Gravity Investigation

The basics

Definition: **Gravity** is a (relatively weak) force of attraction felt between any two objects. The more massive the objects, and the closer together, the greater the force. It is usually measured in Newtons (N).

Definition: **Mass** is the amount of stuff an object is made from, so it doesn't change if we stand on the moon or space-walk. Even in 'weightless' situations, the mass of an object gives it inertia (a measure of the force required to get it moving or slow it down). It is usually measured in kilograms (kg).

Definition: **Weight** is the name given to the force of gravity acting on an object. In everyday language, we often use the word 'weight' when talking about 'mass', but since the only way to change our weight without leaving the planet is to change our mass, it doesn't make much difference. However, technically, weight is a force and is therefore measured in Newtons (N).

On Earth:

Mass: $160kg$

Weight: $1570N$

Most likely to say: "Built-in waste recycling unit: Best. Idea. Ever."

On The Moon:

Mass: $160kg$

Weight: $260N$

Most likely to say: "One small step for man..."

In Deep Space:

Mass: $160kg$

Weight: $0N$

Most likely to say: "How did I get here?"

The investigation

The aim is to develop an understanding of how we experience the force of gravity, and investigate how this would change under different circumstances.

Getting started

The formula for the gravitational force between two objects is:

$$F = \frac{Gm_1m_2}{r^2}$$

Where $G = 6.67384 \times 10^{-11}$ (the Gravitational Constant), m_1 and m_2 are the masses of the two objects (measured in $kilograms$), and r is the distance between them (measured in $metres$).

Use this formula to calculate the gravitational attraction between two astronauts hanging motionless in deep space exactly 2 metres apart.

Another important formula, known as Newton's Second Law, states that:

$$F = ma$$

where F is the overall force acting on an object, m is its mass and a is the resulting acceleration.

The more mass an object has, the greater the force required to achieve the same acceleration.

By combining these two formulae we get: $ma = \frac{GmM}{r^2}$ where m is the mass of the person or object, M the mass of the planet, and r is the radius of the planet (we assume the person is on the surface of the planet).

This gives the result: $a = \frac{GM}{r^2}$ since the mass of the person/object doesn't affect acceleration due to gravity.

Calculate the *acceleration due to gravity* for objects close to Earth's surface. You will need the following:

$$Mass\ of\ Earth = 5.972 \times 10^{24}\ kg \qquad Radius\ of\ Earth = 6.371 \times 10^{6}\ m$$
$$Gravitational\ Constant:\ G = 6.67384 \times 10^{-11}$$

Then complete the table to show the acceleration due to gravity on the surface of our nearest neighbours:

