Carrying the Heavy End

A tale of two settees...

When you're holding the lower end, the sofa does seem to be heavier. Clearly the weight of the sofa (and the direction the weight acts) hasn't changed, and the centre of mass is still halfway between you both. So why is it so much easier for the other guy?

By modelling the sofa as a uniform rod, and the two people as supports at each end of the rod, draw a force diagram illustrating all the forces acting on the rod for the uneven scenario.

Next, by resolving and considering moments, find the overall resultant force exerted by each person, in terms of the sofa's weight and angle.

Carrying the Heavy End - SOLUTIONS

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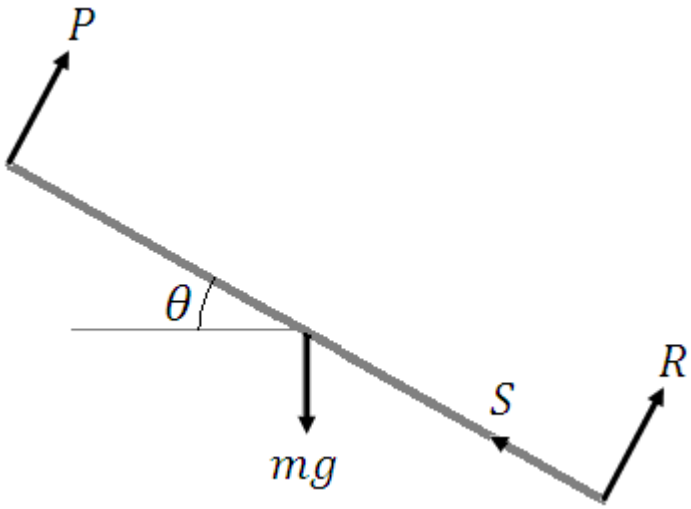

Note: If we assume the only forces provided by the carriers are at right angles to the sofa, we would end up with a resultant sideways force (back towards the person at the lower end). This means that, in practice, this person has to exert not only a reaction force on the sofa, at right angles to it, but also a force along the sofa to stop it moving sideways. The resultant direction of this force will, of course, be somewhere between the two. If the person at the higher end wanted to help, they could pull the sofa towards themselves.

Next, by resolving and considering moments, find the overall resultant force exerted by each person, in terms of the sofa's weight and angle.

Resolving along the rod (1):

$$\boldsymbol{mg\sin\theta = S}$$

Resolving perpendicular to the rod (2):

$$\boldsymbol{R + P = mg\cos\theta}$$

Taking moments about the lower end (taking the length to be $2l$) (3):

$$\boldsymbol{lmg\cos\theta = 2lP} \quad \Longrightarrow \quad \boldsymbol{\frac{1}{2}mg\cos\theta = P}$$

Combining (2) and (3):

$$\boldsymbol{R = \frac{1}{2}mg\cos\theta}$$

Finding the resultant of R and S for various angles, compared to P, gives some insight into the discrepancy:

Angle	Upper end (as a % of weight)	Lower end force (as a % of weight)
$0°$	50%	50%
$15°$	48%	55%
$30°$	43%	66%
$45°$	35%	79%
$60°$	25%	90%
$75°$	13%	97%
$90°$	0%	100%

Notice that the total force exerted is often more than 100% of the weight. The most extreme occurs at $60°$, and at $0°$ or $90°$ the effect disappears. It is, in fact, a consequence of the two people working against one another. The net force of the person at the top isn't purely vertical: it has a horizontal component, too, and the person at the bottom is not only helping hold up the sofa, but is pushing back against this lateral force.