Cp1Ch2 XMQs and MS

(Total: 67 marks)

1. CP2_2019 Q6 . 9 marks - CP1ch2 Argand diagrams
2. CP2_2020 Q2 . 9 marks - CP1ch2 Argand diagrams
3. CP2_2021 Q1 . 5 marks - CP1ch2 Argand diagrams
4. CP(AS)_2018 Q3 . 9 marks - CP1ch2 Argand diagrams
5. CP(AS)_2020 Q2 . 8 marks - CP1ch2 Argand diagrams
6. CP(AS)_2020 Q10. 7 marks - CP1ch2 Argand diagrams
7. CP(AS)_2021 Q5 . 10 marks - CP1ch2 Argand diagrams
8. CP(AS)_2022 Q2 . 10 marks - CP1ch2 Argand diagrams

6. In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin. The point A represents the complex number $6 + 2\text{i}$.

(a) Find the complex numbers represented by the points B and C, giving your answers in the form $x + \text{i}y$, where x and y are real and exact.

(6)

The points D, E and F are the midpoints of the sides of triangle ABC.

(b) Find the exact area of triangle DEF.

(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
6(a)	Examples: $\begin{pmatrix}\cos 120 & -\sin 120\\ \sin 120 & \cos 120\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix}=\ldots$ **or** $(6+2\text{i})\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\text{i}\right)$ **or** $\sqrt{40}\left(\cos\arctan\left(\frac{2}{6}\right)+\text{i}\sin\arctan\left(\frac{2}{6}\right)\right)\left(\cos\left(\frac{2\pi}{3}\right)+\text{i}\sin\left(\frac{2\pi}{3}\right)\right)$ **or** $\sqrt{40}\left(\cos\left(\arctan\left(\frac{2}{6}\right)+\frac{2\pi}{3}\right)+\text{i}\sin\left(\arctan\left(\frac{2}{6}\right)+\frac{2\pi}{3}\right)\right)$ **or** $\sqrt{40}\text{e}^{\text{i}\arctan\left(\frac{2}{6}\right)}\text{e}^{\text{i}\left(\frac{2\pi}{3}\right)}$	M1	3.1a
	$\left(-3-\sqrt{3}\right)$ **or** $\left(3\sqrt{3}-1\right)\text{i}$	A1	1.1b
	$\left(-3-\sqrt{3}\right)+\left(3\sqrt{3}-1\right)\text{i}$	A1	1.1b
	Examples: $\begin{pmatrix}\cos 240 & -\sin 240\\ \sin 240 & \cos 240\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix}=\ldots$ **or** $(6+2\text{i})\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\text{i}\right)$ **or** $\sqrt{40}\left(\cos\arctan\left(\frac{2}{6}\right)+\text{i}\sin\arctan\left(\frac{2}{6}\right)\right)\left(\cos\left(\frac{4\pi}{3}\right)+\text{i}\sin\left(\frac{4\pi}{3}\right)\right)$ **or** $\sqrt{40}\left(\cos\left(\arctan\left(\frac{2}{6}\right)+\frac{4\pi}{3}\right)+\text{i}\sin\left(\arctan\left(\frac{2}{6}\right)+\frac{4\pi}{3}\right)\right)$ **or** $\sqrt{40}\text{e}^{\text{i}\arctan\left(\frac{2}{6}\right)}\text{e}^{\text{i}\left(\frac{4\pi}{3}\right)}$	M1	3.1a
	$\left(-3+\sqrt{3}\right)$ **or** $\left(-3\sqrt{3}-1\right)\text{i}$	A1	1.1b
	$\left(-3+\sqrt{3}\right)+\left(-3\sqrt{3}-1\right)\text{i}$	A1	1.1b
		(6)	
(b) **Way 1**	Area ABC = $3\times\frac{1}{2}\sqrt{6^2+2^2}\sqrt{6^2+2^2}\sin 120°$ or Area AOB = $\frac{1}{2}\sqrt{6^2+2^2}\sqrt{6^2+2^2}\sin 120°$	M1	2.1
	Area DEF = $\frac{1}{4}ABC$ or $\frac{3}{4}AOB$	**d**M1	3.1a
	$=\frac{3}{8}\times 40\times\frac{\sqrt{3}}{2}=\frac{15\sqrt{3}}{2}$	A1	1.1b
		(3)	

(b) Way 2	$D\left(\frac{3-\sqrt{3}}{2},\frac{3\sqrt{3}+1}{2}\right)$ $OD=\sqrt{\left(\frac{3-\sqrt{3}}{2}\right)^2+\left(\frac{3\sqrt{3}+1}{2}\right)^2}=\sqrt{10}$ Area DOF = $\frac{1}{2}\sqrt{10}\sqrt{10}\sin 120°$	M1	2.1
	Area DEF = $3DOF$	**d**M1	3.1a
	$=3\times\frac{1}{2}\times\sqrt{10}\sqrt{10}\times\frac{\sqrt{3}}{2}=\frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 3	$AB=\sqrt{\left(9+\sqrt{3}\right)^2+\left(3-3\sqrt{3}\right)^2}=\sqrt{120}$ Area ABC = $\frac{1}{2}\sqrt{120}\sqrt{120}\sin 60°\left(=30\sqrt{3}\right)$	M1	2.1
	Area DEF = $\frac{1}{4}ABC$	**d**M1	3.1a
	$=\frac{1}{4}\times 30\sqrt{3}=\frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 4	$D\left(\frac{3-\sqrt{3}}{2},\frac{3\sqrt{3}+1}{2}\right),\ E(-3,-1),\ F\left(\frac{3+\sqrt{3}}{2},\frac{-3\sqrt{3}+1}{2}\right)$ $DE=\sqrt{\left(\frac{3-\sqrt{3}}{2}+3\right)^2+\left(\frac{3\sqrt{3}+1}{2}+1\right)^2}\ \left(=\sqrt{30}\right)$ Area DEF = $\frac{1}{2}\sqrt{30}\sqrt{30}\sin 60°$	M1 **d**M1	2.1 3.1a
	$=\frac{15\sqrt{3}}{2}$	A1	1.1b
(b) Way 5	Area ABC = $\frac{1}{2}\begin{vmatrix} 6 & -3-\sqrt{3} & \sqrt{3}-3 & 6 \\ 2 & 3\sqrt{3}-1 & -3\sqrt{3}-1 & 2 \end{vmatrix}=30\sqrt{3}$	M1	2.1
	Area DEF = $\frac{1}{4}ABC$	**d**M1	3.1a
	$=\frac{1}{4}\times 30\sqrt{3}=\frac{15\sqrt{3}}{2}$	A1	1.1b