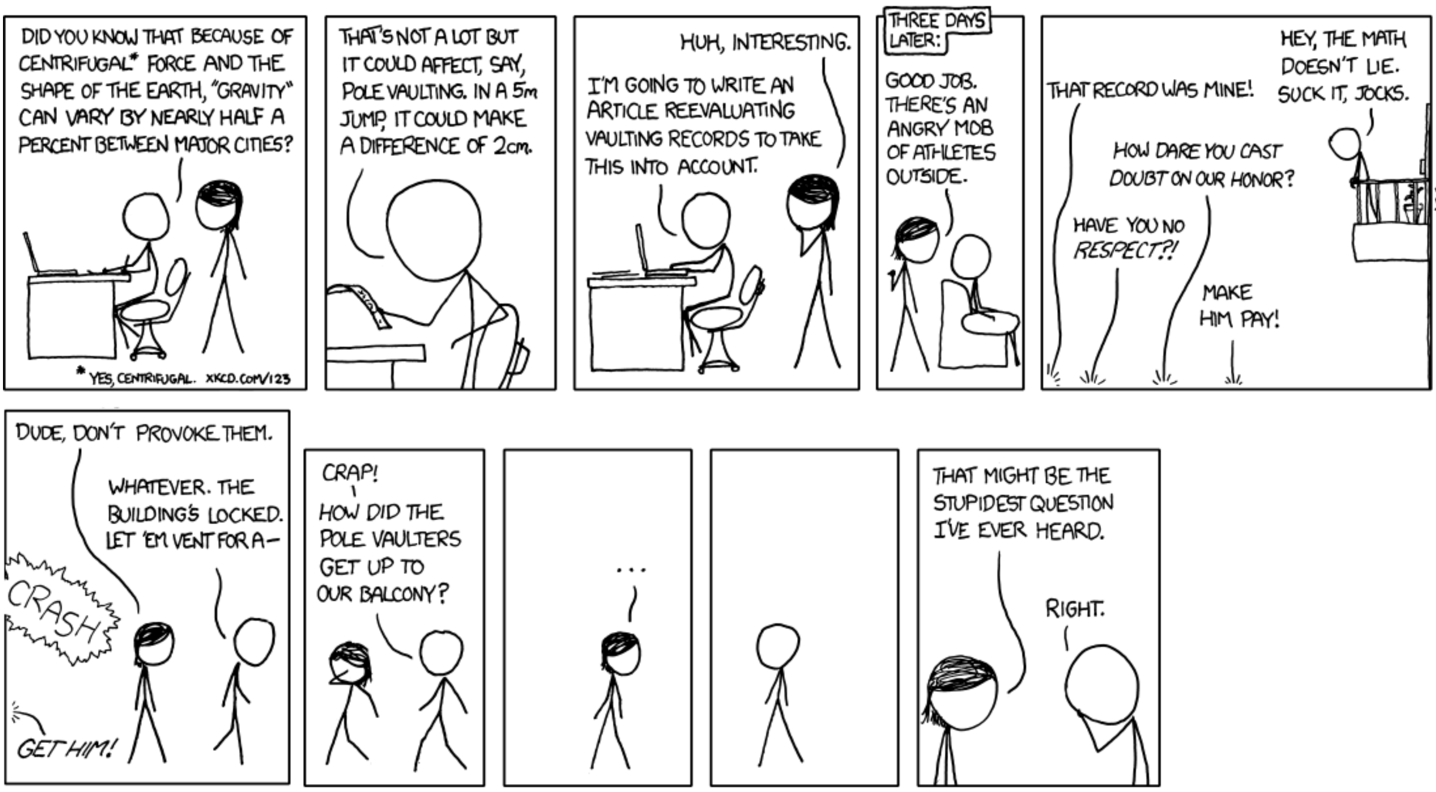
Solar Body	**Radius (m)**	**Mass (kg)**	**Acceleration (ms^{-2})**
Earth	6.371×10^{6}	5.972×10^{24}	
Moon	1.738×10^{6}	7.35×10^{22}	
Mars	3.397×10^{6}	6.42×10^{23}	
Jupiter	7.1492×10^{7}	1.9×10^{27}	
Sun	6.95×10^{8}	1.99×10^{30}	

Challenges

We take gravity for granted so much of the time, but this can lead to some odd misconceptions. See if you can unravel the following apparent contradictions. Discuss, and write down any thoughts you have:

1. "When a lift starts moving upwards, you get heavier, and when it slows down you get lighter." **But your mass doesn't change, and your distance from the centre of the planet has barely changed! So how can your weight be different??**	2. "The astronauts in the International Space Station experience weightlessness, so they can float around in zero gravity." **The ISS orbits at a height of $330km$, which means the distance to the centre of the Earth is $6700km$ compared to $6370km$ on the surface. That gives $8.9ms^{-2}$ instead of $9.8ms^{-2}$. How can they feel weightless??**	3. "The Moon is only a few thousand kilometres from Earth, and it orbits us due to our gravitational pull on it." **The Moon is closer to Earth than it is to the Sun, but the Sun is $300,000$ times more massive than Earth, and it works out to around twice the gravitational attraction. So how come the Moon isn't falling into the sun??**

Once you've discussed these, you might want to watch a musical answer to 3: youtu.be/WuqTHLjhlik



In Rio de Janeiro in 2016, the same jump will get an athlete 0.25% higher (>1cm) than in London four years prior.

