Nerf Energy

A nerf gun has a maximum range of **10m**.

The energy is provided by a spring of natural length **4cm**, compressing down to a length of **2cm**.

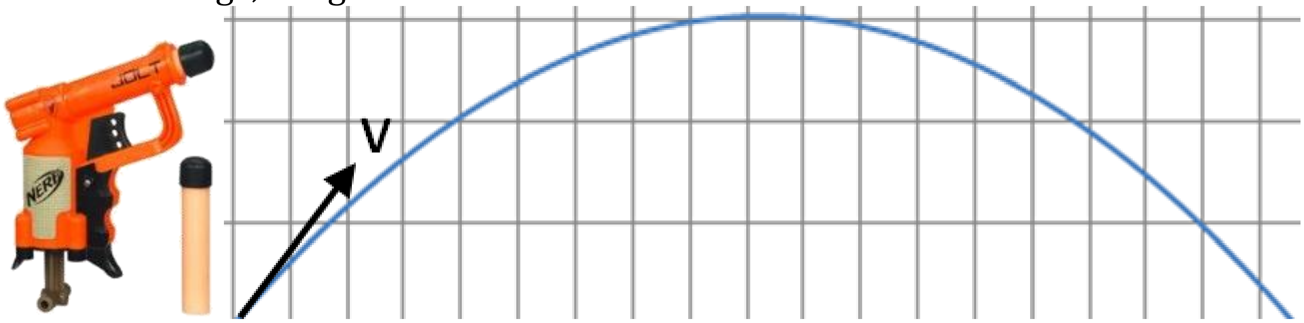
A standard dart weighs **1g**.

The gun can be assumed to be 100% energy efficient.

Using energy considerations, calculate the spring modulus, λ.

Nerf Energy Answers

For maximum range, the gun must be fired at 45° to the horizontal:



Given a range of 10m, we can use the horizontal motion to link the initial velocity and the total flight time:

$$V\cos 45 = \frac{10}{t} \quad \Longrightarrow \quad t = \frac{10}{V\cos 45} = \frac{10\sqrt{2}}{V}$$

Using SUVAT equations on the vertical motion we can find a second link, and solve for V:

$$u = V\sin 45 = \frac{V}{\sqrt{2}} \qquad v = 0 \qquad a = -9.8 \qquad t = \frac{T}{2} = \frac{5\sqrt{2}}{V}$$

$$v = u + at \quad \Longrightarrow \quad 0 = \frac{V}{\sqrt{2}} - 9.8 \times \frac{5\sqrt{2}}{V} = \frac{V}{\sqrt{2}} - \frac{49\sqrt{2}}{V}$$

$$\Longrightarrow \quad 0 = \frac{V^2}{\sqrt{2}} - 49\sqrt{2} \quad \Longrightarrow \quad \frac{V^2}{\sqrt{2}} = 49\sqrt{2} \quad \Longrightarrow \quad V^2 = 98 \quad \Longrightarrow \quad \boldsymbol{V = 7\sqrt{2}}$$

Given the initial speed and the mass of the dart we can find the initial kinetic energy:

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.001)(98) = \boldsymbol{0.049\ Joules}$$

Since this energy is provided entirely by the spring, it must be equal to Elastic Potential:

$$0.049 = EPE = \frac{\lambda e^2}{2l} = \frac{\lambda(0.02^2)}{2(0.04)} = 0.005\lambda \quad \Longrightarrow \quad \lambda = \frac{0.049}{0.005} = \boldsymbol{9.8\ Newtons}$$

*Therefore the modulus of the spring is **10 Newtons** to 1 significant figure.*