(9 marks)

Notes

(a)
M1: Identifies a suitable method to rotate the given point by 120° (or equivalent) about the origin. May see equivalent work with modulus/argument or exponential form e.g. an attempt to multiply by $\cos\frac{2\pi}{3}+\mathrm{i}\sin\frac{2\pi}{3}$ or $\mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}$
A1: Correct real part or correct imaginary part
A1: Completely correct complex number
M1: Identifies a suitable method to rotate the given point by 240° (or equivalent e.g. rotate their *B* by 120°) about the origin

May see equivalent work with modulus/argument or exponential form e.g. an attempt to multiply $6 + 2\text{i}$ by $\cos\frac{4\pi}{3} + \text{i}\sin\frac{4\pi}{3}$ or $\text{e}^{\frac{4\pi}{3}\text{i}}$ or their *B* by $\cos\frac{2\pi}{3} + \text{i}\sin\frac{2\pi}{3}$ or $\text{e}^{\frac{2\pi}{3}\text{i}}$

A1: Correct real part or correct imaginary part

A1: Completely correct complex number

(b)

In general, the marks in (b) should be awarded as follows:

M1: Attempts to find the area of a relevant triangle

dM1: completes the problem by multiplying by an appropriate factor to find the area of *DEF*

Dependent on the first method mark

A1: Correct exact area

In some cases it may not be possible to distinguish the 2 method marks. In such cases they can be awarded together for a direct method that finds the area of *DEF*

Examples:

Way 1

M1: A correct strategy for the area of a relevant triangle such as *ABC* or *AOB*

dM1: Completes the problem by linking the area of *DEF* correctly with *ABC* or with *AOB*

A1: Correct value

Way 2

M1: A correct strategy for the area of a relevant triangle such as *DOF*

dM1: Completes the problem by linking the area of *DEF* correctly with *DOF*

A1: Correct value

Way 3

M1: A correct strategy for the area of a relevant triangle such as *ABC*

dM1: Completes the problem by linking the area of *DEF* correctly with *ABC*

A1: Correct value

Way 4

M1**d**M1: A correct strategy for the area of *DEF*. Finds 2 midpoints and attempts one side of *DEF* and uses a correct triangle area formula. By implication this scores both M marks.

A1: Correct value

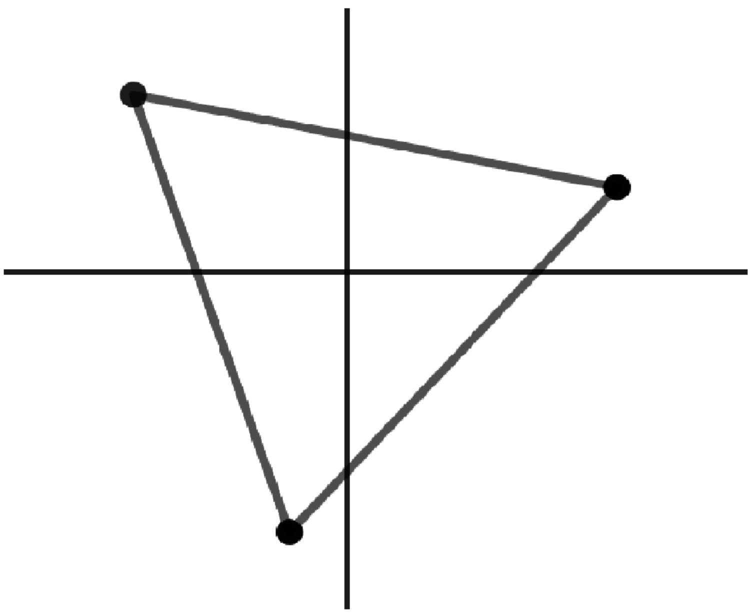
Way 5

M1: A correct strategy for the area of *ABC* using the "shoelace" method.

dM1: Completes the problem by linking the area of *DEF* correctly with *ABC*

A1: Correct value

Note the marks in (b) can be scored using inexact answers from (a) and the A1 scored if an exact area is obtained.



2. In an Argand diagram, the points A and B are represented by the complex numbers $-3 + 2\text{i}$ and $5 - 4\text{i}$ respectively. The points A and B are the end points of a diameter of a circle C.

