Using GeoGebra to discover Circle Theorems

GeoGebra is a completely free program which allows the user to draw geometric and algebraic objects (shapes and graphs), and investigate their properties quickly and easily. It can be downloaded from www.geogebra.org, where you can also use a web-based version of the program, or browse GeoGebra files others have created.

If you've never used GeoGebra before, see Introduction to GeoGebra to learn some of the main features.

The investigation

Because of their special properties, there are many angle rules that apply to lines and shapes connected to circles. Use the prompts in this document to investigate them for yourself.

By creating and modifying shapes, you should come up with **conjectures** (ideas for what the rules might be), and then try to **justify** your conjectures by using the angle rules you already know as well as the properties of circles.

Hint: when constructing a proof, you may find it useful to connect points to the centre of the circle and use the fact that any radius is the same length.

A note on notation

It is often easier to see a pattern than describe it. In geometry, there are certain ways of describing points, lines and angles that will make it easier to clearly explain the results you've noticed:

A, B, etc (usually capital letters starting from A)	A point. This is a corner or the end point of a line segment, not an angle.
AB, BC, etc (two letters)	A line segment. AB means the straight line from point A to point B.
ABC or $A\hat{B}C$ (three letters, sometimes with a ^)	An angle. ABC means the angle at B, between the line segments AB and BC.
ABC or ΔABC (three letters, sometimes with a Δ)	A triangle. ABC means the triangle made by the three line segments AB, BC and CA.

Since the symbols used for angle and triangle can be similar, some people use the alternative notation. However, within context it is usually clear (eg '$ABC = CDA$' must refer to angles, since triangles are not numbers and therefore cannot be 'equal', while 'ABC is isosceles' or 'ABC is similar to CDA' must refer to triangles).

Vocabulary

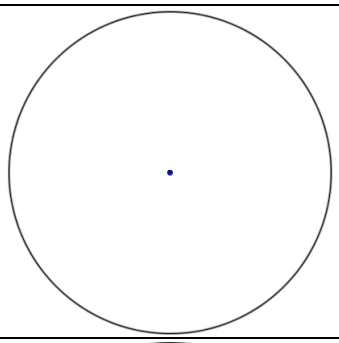
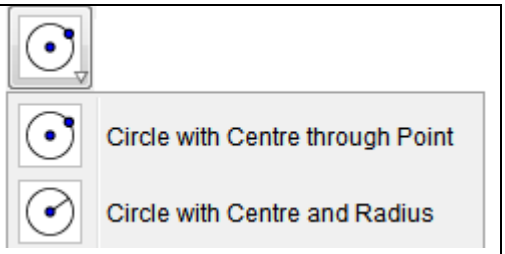
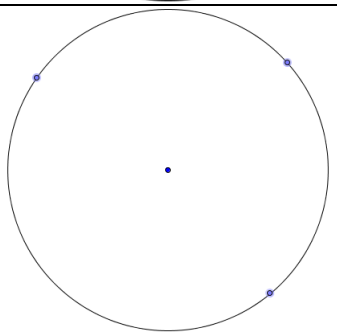
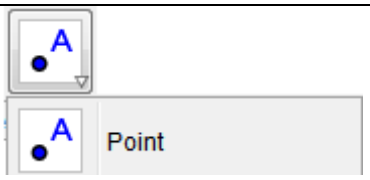
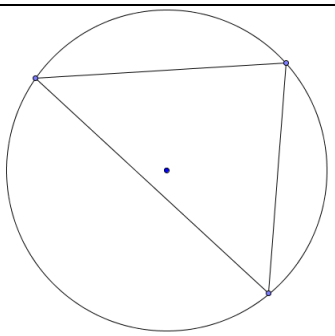
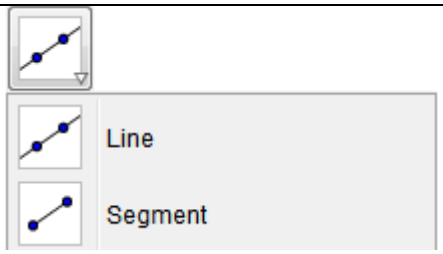
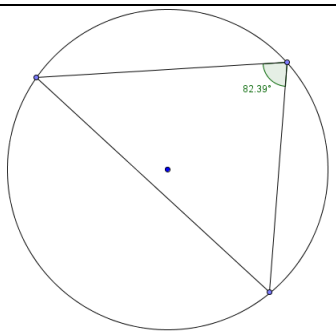
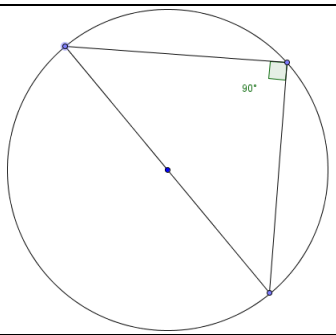
To best describe specific lines or shapes within a circle, the following words should be used:

Centre	The point at the middle of the circle
Circumference	The curved edge of the circle (or the length of this, depending on context)
Diameter	A line segment crossing the circle through the centre (or the length of this)
Radius	A line segment from the centre to anywhere on the circumference
Chord	A line segment from anywhere on the circumference to anywhere else (therefore a diameter is just a chord that has the added property of passing through the centre)
Arc	The curved line made by following the circumference from one point on the circumference to any other
Sector	The shape bounded by an arc and the two radii from each end point of the arc
Segment	The shape bounded by an arc and the chord between the end points of the arc
Tangent	A line which touches the circumference at exactly one point

Since any two radii which form a sector with an arc also form a complementary sector with the rest of the circumference, the larger one (more than half the circle) is sometimes described as the 'major sector', and the smaller one as the 'minor sector'. Similarly, the 'major segment' is one which takes up more than half of the circle.

Turn over for hints and suggestions to get you started

Getting started

1. Construct a circle by selecting one of the 'Circle' tools (pretty much any will do, but 'Circle with Centre and Radius' is probably best since you won't need the size to change).		
2. Add some points to the circumference by selecting the 'Point' tool and clicking on the circumference. They will automatically be constrained to the circumference if you put them there.		
3. Draw some line segments between your points by selecting the 'Segment' tool and clicking on a start and end point.		
4. Measure angles by selecting the 'Angle' tool and clicking on two lines or three points (angles are measured anticlockwise, so if you end up with the opposite angle to what you wanted, use Ctrl+Z to undo, then try selecting the points or lines in the opposite order).		
5. Move some points around. Click the 'Select' tool to allow moving and changing of objects. See what kind of interesting results you spot, write them down, and – if possible – try to work out exactly when they are true and why.		

Suggestions for investigation

Angles in triangles where one corner is at the centre and two are on the circumference, or all three are on the circumference.

Angles in quadrilaterals where one corner is at the centre and three are on the circumference, or all four are on the circumference

The relationship between a radius and a chord

The relationship between a chord and a tangent