt $0.976\mathbf{i}+$ awrt $0.920\mathbf{k}$ or $=$ awrt $0.976\mathbf{i}+0.92\mathbf{k}$ (to 3 s.f.) or $=$ awrt $0.976\mathbf{i}+\frac{3311}{3600}\mathbf{k}$	A1	1.1b
		(3)	
(e)	Modelling as a line, height of net is 0.9m along its length so as **0.92 > 0.9** the ball **will** pass over the net according to the model.	B1ft	3.2a
		(1)	
(f)	Identifies a suitable feature of the model that affects the outcome And uses it to draw a compatible conclusion. For example • The ball is **not a particle** and will have **diameter/radius**, therefore it will hit the net and not pass over. • As above, but so the **ball will clip the net** but it's momentum will take it over as it is mostly above the net. • The **model** says that the ball will clear the net by 2cm which may be smaller than the balls **diameter** • The **net will not be a straight line/taut** so will not be 0.9m high, so the ball **will** have enough clearance to **pass** over the net.	M1 A1	3.2b 2.2b
		(2)	
			(13 marks)

Notes:

Accept any alternative vector notations throughout.

(a)

M1: Attempts to solve the quadratic from equating the **k** component to zero.

A1: Correct value, must select positive root, so accept 1.4 oe.

Correct answer only M1 A1

(b)

B1ft: For $(2.56,15,-2)$ o.e or follow through $(9-4.6\times'\lambda',15,0.8-2\times'\lambda')$ for their λ.

(c)

M1: Recognises the angle between the perpendicular and direction vector is needed, and identifies the perpendicular as $a\mathbf{k}$ for any non-zero a (including 1), and attempts dot product

Alternatively recognises the dot product of $(2.56,15,-2)$ and $(2.56,15,0)$

M1: Applies the dot product formula $\frac{a \bullet b}{|a||b|}$ correctly between *any* two vectors, but must have dot product and modulus evaluated.

ddM1: Dependent on both previous marks. A correct method to proceed to the required angle, usually $90° - \arccos('-0.130...')$ as shown in scheme but may e.g. use sin θ instead of cos θ in formula.

Alternatively is using dot product of $(2.56,15,-2)$ and $(2.56,15,0)$ finds arccos(0.991…)

A1: For 7.5° cao

Alternative

M1: Finds the length of the vector in the **ij** plane.

M1: Finds the tan of any angle the

ddM1: Dependent on both previous marks. Finds the required angle

A1: For 7.5° cao

(d)

M1: Attempts to find value of λ that gives zero **j** component.

M1: Uses their value of λ in the equation of the path to find position.

A1: Correct position.

(e)

B1ft: States that 0.920 > 0.9 so according to the model the ball will pass over the net. Follow through on their **k** component and draws an appropriate conclusion. May stay the value of k > 0.92

(f)

M1: There must be some reference to the model to score this mark. See scheme for examples. It is likely to be either the ball is not a particle, or the top of the net is not a straight line. Accept references to the ball crossing a long way from the middle.

Do not accept reasons such as "there may be wind/air resistance" as these are not referencing the given model.

A1: For a reasonable conclusion based on their reference to the model.

For example

The ball is not a particle; therefore, it will not go over the net is M1A0 as not explained why – needs reference to radius/diameter