(a) Find the equation of C, giving your answer in the form

$$|z - a| = b \qquad a \in \mathbb{C},\ b \in \mathbb{R}$$

(3)

The circle D, with equation $|z - 2 - 3\text{i}| = 2$, intersects C at the points representing the complex numbers z_1 and z_2

(b) Find the complex numbers z_1 and z_2

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
2(a)	Centre of circle C is (1, –1)	B1	1.1b
	$r=\sqrt{(5-1)^2+(-4+1)^2}=5$ or $r=\sqrt{(-3-1)^2+(2+1)^2}=5$ or $r=\frac{1}{2}\sqrt{(-3-5)^2+(2+4)^2}=5$	M1	3.1a
	$\lvert z-1+\mathrm{i}\rvert=5$ or $\lvert z-(1-\mathrm{i})\rvert=5$	A1	2.5
		(3)	
(b)	$(x-1)^2+(y+1)^2=25,\quad (x-2)^2+(y-3)^2=4$ $x^2-2x+1+y^2+2y+1=25$ $x^2-4x+4+y^2-6y+9=4$ $\Rightarrow 2x+8y=32$	M1	3.1a
	$(16-4y)^2-4(16-4y)+4+y^2-6y+9=4$ or $x^2-4x+4+\left(\frac{16-x}{4}\right)^2-6\left(\frac{16-x}{4}\right)+9=4$	M1	1.1b
	$17y^2-118y+201=0$ or $17x^2-72x+16=0$	A1	1.1b
	$17y^2-118y+201=0\Rightarrow(17y-67)(y-3)=0\Rightarrow y=\frac{67}{17},3$ or $17x^2-72x+16=0\Rightarrow(17x-4)(x-4)=0\Rightarrow x=\frac{4}{17},4$	M1	1.1b
	$y=\frac{67}{17},3\Rightarrow x=\frac{4}{17},4$ or $x=\frac{4}{17},4\Rightarrow y=\frac{67}{17},3$	M1	2.1
	$4+3\mathrm{i},\ \frac{4}{17}+\frac{67}{17}\mathrm{i}$	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)
B1: Correct coordinates of centre
M1: Fully correct strategy for identifying the radius. If the diameter is calculated this must be halved to achieve this mark.
A1: Correct equation using the required notation
(b)
M1: Begins the process of finding z_1 and z_2 by using the Cartesian equations to obtain the equation of the line of intersection
M1: Substitutes back into the equation of one of the circles to obtain an equation in one variable
A1: Correct 3 term quadratic
M1: Solves their 3TQ
M1: Substitutes to find values of the other variable to complete the process of finding z_1 and z_2
A1: Correct complex numbers

1. Given that

$$z_1 = 3\left(\cos\left(\frac{\pi}{3}\right) + \mathrm{i}\sin\left(\frac{\pi}{3}\right)\right)$$

$$z_2 = \sqrt{2}\left(\cos\left(\frac{\pi}{12}\right) - \mathrm{i}\sin\left(\frac{\pi}{12}\right)\right)$$

(a) write down the exact value of

(i) $|z_1 z_2|$

(ii) $\arg(z_1 z_2)$

(2)

Given that $w = z_1 z_2$ and that $\arg(w^n) = 0$, where $n \in \mathbb{Z}^+$

(b) determine

(i) the smallest positive value of n

(ii) the corresponding value of $|w^n|$

(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 6 6 7 9 7 A 0 2 3 2

Question	Scheme	Marks	AOs
1(a) (i)	$\lvert z_1 z_2 \rvert = 3\sqrt{2}$	B1	1.1b
(ii)	$\arg(z_1 z_2) = \frac{\pi}{3} + \left(-\frac{\pi}{12}\right) = \frac{\pi}{4}$ o.e.	B1	1.1b
		(2)	
(b) (i)	$n = 8$	B1ft	2.2a
(ii)	$\lvert w^n \rvert = (\text{'their} \lvert z_1 z_2 \rvert\text{'})^{\text{their } n}$	M1	1.1b
	$\lvert w^n \rvert = 104\,976$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a)

(i)

B1: Deduces $\lvert z_1 z_2 \rvert = 3\sqrt{2}$

(ii)

B1: Deduces $\arg(z_1 z_2) = \frac{\pi}{4}$ o.e

These marks may be awarded for $z_1 z_2 = 3\sqrt{2}\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right)$

(b)

(i)

B1ft: 2π divided by their $\arg(z_1 z_2)$ found in part (a) (ii) to give an integer

Alternatively smallest positive integer multiple required to make their argument a multiple of 2π

(ii)

M1: Their answer to (a) (i) to the power of their *n*.

A1: 104 976

3. (a) Shade on an Argand diagram the set of points

$$\{z \in \mathbb{C} : |z - 1 - \mathrm{i}| \leqslant 3\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leqslant \arg(z - 2) \leqslant \frac{3\pi}{4}\right\}$$

(5)

The complex number w satisfies

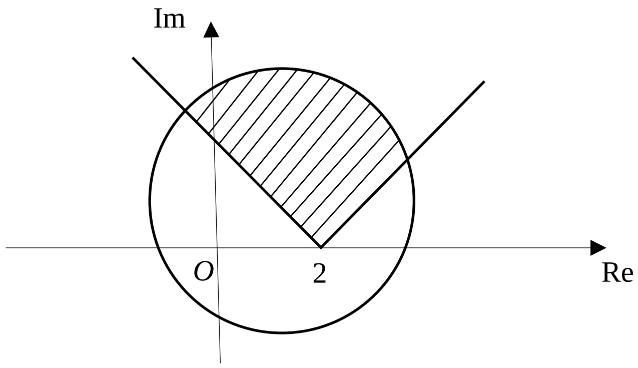
$$|w - 1 - \mathrm{i}| = 3 \text{ and } \arg(w - 2) = \frac{\pi}{4}$$