XKCD.com

Gravity Investigation SOLUTIONS

The formula for the gravitational force between two objects is:

$$F = \frac{Gm_1m_2}{r^2}$$

Where $G = 6.67384 \times 10^{-11}$ (the Gravitational Constant), m_1 and m_2 are the masses of the two objects (measured in $kilograms$), and r is the distance between them (measured in $metres$).

Use this formula to calculate the gravitational attraction between two astronauts hanging motionless in deep space exactly 2 metres apart.

Taking the mass of each astronaut (including space suit) to be $160kg$ as in the pictures, we have:

$$F = \frac{6.67384 \times 10^{-11} \times 160 \times 160}{2^2} = \mathbf{4.27 \times 10^{-7}\ Newtons}$$

Note: using Newton's Second Law we can find the acceleration:

$$F = ma \quad \Rightarrow \quad 4.27 \times 10^{-7} = 160a \quad \Rightarrow \quad a = 2.67 \times 10^{-9}$$

And using SUVAT equations we can calculate how long it would take the astronauts to meet. (Note that each of them is moving towards the other at the same rate, so they only need to move $1m$ each).

$$s = 1$$
$$u = 0$$
$$v = ?$$
$$a = 2.67 \times 10^{-9}$$
$$t = t$$

$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad 1 = 0 + (2.67 \times 10^{-9})t^2 \quad \Rightarrow \quad t^2 = \frac{1}{2.67 \times 10^{-9}} \quad \Rightarrow \quad t = 19355s \approx \mathbf{5\frac{1}{2}\ hrs}$$

Calculate the *acceleration due to gravity* for objects close to Earth's surface.

$$F = ma \quad and \quad F = \frac{GM_Em}{r^2} \quad \Rightarrow \quad ma = \frac{GM_Em}{r^2} \quad \Rightarrow \quad a = \frac{GM_E}{r^2} = \frac{6.67384 \times 10^{-11} \times 5.972 \times 10^{24}}{6.371 \times 10^6} = \mathbf{9.82ms^{-2}}$$

Solar Body	Radius (m)	Mass (kg)	Acceleration (ms^{-2})
Earth	6.371×10^6	5.972×10^{24}	9.82
Moon	1.738×10^6	7.35×10^{22}	**1.62**
Mars	3.397×10^6	6.42×10^{23}	**3.71**
Jupiter	7.1492×10^7	1.9×10^{27}	**24.81**
Sun	6.95×10^8	1.99×10^{30}	**274.95**

1. "When a lift starts moving upwards, you get heavier, and when it slows down you get lighter."
But your mass doesn't change, and your distance from the centre of the planet has barely changed! So how can your weight be different??

We never directly experience weight, but unless we're in free-fall we will experience a contact force (the force from the ground pushing back against us to counter-act gravity). When we accelerate upwards, that is by a larger than usual contact force, and when we accelerate downwards, it is smaller than usual.

At the highest point of a swing, you're barely in contact with the seat – that's your contact force dropping to zero.

2. "The astronauts in the International Space Station experience weightlessness, so they can float around in zero gravity."
The ISS orbits at a height of $330km$, which means the distance to the centre of the Earth is $6700km$ compared to $6370km$ on the surface. That gives $8.9ms^{-2}$ instead of $9.8ms^{-2}$. How can they feel weightless??

They are technically in freefall (so there's no force other than weight acting on them). Because they're travelling $8km$ every second it's fast enough that they're always falling past the Earth. That's what it means to be in orbit.

Did you know that a geostationary orbit is only possible at a height of around $36,000km$? At that height the orbital speed matches the rotation of the Earth.

3. "The Moon is only a few thousand kilometres from Earth, and it orbits us due to our gravitational pull on it."
The Moon is closer to Earth than it is to the Sun, but the Sun is $300,000$ times more massive than Earth, and it works out to around twice the gravitational attraction. So how come the Moon isn't falling into the sun??

Actually, it is. Just like the ISS, it's constantly falling towards the sun, but going fast enough sideways that it is always falling past it. It is in orbit around the sun just as we are (otherwise we'd leave it behind), it's just that it's also orbiting Earth at the same time.

Seriously, watch the song:
youtu.be/WuqTHLjhIik