(b) Find, in simplest form, the exact value of $|w|^2$

(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 5 8 3 0 2 A 0 8 3 6

Question	Scheme	Marks	AOs
3(a)		M1 M1 A1 M1 A1	1.1b 1.1b 2.2a 3.1a 1.1b
		(5)	
(b)	$(x-1)^2+(y-1)^2=9,\ y=x-2 \Rightarrow x=...,\text{ or } y=...$	M1	3.1a
	$x=2+\frac{\sqrt{14}}{2}, y=\frac{\sqrt{14}}{2}$	A1	1.1b
	$\lvert w\rvert^2=\left(2+\frac{\sqrt{14}}{2}\right)^2+\left(\frac{\sqrt{14}}{2}\right)^2$	M1	1.1b
	$=11+2\sqrt{14}$	A1	1.1b
		(4)	
			(9 marks)

Notes

(a)

M1: Circle or arc of a circle with centre in first quadrant and with the circle in all 4 quadrants or arc of circle in quadrants 1 and 2

M1: A "V" shape i.e. with both branches above the *x*-axis and with the vertex on the positive real axis. Ignore any branches below the *x*-axis.

A1: Two half lines that meet on the positive real axis where the right branch intersects the circle or arc of a circle in the first quadrant and the left branch intersects the circle or arc of a circle in the second quadrant but not on the *y*-axis.

M1: Shades the region between the half-lines and within the circle

A1: Cso. A fully correct diagram including 2 marked (or implied by ticks) at the vertex on the real axis with the correct region shaded and all the previous marks scored.

(b)

M1: Identifies a suitable strategy for finding the *x* or *y* coordinate of the point of intersection.

Look for an attempt to solve equations of the form $(x\pm1)^2+(y\pm1)^2=9\text{ or }3$ and $y=\pm x\pm2$

A1: Correct coordinates for the intersection (there may be other points but allow this mark if the correct coordinates are seen). (The correct coordinates may be implied by subsequent work.)

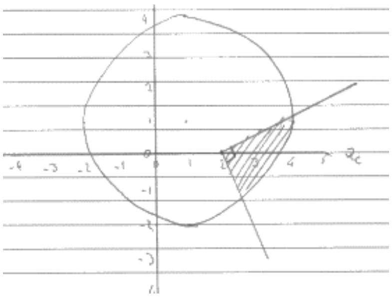
Allow equivalent exact forms and allow as a complex number e.g. $2+\frac{\sqrt{14}}{2}+\frac{\sqrt{14}}{2}\text{i}$

M1: Correct use of Pythagoras on their coordinates (There must be no i's)

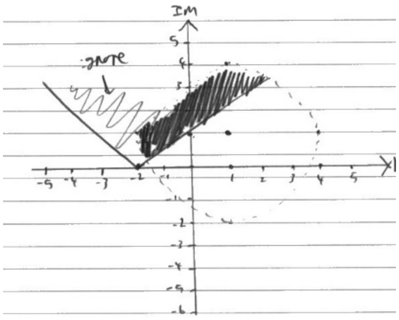
A1: Correct **exact** value by cso

Note that solving $(x-1)^2+(y-1)^2=9,\ y=x+2$ gives $x=\frac{\sqrt{14}}{2}, y=2+\frac{\sqrt{14}}{2}$ and hence the correct answer fortuitously so scores M1A0M1A0

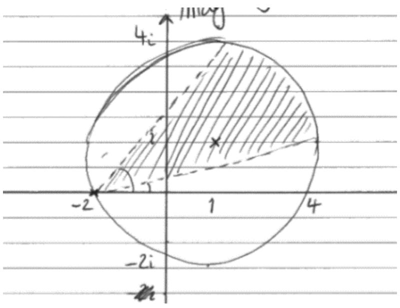
Example marking for 3(a)



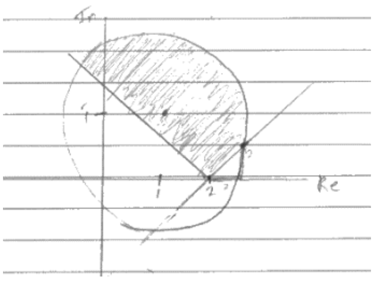
M1: Circle with centre in first quadrant
M0: The branches of the "V" must be above the x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle
A0: Depends on all previous marks



M1: Circle with centre in first quadrant
M0: The vertex of the "V" must be on the positive x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle (BOD)
A0: Depends on all previous marks



M1: Circle with centre in first quadrant
M0: The vertex of the "V" must be on the positive x-axis
A0: Follows M0
M1: Shades the region between the half-lines and within the circle
A0: Depends on all previous marks



M1: Circle with centre in first quadrant
M1: A "V" shape i.e. with both branches above the x-axis and with the vertex on the positive real axis. Ignore any branches below the x-axis.
A1: Two half lines that meet on the positive real axis where the right branch intersects the circle in the first quadrant and the left branch intersects the circle in the second quadrant.
M1: Shades the region between the half-lines and within the circle
A1: A fully correct diagram including 2 marked at the vertex on the real axis with the correct region shaded and all the previous marks scored.

2. Given that

$$z_1 = 2 + 3\text{i}$$
$$|z_1 z_2| = 39\sqrt{2}$$
$$\arg(z_1 z_2) = \frac{\pi}{4}$$

where z_1 and z_2 are complex numbers,

(a) write z_1 in the form $r(\cos\theta + \text{i}\sin\theta)$

Give the exact value of r and give the value of θ in radians to 4 significant figures.

(2)

(b) Find z_2 giving your answer in the form $a + \text{i}b$ where a and b are integers.

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
2(a)	$\lvert z_1 \rvert = \sqrt{13}$ and $\arg z_1 = \tan^{-1}\left(\frac{3}{2}\right)$	B1	1.1b
	$z_1 = \sqrt{13}\left(\cos 0.9828 + \mathrm{i}\sin 0.9828\right)$	B1ft	1.1b
		(2)	
(b)	A complete method to find the modulus of z_2 e.g. $\lvert z_1 \rvert = \sqrt{13}$ and uses $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \times \lvert z_2 \rvert = 39\sqrt{2} \Rightarrow \lvert z_2 \rvert = 3\sqrt{26}$ or $\sqrt{234}$	M1 A1	3.1a 1.1b
	A complete method to find the argument of z_2 e.g. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) = \frac{\pi}{4} \Rightarrow \arg(z_2) = ...$ $\arg(z_2) = \frac{\pi}{4} - \tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4} - 0.9828$ or $-0.1974...$	M1 A1	3.1a 1.1b
	$z_2 = 3\sqrt{26}\left(\cos('-0.1974...') + \mathrm{i}\sin('-0.1974...')\right)$ or $z_2 = a + b\mathrm{i} \Rightarrow a^2 + b^2 = 234$ and $\tan^{-1}(-0.1974) = \frac{b}{a} \Rightarrow \frac{b}{a} = -0.2$ $\Rightarrow a = ...$ and $b = ...$	ddM1	1.1b
	Deduces that $z_2 = 15 - 3\mathrm{i}$ only	A1	2.2a
	Alternative $z_1 z_2 = (a + b\mathrm{i})(2 + 3\mathrm{i}) = (2a - 3b) + (3a + 2b)\mathrm{i}$ $(2a - 3b)^2 + (3a + 2b)^2 = \left(39\sqrt{2}\right)^2$ or 3042 $\Rightarrow a^2 + b^2 = 234$ or $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \times \lvert z_2 \rvert = 39\sqrt{2} \Rightarrow \lvert z_2 \rvert = 3\sqrt{26}$ or $\sqrt{234}$ $\Rightarrow a^2 + b^2 = 234$	M1 A1	3.1a 1.1b
	$\arg\left[(2a - 3b) + (3a + 2b)\mathrm{i}\right] = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{3a + 2b}{2a - 3b}\right) = \frac{\pi}{4} \Rightarrow \frac{3a + 2b}{2a - 3b} = 1$ $\Rightarrow a = -5b$	M1 A1	3.1a 1.1b
	Solves $a = -5b$ and $a^2 + b^2 = 234$ to find values for a and b	ddM1	1.1b
	Deduces that $z_2 = 15 - 3\mathrm{i}$ only	A1	2.2a
		(6)	
			(8 marks)

Notes:
(a) **B1:** Correct exact value for $\lvert z_1\rvert=\sqrt{13}$ and $\arg z_1=\tan^{-1}\left(\frac{3}{2}\right)$. The value for $\arg z_1$ can be implied by sight of awrt 0.98 or awrt 56.3° **B1ft**: Follow through on $r=\lvert z_1\rvert$ and $\theta=\arg z_1$ and writes $z_1=r(\cos\theta+\mathrm{i}\sin\theta)$ where r is exact and θ is correct to 4 s.f. do not follow through on rounding errors.
(b) **M1:** A complete method to find the modulus of z_2 **A1:** $\lvert z_2\rvert=3\sqrt{26}$ **M1:** A complete method to find the argument of z_2 **A1:** $\arg(z_2)=\frac{\pi}{4}-\tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4}-0.9828$ or $-0.1974...$ **ddM1:** Writes z_2 in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, dependent on both previous M marks. Alternative forms two equations involving *a* and *b* using the modulus and argument of z_2 and solve to find values for *a* and *b* **A1**: Deduces that $z_2=15-3\mathrm{i}$ only
(b) Alternative: $z_1z_2=(a+b\mathrm{i})(2+3\mathrm{i})=(2a-3b)+(3a+2b)\mathrm{i}$ **M1:** A complete method to find an equation involving *a* and *b* using the modulus **A1:** Correct simplified equation $a^2+b^2=234$ o.e. **M1:** A complete method to find an equation involving *a* and *b* using the argument. Note $\tan^{-1}\left(\frac{2a-3b}{3a+2b}\right)=\frac{\pi}{4}$ this would score **M0 A0 ddM0 A0** **A1:** Correct simplified equation $a=-5b$ o.e. **ddM1:** Dependent on both the previous method marks. Solves their equations to find values for *a* and *b* **A1**: Deduces that $z_2=15-3\mathrm{i}$ only

10. Given that there are two distinct complex numbers z that satisfy

$$\left\{z : |z - 3 - 5\mathrm{i}| = 2r\right\} \cap \left\{z : \arg(z - 2) = \frac{3\pi}{4}\right\}$$

determine the exact range of values for the real constant r.

(7)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
10	$(x-3)^2+(y-5)^2=(2r)^2$ and $y=-x+2$	B1	1.1b
	$(x-3)^2+(-x+2-5)^2=(2r)^2$ or $(-y+2-3)^2+(y-5)^2=(2r)^2$	M1	3.1a
	$2x^2+18-4r^2=0$ or $2y^2-8y+26-4r^2=0$	A1	1.1b
	$b^2-4ac>0 \Rightarrow 0^2-4(2)(18-4r^2)>0 \Rightarrow r>...$ or $x^2=9-2r^2 \Rightarrow 9-2r^2>0 \Rightarrow r>...$ or $b^2-4ac>0 \Rightarrow (-8)^2-4(2)(26-4r^2)>0 \Rightarrow r>...$	dM1	3.1a
	Finds a maximum value for r $(2r)^2=5^2+(3-2)^2 \Rightarrow r=...$	M1	3.1a
	$\frac{3\sqrt{2}}{2}<r<\frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
	Alternative Using a circle with centre (3, 5) and radius $2r$ and $y=-x+2$	B1	1.1b
	$y-5=1(x-3) \Rightarrow y=x+2$ $x+2=-x+2 \Rightarrow x=...$	M1	3.1a
	$(0,2)$	A1	1.1b
	$2r>\sqrt{(3-0)^2+(5-2)^2} \Rightarrow r>...$	dM1	3.1a
	Finds a maximum value for r $(2r)^2=5^2+(3-2)^2 \Rightarrow r=...$	M1	3.1a
	$\frac{3\sqrt{2}}{2}<r<\frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
		(7)	
			(7 marks)

Notes:

B1: Correct equations for each loci of points
M1: A complete method to find a 3TQ involving one variable using equations of the form
$(x\pm3)^2+(y\pm5)^2=(2r)^2$ or $2r^2$ or r^2 and $y=\pm x\pm2$
A1: Correct quadratic equation

dM1: Dependent on previous method mark. A complete method uses $b^2 - 4ac > 0$ or rearranges to find $x^2 = \mathrm{f}(r)$ and uses $\mathrm{f}(r) > 0$ to the minimum value of *r*. **M1:** Realises there will be an upper limit for *r* and uses Pythagoras theorem $(2r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$ condone $(r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$ **A1:** One correct limit, either $\frac{3\sqrt{2}}{2} < r$ or $r < \frac{\sqrt{26}}{2}$ o.e. **A1:** Fully correct inequality
Alternative **B1:** Using a circle with centre (3, 5) and radius 2*r* and $y = -x + 2$ **M1:** A complete method to find the point of intersection of the line $y = \pm x \pm 2$ and circle where the line is a tangent to the circle. **A1:** Correct point of intersection **dM1:** Finds the distance between the point of intersection and the centre and uses this to find the minimum value of *r*. Condone radius of r. **M1:** Realises there will be an upper limit for *r* and uses Pythagoras theorem $(2r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$ **A1:** One correct limit, either $\frac{3\sqrt{2}}{2} < r$ or $r < \frac{\sqrt{26}}{2}$ o.e. **A1:** Fully correct inequality

5.

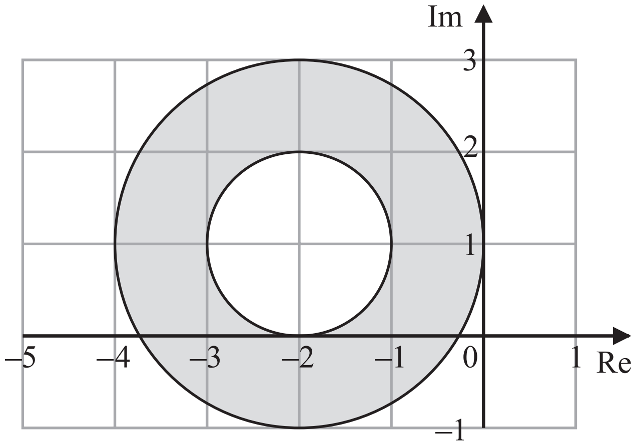


Figure 1

Figure 1 shows an Argand diagram.

The set P, of points that lie within the shaded region including its boundaries, is defined by

$$P = \{z \in \mathbb{C} : a \leqslant |z + b + c\text{i}| \leqslant d\}$$

where a, b, c and d are integers.

(a) Write down the values of a, b, c and d.

(3)

The set Q is defined by

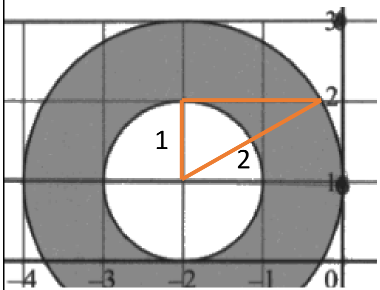
$$Q = \{z \in \mathbb{C} : a \leqslant |z + b + c\text{i}| \leqslant d\} \cap \{z \in \mathbb{C} : |z - \text{i}| \leqslant |z - 3\text{i}|\}$$

(b) Determine the exact area of the region defined by Q, giving your answer in simplest form.

(7)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 6 6 7 9 0 A 0 1 2 3 2

Question	Scheme	Marks	AOs
5(a)	$a = 1, d = 2$	B1	1.1b
	$b = 2$	B1	1.1b
	$c = -1$	B1	1.1b
		(3)	
(b)	$\lvert z - \mathrm{i}\rvert = \lvert z - 3\mathrm{i}\rvert \Rightarrow y = 2$	B1	2.2a
	Area between the circles $= \pi \times 2^2 - \pi \times 1^2$	M1	1.1a
	 Angle subtended at centre = $2 \times \cos^{-1}\left(\frac{1}{2}\right)$ Alternatively $(x+2)^2 + (y-1)^2 = 4,\ \ y = 2 \Rightarrow x = ...$ Or $x = \sqrt{2^2 - 1^2}$ Leading to Angle subtended at centre = $2 \times \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$	M1	3.1a
	Segment area = $\frac{1}{3} \times \pi \times 2^2 - \frac{1}{2} \times 2^2 \times \sin\left(\text{"}\frac{2\pi}{3}\text{"}\right) \left\{ = \frac{4}{3}\pi - \sqrt{3} \right\}$	M1 A1	2.1 1.1b
	Area of Q: $\pi \times 2^2 - \pi \times 1^2 - \left(\frac{1}{3} \times \pi \times 2^2 - \frac{1}{2} \times 2^2 \times \sin\left(\frac{2\pi}{3}\right)\right)$	M1	3.1a
	$= \frac{5\pi}{3} + \sqrt{3}$	A1	1.1b
		(7)	
		(10 marks)	

Notes

(a)
B1: Correct values for *a* and *d*
B1: Correct value for *b*
B1: Correct value for *c*
(b)
B1: Deduces that $\lvert z - \mathrm{i}\rvert = \lvert z - 3\mathrm{i}\rvert$ is a perpendicular bisector with equation $y = 2$, this may be drawn on a diagram.
M1: Selects the correct procedure to find the area of the large circle – the area of the small circle.
M1: Correct method to find the angle at the centre (or half this angle).
Recognises that the hypotenuse is the radius of the larger circle and the adjacent is the radius if the smaller circle and using cosine
Alternatively find where the perpendicular bisector intersects the larger circle so uses their $y = 2$ and the equation of the larger circle in an attempt to establish the x values for the intersection points or uses geometry and Pythagoras to identify the required length and then uses tangent.
M1: Correct method for the area of the minor segment (allow equivalent work)

A1: Correct expression
M1: Fully correct strategy for the required area. Must be subtracting the area of the minor segment from the annulus area.
A1: Correct exact answer
Note: 6.968

2. (a) Express the complex number $w = 4\sqrt{3} - 4\text{i}$ in the form $r(\cos\theta + \text{i}\sin\theta)$ where $r > 0$ and $-\pi < \theta \leqslant \pi$

(4)

(b) Show, on a single Argand diagram,

(i) the point representing w

(ii) the locus of points defined by $\arg(z + 10\text{i}) = \frac{\pi}{3}$

(3)

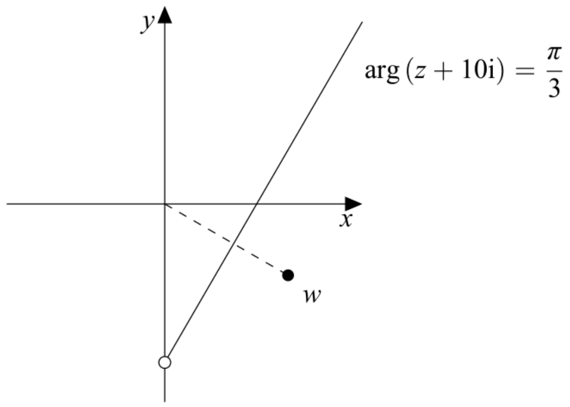
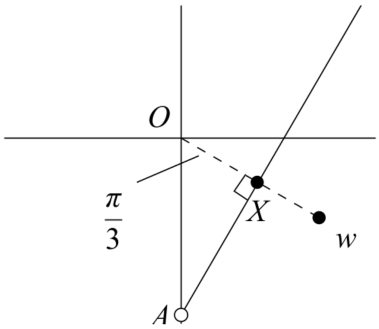
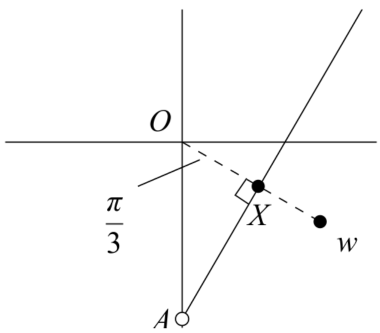
(c) Hence determine the minimum distance of w from the locus $\arg(z + 10\text{i}) = \frac{\pi}{3}$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question	Scheme	Marks	AOs
2(a)	$\lvert w\rvert=\sqrt{\left(4\sqrt{3}\right)^2+\left(-4\right)^2}=8$	B1	1.1b
	$\arg w=\arctan\left(\dfrac{\pm4}{4\sqrt{3}}\right)=\arctan\left(\pm\dfrac{1}{\sqrt{3}}\right)$	M1	1.1b
	$=-\dfrac{\pi}{6}$	A1	1.1b
	So $(w=)8\left(\cos\left(-\dfrac{\pi}{6}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{6}\right)\right)$	A1	1.1b
		(4)	
(b)	 (i) w in 4th quadrant with either $\left(4\sqrt{3},-4\right)$ seen or $-\dfrac{\pi}{4}<\arg w<0$	B1	1.1b
	(ii) half line with positive gradient emanating from imaginary axis.	M1	1.1b
	The half line should pass between O and w starting from a point on the imaginary axis below w	A1	1.1b
		(3)	
(c)	 ΔOAX is right angled at X so $OX=10\sin\dfrac{\pi}{6}=5$ (oe)	M1	3.1a
	So shortest distance is $WX=OW-OX=$ '8' $-5=\ldots$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 1  A complete method to find the coordinates of X. Finds the equation of the line from O to w, $y=-\dfrac{1}{\sqrt{3}}x$ and the equation of the half line $y=\sqrt{3}x-10$, solves to find the point of intersection $X\left(\dfrac{5\sqrt{3}}{2},-\dfrac{5}{2}\right)$	M1	3.1a
	Finds the length WX $\sqrt{\left(4\sqrt{3}-\dfrac{5\sqrt{3}}{2}\right)^2+\left(-4--\dfrac{5}{2}\right)^2}$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 2	M1	3.1a

	Finds the length $AW = \sqrt{\left(4\sqrt{3}-0\right)^2+\left(-4--10\right)^2} = ...\left\{\sqrt{84}\right\}$ Finds the angle between the horizontal and the line AW $= \tan^{-1}\left(\frac{-4--10}{4\sqrt{3}}\right) = ...\left\{0.7137...\text{radians or } 40.89...^\circ\right\}$		
	Finds the length of $WX = \sqrt{84}\times\sin\left(\frac{\pi}{3}-0.7137\right) = ...$ Or $= \sqrt{84}\times\sin\left(60-40.89\right) = ...$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 3 Vector equation of the half line $r = \begin{pmatrix}0\\-10\end{pmatrix}+\lambda\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}$ $XW = \begin{pmatrix}4\sqrt{3}-\lambda\\-4-\lambda\sqrt{3}-(-10)\end{pmatrix}$ Then either $\begin{pmatrix}4\sqrt{3}-\lambda\\6-\lambda\sqrt{3}\end{pmatrix}\bullet\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = 4\sqrt{3}-\lambda+6\sqrt{3}-3\lambda = 0 \Rightarrow \lambda = ...\left\{\frac{5}{2}\sqrt{3}\right\}$ $r = \begin{pmatrix}0\\-10\end{pmatrix}+\frac{5}{2}\sqrt{3}\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} = ...$ Or $XW^2 = \left(4\sqrt{3}-\lambda\right)^2+\left(6-\lambda\sqrt{3}\right)^2 = 48-8\lambda\sqrt{3}+\lambda^2+36-12\lambda\sqrt{3}+3\lambda^2$ $xw^2 = 84-20\lambda\sqrt{3}+4\lambda^2$ leading to $\frac{d\left(XW^2\right)}{d\lambda} = -20\sqrt{3}+8\lambda = 0 \Rightarrow \lambda = ...$	M1	3.1a
	Finds the length WX $\sqrt{\left(4\sqrt{3}-\frac{5\sqrt{3}}{2}\right)^2+\left(-4--\frac{5}{2}\right)^2}$ Or $XW = \sqrt{\left(4\sqrt{3}-'\frac{5}{2}\sqrt{3}'\right)^2+\left(6-'\frac{5}{2}\sqrt{3}'\sqrt{3}\right)^2}$	M1	1.1b
	So min distance is 3	A1	1.1b
		(3)	
			(10 marks)

Notes:

(a)

B1: Correct modulus

M1: Attempts the argument. Allow for $\arctan\left(\frac{\pm4}{\pm4\sqrt{3}}\right)$ or equivalents using the modulus (may be in wrong quadrant for this mark).

A1: Correct argument $-\frac{\pi}{6}$ (must be in fourth quadrant but accept $\frac{11\pi}{6}$ or other difference of 2π for this mark).

A1: Correct expression found for w, in the correct form, must have positive r= 8 and $\theta = -\frac{\pi}{6}$.

Note: using degrees B1 M1 A0 A0

(b)(i)&(ii)

B1: w plotted in correct quadrant with either the correct coordinate clearly seen or above the line $y = -x$

M1: Half line drawn starting on the imaginary axis away from O with positive gradient (need not be labelled)

A1: Sketch on **one diagram**– both previous marks must have been scored and the half line should pass between O and w starting from a point on the imaginary axis below w. (You may assume it starts at −10i unless otherwise stated by the candidate)

Note: If candidates draw the loci on separate diagrams the maximum they can score is B1 M1 A0

(c)

M1: Formulates a correct strategy to find the shortest distance, e.g. uses right angle OXA where X is where the lines meet and proceeds at least as far as OX.

M1: Full method to achieve the shortest distance, e.g. for $WX = OW - OX$.

A1: cao shortest distance is 3

Alternative 1:

M1: Uses a correct method to find the equation of the line from O to w, $y = -\frac{1}{\sqrt{3}}x$ and the equation of the half line $y = \sqrt{3}x - 10$, solves to find the point of intersection $X\left(\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$

If the incorrect gradient(s) is used with no valid method seen this is M0

M1: Finds the length $WX = \sqrt{\left(\text{their}\frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their} -\frac{5}{2} - -4\right)^2} = ...$ condone a sign slip in the brackets.

A1: cao shortest distance is 3

Alternative 2:

M1: Uses a correct method to find the length AW and a correct method to find the angle between the horizontal and the line AW

M1: Finds the length of $WX = \text{their}\sqrt{84} \times \sin\left(\frac{\pi}{3} - \text{their } 0.7137\right) = ...$

A1: cao shortest distance is 3

Alternative 3

M1: Finds the vector equation of the half line, then XW.

Then either: Sets dot product XW and the line = 0 and solves for λ. Substitutes their λ into the equation of the half line to find the point of intersection.

Or finds the length of XW and differentiates, set = 0 and solve for λ

M1: Finds the length $WX = \sqrt{\left(\text{their}\frac{5\sqrt{3}}{2} - 4\sqrt{3}\right)^2 + \left(\text{their} -\frac{5}{2} - -4\right)^2} = ...$ condone a sign slip in the brackets.

Or substitutes their value for λ into the length of *(d)*

A1: cao shortest distance is 3